THE ROLE OF GAME THEORY IN THE ANALYSIS OF SOFTWARE RADIO NETWORKS

James Neel (MPRG, Virginia Tech, Blacksburg, VA, USA, janeel@vt.edu)
Jeffrey H. Reed (MPRG, Virginia Tech)
Robert P. Gilles (Department of Economics, Virginia Tech)

ABSTRACT

The development of software radio technology facilitates the adoption of the adaptive modulation schemes that have recently been proposed in literature which promise significant performance improvements over a link. However, alterations to one link's waveform will alter the interference seen by the other links in the network, which will in turn affect their adaptation schemes. The dynamic and interdependent nature of this network makes analysis and management of such a network difficult to perform. As part of this paper we propose a new approach, based on the theory of potential games for analyzing and managing a wide variety of adaptive networks.

1. INTRODUCTION

Currently wireless networks are becoming increasingly less-structured, assuming many of the characteristics of ad-hoc networks. There are three impetuses driving this paradigm shift: the increased emphasis on providing a multitude of services over heterogeneous networks [1], the recognition that dynamic decentralized networks have the capacity to outperform traditional static centralized networks [2][3][4] and the emergence of software radio technology.*

These networks consist of smart and power efficient devices that can dynamically reconfigure themselves to handle any air-interface or data format, controllable QoS, global roaming, and integrated services. This has the potential to radically alter communication networks so services and performance can be reconfigured to best meet the needs of the system (based on traffic and congestion) and the user.

In order to perform these activities, a framework needs to be developed such that a radio can evaluate its capabilities, the requirements of its services, its potential waveforms, and the environment to then decide and act in a way that, to the limit of its knowledge, best satisfies the needs of the situation. Cognitive radio is an enhancement on traditional software radio design that attempts to establish such a framework. As illustrated below in Figure 1, cognitive radios employ a cognition cycle to alter their actions in response to changes in the environment through the use of state machines. The cognition cycle may perform a detailed analysis that predicts future changes in the environment or may make simple adjustments in immediate response to environmental changes. With this framework, it is possible to construct software radios that can intelligently adapt to their environment.

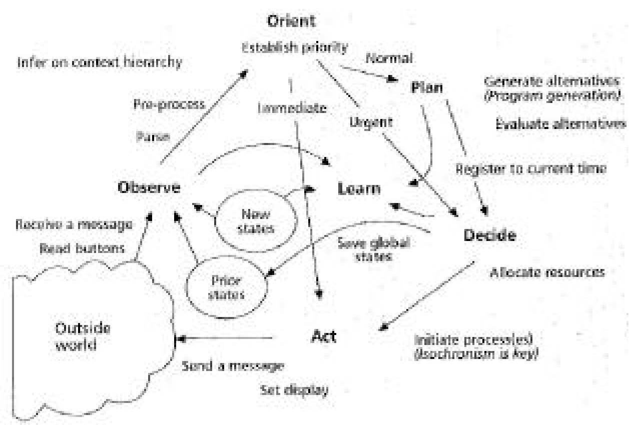

Figure 1 Cognition Cycle [5]

However, resources in these networks are often allocated on a contention basis, which means that it will be difficult to guarantee that all radios will be able to achieve an adequate level of performance. Radio etiquette has been proposed as a solution to this problem [5] wherein resources are shared by an agreed upon set of rules dictated by a hierarchical structure. While an important approach for establishing priorities, such as in the case of emergencies, radio etiquette does not fully address the general problems of contention based wireless networks, specifically, the modeling, analyzing, and control of these networks in such a manageable manner even when a

* This material is based on work sponsored by Motorola under the University Partnerships in Research program.

hierarchy of users is not clear. A smart network presents particularly difficult challenges to the analysis of radio resource management (RRM), as changes that one node makes may influence the decisions that other nodes make so network planning remains a difficult, if not impossible, task.

A number of authors have proposed game theory as a viable approach towards solving this problem [6], [7]. However, until now these approaches have treated each network separately requiring a fresh analysis for each network, a process that can be quite involved. Additionally, these approaches rest upon the assumption of higher-order rationality, i.e. the ability of a node to independently and recursively analyze a best response to other network nodes' best response to the original node's best response.

Fundamentally, most networks that perform adaptation at the physical layer modify either power levels or waveform as a function of measured SINR. It is the belief of the authors of this paper that it is possible to group adaptive networks into classes of adaptive networks. This paper begins the process of classification and analysis by describing a particular class of game, known as a *potential game*, that can be applied to a variety of different networks of cognitive radios that alter their power level or their waveform in response to environmental changes without the need for higher-order rationality. As an added benefit of the potential game approach, it is relatively straight forward to implement a network management system to constrain the operation of the network without damaging the potentially beneficial aspects of the adaptive algorithms.

This paper is organized as follows: in Section 2 we discuss some fundamental elements of game theory; in Section 3 we describe the properties of potential games; in Section 4 we show how a wide variety of adaptive networks can be modeled as a potential game; in Section 5, we describe how a network planner can change the steady state of the network to any point of their choosing; and finally, we summarize the key results presented in this paper and describe future directions for research.

2. GAME THEORY

Game theory is a set of tools developed in economics for the purposes of analyzing the complexities of human interactions. Game theory can be used to predict the outcome of these interactions and to identify strategies that are optimal and others that are deleterious. Game theoretic analyses are traditionally predicated on three assumptions; the decision makers are *rational*, i.e. expected to act in its own self-interest at all times by optimizing some objective function known to each decision maker and the analyst (although not necessarily to the other decision makers), and each decision maker has at least some short-sighted knowledge as to how their actions will affect themselves given that the environment remains constant.

2.1. Fundamental Components of Game Theory

The fundamental component of game theory is the notion of a game, expressed in normal form as $G = \langle M, A, \{u_i\} \rangle$ where G is a particular game, M is a finite set of players (decision makers) $\{1,2,\ldots,m\}$, A_i is the set of actions available to player i, $A = A_1 \times A_2 \times \cdots \times A_m$ is the action space, and $\{u_i\} = \{u_1, u_2, \ldots, u_m\}$ is the set of objective functions that the players wish to maximize. For every player i, the objective function, u_i, is a function of the particular action chosen by player i, a_i, and the particular actions chosen by all of the other players in the game, a_{-i}. From this model, steady-state conditions, known as *Nash Equilibria* are identified wherein no player would rationally choose to deviate from their chosen action as this would diminish their payoff, i.e. $u_i(a) \geq u_i(b_i, a_{-i})$ for all $i, j \in M$. The action tuples (a unique choice of actions by each player) corresponding to the Nash Equilibria are then predicted as the most probable outcomes. Note that in a game, the steady-state condition (Nash Equilibrium) need not be the optimal (Pareto Efficient) operating point. An example analysis of a game expressed in normal form can be found in [8].

2.2. Improvement Paths

Another way this game can be analyzed is by following preferable deviations from each action tuple. An improvement deviation is the relative change in the value of the objective function of the deviating player, or $\Delta = u_i(a_i, a_{-i}) - u_i(b_i, a_{-i})$. Action tuples from which all improvement deviations are negative are Nash Equilibria. A sequence of preferable (positive) deviations is known as an *improvement path*. When all conceivable improvement paths are finite, there will not be oscillation between any sequence of action tuples in the game.

2.3. Game Theory and Radio Networks

A radio network can be modeled as a game if the following conditions hold.

<u>Conditions for Rationality</u>

1. The decision-making process must be well-defined, i.e., each of the radios must follow a well-defined set of rules for selecting an action with respect to environmental factors.
2. A decision to change an action must result in a positive improvement deviation.

These rules may be implemented with a well-defined objective function that each radio maximizes, or a simple state machine that determines the radio's operating parameters in response to changes in its environment – an example of a myopic decision maker. This single condition effectively combines the need for definable action sets and objective functions.

Conditions for a Nontrivial Game

1. There must be more than one decision making entity in the network.
2. More than one decision maker has a nonsingleton action set.

3. POTENTIAL GAMES

Within game theory literature, many special classes of games have been developed. One of the most powerful of these classes of games is the potential game. Potential games are characterized by a potential function, $P : A \rightarrow \mathbb{R}$, whose change in value when any player deviates in their action is related to the change in the objective function of that player. Rather than modeling the exact payoffs of the game, P instead models the information associated with the improvement paths of a game. There are a number of different kinds of potential games, most notably exact potential games, weighted potential games, and ordinal potential games. Each of these games are named for the relationship between the value of deviations in the potential function and the improvement deviations of the game. In the exact potential game, for a change in actions of a single player, the change in the potential function is equal to the value of the improvement deviation. For weighted potential games, the change in the potential function is equal to a (possibly different) scalar multiple of the improvement deviations of each player. For ordinal potential games, the sign of the change in the potential function is the same as the sign of the improvement deviation.

3.1. Exact Potential Games

An exact potential game is most strictly defined as a game with a potential function P which satisfies

$$P(a_i, a_{-i}) - P(b_i, a_{-i}) = u_i(a_i, a_{-i}) - u_i(b_i, a_{-i})$$

for any $i \in M$, a, $a_{-i} \in A$. Note that by this definition, it is possible for many potential functions to exist for the same game. However, the difference between any two potential functions for the same exact potential game must be a constant [9]. Exact potential games have the following attractive properties:

- All improvement paths in the game are finite (no improvement cycles). This is known as the Finite Improvement Path Property (FIP). It has also been shown that any game satisfying FIP is an exact potential game.
- Existence of at least one Nash Equilibirum is ensured.
- All improvement paths lead to a Nash Equilibrium.
- Relative maximums of $P(a)$ (local or global) are the Nash Equilibria of the game.

Proofs of these properties are given in [10]. Thus, if a network can be identified as an exact potential game then convergence to a steady-state is ensured, even if the radios do not have higher-order rationality. Additionally, the analysis and management of a network that is an exact potential game is more straight forward than if the network is a game and not an exact potential game. Although, ordinal and weighted games also have attractive properties, for the remainder of this document, we will only consider exact potential games.

3.2. Exact Potential Games with Interval Action Sets

If for all $i \in N$, A_i is an interval of real numbers, and u_i is twice continuously differentiable, then the potential function satisfies the following properties

$$\frac{\partial P}{\partial a_i} = \frac{\partial u_i}{\partial a_i}$$

$$\frac{\partial^2 P}{\partial a_i \partial a_j} = \frac{\partial^2 u_i}{\partial a_i \partial a_j} = \frac{\partial^2 u_j}{\partial a_i \partial a_j}.$$

As shown in [10], a necessary and sufficient condition for a game with a continuous, bounded action set to be an exact potential game is given by the following

$$\frac{\partial^2 u_i}{\partial a_i \partial a_j} = \frac{\partial^2 u_j}{\partial a_i \partial a_j} \text{ for every } i, j \in N.$$

Also in [10], it is stated that the potential for such a game can be found by evaluating

$$P(a) = \sum_{i \in M} \int_0^1 \frac{\partial u_i}{\partial a_i} x(t) x_i'(t)\, dt$$

where x is a piecewise continuously differentiable path that connects some arbitrary action tuple b to some other action tuple a such that $x : [0,1] \rightarrow A$ ($x(0) = b$, $x(1) = a$). Although quite general, this evaluation can be quite tedious. Thus the following refinements have been introduced, which as we shall see, can be quite useful for modeling radio network games.

3.3. Coordination – Dummy Games

As shown in [11], if all players in the game have an objective function that can be characterized as

$$u_i(a) = V(a) + Q_i(a_{-i}),$$

then the potential for this game can be written as $P(a) = V(a)$. Note that $V(a)$ describes a coordination function wherein all players receive the same payoff for a particular action tuple a. $Q_i(a_{-i})$ describes a *dummy function* – a function where the outcome for player i is not dependent on the actions of i. Note that each player in the coordination – dummy game may have an independent dummy function.

Repeating the proof shown in [11] for the sake of completeness, it is relatively easy to show that $V(a)$ is indeed an exact potential for this game by noting that

$$u_i(a_i, a_{-i}) - u_i(b_i, a_{-i}) = V(a_i, a_{-i}) - V(b_i, a_{-i}) \forall i, j \in N$$

3.4. Bilateral Symmetric Interaction Games

In a bilateral symmetric interaction (BSI) game, every player's objective function can be characterized by the following function

$$u_i(a) = \sum_{j \in M \setminus \{i\}} w_{ij}(a_i, a_j) - h_i(a_i)$$

where $w_{ij} : A_i \times A_j \to \Re$ such that $w_{ij}(a_i, a_j) = w_{ji}(a_j, a_i)$ for any $(a_i, a_j) \in A_i \times A_j$ and $h_i : A_i \to \Re$. As shown in [11], a game of this type is a potential game and has the following potential function

$$P(a) = \sum_{i \in M} \sum_{j=1}^{i-1} w_{ij}(a_i, a_j) - \sum_{i \in M} h_i(a_i).$$

Again, it is relatively straight-forward to demonstrate that this is an exact potential as

$$P(a_i, a_{-i}) - P(b_i, a_{-i}) = u_i(a_i, a_{-i}) - u_i(b_i, a_{-i})$$
$$= \sum_{j \in M} w_{ij}(a_i, a_j) - \sum_{j \in M} w_{ij}(b_i, a_j) - h_i(a_i) + h_i(b_i)$$

4. RADIO MODEL

Consider a software radio capable of arbitrarily altering its transmitted energy level and signature waveform. Let us use the following notational conventions:

E_i – the set of possible energy levels available to radio i

e_i – the energy level chosen by i

e - the tuple of chosen energy levels of all radios in the network

W_i – the set of signature waveforms available to radio i

w_i – the chosen waveform of i

w - the tuple of chosen waveforms of all radios in the network

N_i – noise power at node i

r_{ij} - the correlation between the signature waveform sequences of radios i and j. Note that r_{ij} necessarily equals r_{ji}.

A generalized expression for a digital linear waveform for a radio in this model is given by $s_i = m_i w_i \sqrt{e_i}$ where m_i is the message sequence for radio i. As it is a wireless network, some portion of the energy generated by each node in the network will arrive at every other node in the network. The exact proportion of the energy seen at some other node will be a function of antenna form factor, orientation, and potentially time-varying channel propagation effects. This is illustrated below in Figure 2 where the proportion of the energy transmitted by node i seen by node j is given by $e_i g_{i,j}$.

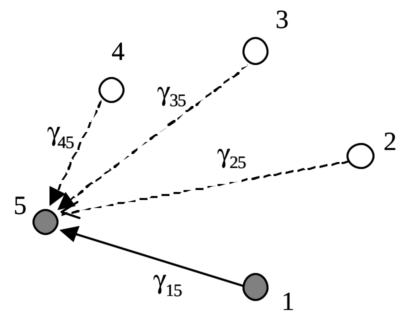

Figure 2 Propagation Losses to Node 5, the Node of Interest of Node 1

For the sake of simplicity, we will assume that each node only desires to communicate with one other node in the network, its *node of interest*. We will denote radio i's node of interest as n_i. A single node may be the node of interest of multiple other nodes.

The SINR seen at n_i is given by the following expression

$$SINR = \frac{e_i g_{i,n_i}}{\sum_{j \in M \setminus \{i, n_i\}} e_j g_{j,n_i} r_{ij} + N_{n_i}}.$$

It is presumed that self-interference is negligible or nonexistent. Each node of interest relays its SINR information back to the nodes transmitting to it. Each transmitting node then adapts its transmission parameters as a function of SINR at its node of interest constrained by a cost function that models the internal costs for a particular energy / waveform pair (battery life, complexity, distortion) and / or a cost function imposed by a network for a particular energy / waveform pair. This describes a game in the following form

$$G = \langle M, A, \{u_i\} \rangle$$

$$\{1,2,\ldots,m\} \in M$$

$$A_i = E_i \times W_i,\ A = A_1 \times A_2 \times \cdots \times A_m$$

$$u_i(a) = f_i\left(\frac{e_i g_{i,n_i}}{\sum_{j \in M \setminus \{i, n_i\}} e_j g_{j,n_i} r_{ij} + N_{n_i}} \right) - c_i(e_i, w_i)$$

Note that in this model, each radio is permitted to have a different objective function and can be applied to a broad class of adaptive wireless networks.

4.1. Separable SINR Games

Let us consider a subclass of the game described above wherein each radio can separate its SINR function into a function of received signal strength less a function of interference such that the radio's objective function takes the following form

$$u_i(a) = f_{i,1}\left(e_i, g_{i,n_i}, w_i\right) - f_{i,2}\left(\sum_{j\in M\setminus\{i,n_i\}} e_j g_{j,n_i} r_{ij} + N_{n_i}\right) - c_i\left(e_i, w_i\right).$$

This is a widely applicable model as many adaptive modulation schemes perform their adaptation based on SINR estimates, and can be readily put in this form by working with SINR estimates in dB or by directly subtracting interference from noise. Also note that the nature of the functions of Received Signal Strength (RSS) and interference can be defined in arbitrary ways. Let us now consider the following two additional refinements to this model.

4.2. SINR Power Games

In this game, we limit each radio to only adjusting its power level in response to changes in SINR at its point of interest. In this model, the waveform selected at link initialization remains fixed. Every radio's performance is impacted by interference and chooses an action to change its power level in response to SINR and maintains some minimum threshold, e_i. Additionally, each radio has some cost function associated with each power level, e_i. In this case each radio's objective function can be written as

$$u_i(a) = f_{i,1}\left(e_i, g_{i,n_i}\right) - f_{i,2}\left(\sum_{j\in M\setminus\{i,n_i\}} e_j g_j r_{ij} + N_{n_i}\right) - e_i - c_i\left(e_i\right).$$

This game can be verified to have a potential as

$$\frac{\partial^2 u_i}{\partial a_i \partial a_j} = \frac{\partial^2 u_j}{\partial a_i \partial a_j} = 0, \forall i, j \in M, i \neq j$$

By recognizing that $f_{i,2}$ is a dummy function, a potential function can be written as

$$P(e) = \sum_{i\in M}\left[f_{i,1}\left(e_i, g_{i,n_i}\right) - c_i\left(e_i\right)\right]$$

As this model was explicitly designed for ad-hoc networks, this model can be extended to any network topology.

4.3. SINR Waveform Games

In this model, let us assume now that the radios are a part of a power controlled star network such that the energy received at the sole access point is the same for each radio. Thus each radio has the same node of interest and only maintains a single link. Adaptive modulation is employed and each radio may select any waveform from its waveform set W_i. This is a network extension of the link model in [12]. Again we assume some cost associated with the use of each waveform. The objective function for each radio now takes the form

$$u_i(a) = f_{i,1}(w_i) - f_2\left(\sum_{j\in M\setminus\{i,n_i\}} \int w_i w_j + N_{n_i}\right) - e_i - c_i(w_i).$$

Here we must constrain f_2 to be linear. By recognizing that f_2 is the sum of BSI terms, an exact potential function can be written as

$$P(a) = \sum_{i\in M}\sum_{j=1, j\neq n_i}^{i-1} f_2\left(\int w_i w_j\right) - \sum_{i\in M}\left[c_i(w_i) - f_{i,1}(w_i)\right].$$

4.4. Sufficient Conditions for an Exact Potential Game

Based on these results, the following can be said about sufficient conditions for when an adaptive radio can be modeled as an exact potential game. Given that a radio network satisfies the conditions for being modeled as a game, if the following conditions are also satisfied then the network is also an exact potential game:

- The network is interference limited. For this model, interference is the mechanism for impacting the players' objective functions.
- Each decision making entity makes decisions that alter either its power level or its waveform, thus changing the interference it introduces to the network making the game nontrivial.
- Each decision making entity makes decisions with respect to the difference between some arbitrary function of its signal level and some arbitrary function of the interference level.
- Each entity may also make decisions with respect to other parameters such as power consumption or memory usage which are solely a function of the decisions of that entity. If present, this function should be additive with respect to the aforementioned functions.
- Each decision making entity has a bounded action set. For potential games, the finite improvement path property must hold, and this could not be guaranteed if for instance the radios could select any power level from 0 to infinity.
- When games are constructed around signature waveform selection, propagation losses in the network must either be negligible or identical at all points where performance is measured. This condition can occur when radios are tightly packed geographically or in a power controlled network (thus interference is only a function of the selected waveform).

5. MANAGEMENT OF ADAPTIVE EXACT POTENTIAL GAME RADIO NETWORKS

While exact potential games are guaranteed to converge to a Potential Maximizing Nash Equilibrum, this state may not be desirable. Fortunately, for exact potential games, it is a relatively straight forward task to move the steady-state of the game, a^* to some other desired valid state, a^{**}. The procedure for doing so is as follows.

Introduce a network cost function, $NC(a)$. Solve the following equation for $NC(a)$

$$\frac{\partial P\left(a^{**}\right)}{\partial a_i} + \frac{\partial NC\left(a^{**}\right)}{\partial a_i} = 0$$

where $P(a)$ is the game's potential function. In other words, at the desired action tuple, the network cost function should have the negative slope of the potential function's slope. After solving for $NC(a)$, "charge" the nodes this cost function. This can either be done as part of the node's development or as a cost imposed by the network on the nodes. Thus the objective function for each node takes the following form

$$u_i(a) = f_{i,1}\left(e_i, g_{i,n_i}, w_i\right) - f_{i,2}\left(\sum_{j \in M \setminus \{i, n_i\}} e_j g_{j,n_i} r_{ij} + N_{n_i}\right) - c_i\left(e_i, w_i\right) + NC(a)$$

The potential function for this modified game is given by $P'(a) = P(a) + NC(a)$ for which a^{**} is clearly a Nash Equilibrium. Note that this process may introduce additional Nash Equilibria depending on the exact topology of $P(e)$ and the choice of $NC(a)$. Care should also be taken so that the original Nash Equilibrium is no longer a Nash Equilibrium. To minimize the creation of new Nash Equilibria, it is suggested that the function be of low order or a piecewise function. Also note that it is possible to impose arbitrary cost functions to create arbitrary potential functions. However, this will not in general be desirable, as significant alterations in the potential function will result in significant changes to the behavior of the network, perhaps negating the original advantages of the adaptation scheme. Further, while this solution is deterministic, the actual channel conditions will be stochastic and the stability of the Nash Equilibria should also be considered. Thus in general, it is anticipated that small changes in the neighborhood of the original Nash Equilibrium will be more desirable than more significant alterations to the game.

6. SUMMARY AND FUTURE DIRECTIONS

We have introduced novel techniques for analyzing and managing a wide variety of adaptive networks through the use of potential game models for which we have delineated specific conditions for which the models apply. These models can be used to quickly identify the steady-state conditions of those networks as the first stage of network planning. Further, when an adaptive network satisfies the conditions of the models introduced in this due to the finite improvement path property of potential games, even when the radios do not possess higher order reasoning and are only able to make myopic decisions, the radio networks will still converge to the steady state.

In the future, we will begin examining the stability of Nash Equilibria and potential games when operating in a stochastic environment. To aid in this study, we are currently developing a generalizable simulation of a wireless network with resource contention in C++ and OPNET. We will also explore other wireless network resource contention issues such as traffic loading in a wireless hot-spot, and ad-hoc network formation.

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