Game Theory

Department of Electronics

EL-766

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Course Information

- Part I: Introduction to Game Theory

  Introduction to game theory, games with perfect information, Nash equilibrium and its properties, mixed strategy equilibrium, extensive games, conditional games, coalition games, games with imperfect information, Bayesian games, cooperative and non cooperative games, repeated games, bargaining.

- Part II: Applications
- Application to networks, intelligent decision making, role of information and distributed decisions, pricing of network services, network layers adaptation, cross layer framework, access control, power control, interference avoidance, best reply and better reply dynamics, game theoretic frameworks for cellular networks, cognitive radios, and ad hoc networks.

Course Material

- Textbook:

  Game Theory, D. Fudenberg and J. Tirole, MIT press 1991

- Some additional material covered: handouts

Course Information

- Grading:
    - Homework
    - Quiz
    - Two midterms
    - Class presentations and discussions
    - Term paper
- Attendance
  – Absolute requirement!

What is Game Theory?

- A mathematical formalism for understanding, designing and predicting the outcome of games.
- What is a game?
  - Characterized by a number of players (2 or more), assumed to be intelligent and rational, that interact with each other by selecting various actions, based on their assigned preferences.
- Players: decision makers
  - A set of actions available for each player
  - A set of preference relationships defined for each player for each possible action tuple.
- Usually measured by the utility that a particular user gets from selecting that particular action
  - Intelligent and rational

What is Game Theory?

- Game theory is a branch of applied mathematics.
- Economics, biology, engineering, computer science.
  - political science, international relations, social psychology, philosophy and management.
- Mathematically capture behavior in strategic situations games,
  - in which an individual's success in making choices depends on the choices of others.
- Analyze competitions.
- Wide class of interactions (non human: animals, plants, computers)
- Equilibrium:
  - each player of the game has adopted a strategy that they are unlikely to change.
- The Nash equilibrium.

History

- Émile Borel's researches in his 1938 book Applications aux Jeux de Hasard,
- John von Neumann and Oskar Morgenstern in 1944, book: Theory of Games and Economic Behavior.
- This theory was developed extensively in the 1950s by many scholars.
- Game theory was later explicitly applied to biology in the 1970s.
- Eight game theorists have won the Nobel Memorial Prize in Economic Sciences.
- John Maynard Smith was awarded the Crafoord Prize for his application of game theory to biology.

Simple game example: stag or hare (Philosophy)

- 2 hunters, have choices to hunt stag or hare; successful stag hunting requires cooperation, but it is more rewarding (higher utility, qualm)
- Game model
- What is the outcome of the game?
- Equilibrium (Nash equilibrium) = neither player has a unilateral incentive to change its strategy.
- One equilibrium is payoff dominant The other is risk dominant

Player 1 \ Player2	Stag	Hare
Stag	2, 2	0, 1
Hare	1, 0	1, 1

Strategic-Form Games

- Three elements:
  - The set of players *i*: {1, 2, …, I } (finite set)
  - The pure strategy space for each player *i*: S_i
  - Payoff function $u_i(s)$ (utility functions) for each profile of strategies: **s**= $(s_1, \ldots, s_I)$
  - *-i* (player *i's* opponents)
  - Players may not always try to beat other players, rather they maximize their payoff buy helping or hurting other players.
  - Zero sum games –players are indeed pure opponents:
  - sum of utilities is a constant

$$\sum_{i=1}^{2} u_i(s) = 0$$

Mixed Strategy Definitions

- Pure strategy
- Structure of the strategic form is known (common knowledge)
- Mixed strategy: σ_i is the probability distribution over pure strategies
- $\sigma_i(s_i)$ = probability that σ_i assigns to s_i
- Σ_i = the space of mixed strategies for player *i*
- Randomization is statistically independent from each other
- Payoff: expected values of the corresponding pure strategy payoffs

$$\sum_{s \in S} \left(\prod_{j=1}^{I} \sigma_j(S_j) \right) u_i(s)$$

Example: Mixed Strategy

- Mixed strategy for player 1 is *$\sigma_1(U)$,$\sigma_1(M)$, σ_1 (D). (non negative, sum=1)*
- *Payoffs*
- *$U_1(\sigma_1, \sigma_2)$ =1/3(0*4+1/2*5+1/2*6)*

  *+1/3(0*2+1/2*8+1/2*3)*

  *+1/3(0*3+1/2*9+1/2*2)= 11/2*

Similarly, $U_2(\sigma_1, \sigma_2)$=27/6

	L	M	R
U	4, 3	5, 1	6, 2
M	2, 1	8, 4	3, 6
D	3, 0	9, 6	2, 8

Dominated Strategy

- Assumptions:
  - Common knowledge: the players know the structure of the strategic game, and know that they opponents know it, and that they know that they know it, etc.
  - Rationality -actions taken by an user are in that user’s self-interest
- Sometimes, some strategy arises as the best strategy, by a process of elimination: iterated dominance
- A dominant strategy for a player is a strategy that is better regardless of the actions chosen by the other players

Examples of Dominated Strategy

	L	M	R
U	4,3	5,1	6,2
M	2,1	8,4	3,6
D	3,0	9,6	2,8

R gives a higher payoff than M → M dominated
If player 1 knows that player 2 does not play M → U
If player 2 knows that player 1 chooses U → L
Nash Equilibrium is (U,L)

Formal Definitions

- **Notations:**

$s_{-i} \in S_{-i}$ ⟵ Strategy selection for all players, but i

$(s'_i, s_{-i}) = (s_1, \ldots, s_{i-1}, s'_i, s_{i+1}, \ldots, s_I)$

$(\sigma'_i, \sigma_{-i}) = (\sigma_1, \ldots, \sigma_{i-1}, \sigma'_i, \sigma_{i+1}, \ldots, \sigma_I)$

- **Definition:**
  - Pure strategy is strictly dominated for player i, if there exists $\sigma'_i \in \Sigma_i$ such that

$$u_i(\sigma'_i, s_{-i}) > u_i(s_i, s_{-i}),\ \forall\ s_{-i} \in S_{-i} \quad (*)$$

  - Weakly dominated, if there exists a $\sigma'_i \in \Sigma_i$ such that (*) holds with weak inequality, and the inequality is strict, for at least one s_{-i}

Note: a mixed strategy that assigns positive probability to a dominated strategy, Is dominated.

Prisoner's Dilemma

- C: cooperate and do not testify
- D: defect and testify

	C	D
C	1,1	-1,2
D	2,-1	0,0

Nash Equilibrium

- Many problems (especially in resource allocation) are not solvable by iterated strict dominance
- A broad class of games are characterized by the Nash equilibrium solution.
- Nash equilibrium is a profile of strategies such that each player strategy is an optimal response to the other players' strategies
- Definition: A mixed strategy profile σ^* is a Nash equilibrium, if for all players *i*
- Similar def. for the pure strategies

References

- Wikipedia
  - http://en.wikipedia.org/wiki/Game_theory
- Game Theory, D. Fudenberg and J. Tirole, MIT press 1991.