Pricing for Enabling Forwarding in Self-Configuring Ad Hoc Networks

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***Abstract*—The assumption that all nodes cooperate to relay packets for each other may not be realistic for commercial wireless ad hoc networks. An autonomous (selfish) node in a wireless network has two disincentives for forwarding for others: energy expenditure (real cost) and possible delays for its own data (opportunity cost). We introduce a mechanism that "fosters cooperation through bribery" in the context of forwarding in ad hoc networks. Using a microeconomic framework based on game theory, we design and analyze a pricing algorithm that encourages forwarding among autonomous nodes by reimbursing forwarding. Taking a joint network-centric and user-centric approach, the revenue maximizing network and utility (measured in bits-per-Joule) maximizing nodes interact through prices for channel use, reimbursements for forwarding, transmitter power control, as well as forwarding and destination preferences. In a three-node (two-sources, one-access-point) network, the network converges to an architecture that induces forwarding only when the network geometries are such that forwarding is likely to increase individual benefits (network revenue and node utilities). For other geometries, the network converges to architectures that do not favor forwarding. We then generalize to a multinode network, where it is seen that the nodes' willingness to forward decrease for large ratios of the average internodal distance to the smallest distance between the access point and any source node. Pricing with reimbursement generally improves the network aggregate utility (or aggregate bits-per-Joule), as well as utilities and revenue compared with the corresponding pricing algorithm without reimbursement.**

***Index Terms*—Cooperation, incentive for forwarding, noncooperative game, pricing, revenue maximization, Stackelberg game, utility.**

I. Introduction

THE AD HOC networks of the emerging pervasive computing and communication environment will likely consist of autonomous users with heterogeneous devices, and possibly operate in the unlicensed band without stringent rules or etiquette. Such networks need to be formed, run, and maintained in an autonomous and distributed fashion. The willingness of each node to relay data for others is required to achieve these objectives. The usual assumption of spontaneous willingness to forward is unrealistic for autonomous users because by forwarding, the forwarder incurs the real cost of battery energy expenditure and the opportunity cost of possible delay for its own data.

Besides being important for the formation and operation of ad hoc networks of autonomous users, cooperation among users in the form of forwarding can also greatly enhance system performance. Reference [1] shows that mobility combined with forwarding can change the capacity per node of an ad hoc network from being not scalable [2] to scalable. In [3] and [4], forwarding allows the exploitation of multichannel diversity. Reference [5] considers content distribution via single-hop multicast. In order to expedite data dissemination, a node also relays packets for other nodes if it has not done so for some time. While these three works implicitly assume willingness to relay data, effective willingness to relay data is created in [6] and [7] with the help of a social contract, based on which, two mobiles exchange files that one another need, leading to significant system capacities. As we will see, the effect of network geometry (in addition to topology) on incentivizing forwarding can only be studied with a more realistic channel model. Previous works on incentivizing forwarding tend to focus on its protocol and security aspects and consider very simplified wireless channel models, for example, ones in which the amount of energy expended to make one hop is a constant as long as the hop is in range.

For instance, [8] proposed a reputation system to indirectly create incentive for forwarding. References [9] and [10] in the context of the Terminodes project [11], use two forms of virtual currencies to create economies in which virtual currencies can be exchanged for data forwarding. More recently, [12] introduces pricing-and-credit based incentives in the context of multihop flow control and analyzed the system dynamics. Reference [13] is more concerned with security and collusion resistance. The approach in [14] appears to be the closest to ours. The authors use the same communication figure of merit bits-per-Joule as we do. Their problem is formulated as a cooperative game, for which the authors show that users would be willing to stay in the network despite being required to forward for others while staying.

In this paper, we use the time-tested paradigm of "fostering cooperation through bribery" in the context of forwarding in wireless ad hoc networks. Based on a microeconomic framework, we design a pricing mechanism in which the network, while charging users for radio-channel usage, also reimburses forwarding behavior as illustrated in Fig. 1. The network announces prices for channel utilization, as well as reimbursements for forwarders in order to maximize its net revenue. The users in response adjust their transmit powers as well as forwarding and destination preferences in order to maximize their net utilities. This interaction between network and users evolves until the network revenue is maximized. Since net

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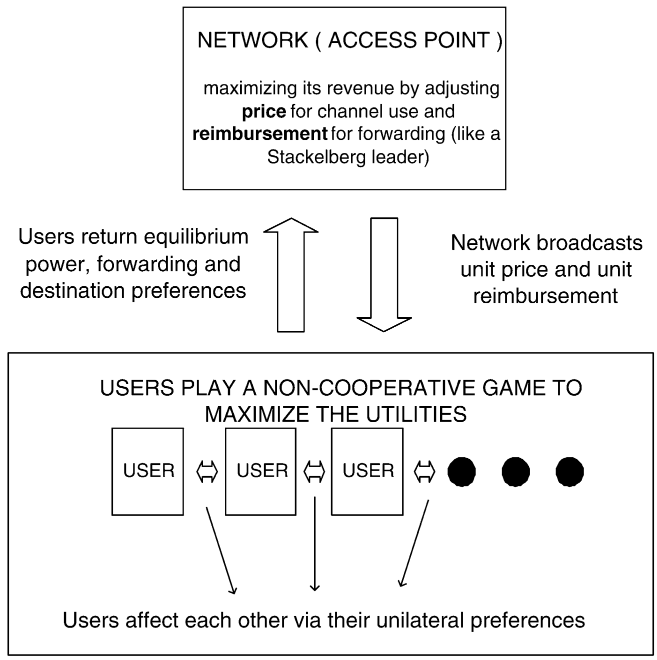

Fig. 1. Illustration of interactions between users and network (access point) in our pricing model. They do so via price for channel use and reimbursement for forwarding, as well as transmit power control, destination preferences, and forwarding preferences.

utilities depend on radio-link qualities and is measured in the physical unit of bits-per-Joule, it is possible for network revenue in a forwarding architecture to be superior to that in a nonforwarding architecture, despite the required reimbursements in the forwarding architecture. Once the physical reality of the radio channel is included, induced forwarding behavior depends not only on network topology, but also on network *geometry*. As far as we know, this is the first attempt to analyze the network-geometric dependence of incentivized cooperation in networks.

In the context of this paper, we view pricing as an inducer of forwarding, or more generally, as an enabler of cooperation among selfish users in an ad hoc network. When applied to network formation, pricing can be viewed as an *enabler* of self-configuring ad hoc networks of autonomous users. Special cases of the current pricing design can be found in [15]–[17], where pricing can be interpreted as mediating and policing mechanisms, respectively.

II. System Model

We consider a fixed number of autonomous wireless data users trying to connect to a unique revenue-seeking access point (AP), sometimes referred to as the network. We assume an "interference free" model where user transmissions are considered to be orthogonal to each other. The network is assumed to be geographically static or quasi-static in the sense that the time scale of algorithm convergence is shorter than those of channel variations and mobility.

A. User Metric

A user's quality-of-service (QoS) is modeled by a utility function. While several choices of utilities are possible (e.g., those in [16]–[20]), we choose the one in [16] and [17] since it combines the three important criteria of wireless *data* communication: throughput, transmission power, and battery life. The utility of user i is defined as the average amount of data received correctly per unit of energy expended, measured in bits-per-Joule

$$u_i(p_i) \triangleq \frac{T_i(p_i)}{p_i} \frac{\text{bits}}{\text{Joule}} \tag{1}$$

where the throughput T_i of user i depends only on its own power p_i through the signal-to-noise ratio (SNR) $\gamma_i \equiv h_i p_i / N_0 W$, with h_i, W, and $N_0/2$ being the path gain of user i, signal bandwidth, and noise spectral density, respectively. An assumption in this model is that data bits are packed into frames of M bits containing $L < M$ information bits per frame, where $M - L$ bits are used for error detection. A frame is assumed to be retransmitted until received correctly in deriving the above expression [16]. In contrast to [15]–[17], an important difference in the current model[1] is that the users here occupy orthogonal additive white Gaussian noise (AWGN) channels with identical bandwidth W. The explicit relationship of throughput to SNR is given by $T_i(p_i) \equiv (L/M) W f(\gamma_i(p_i))$, where the efficiency function $f(\gamma) \triangleq [1 - 2\,\mathrm{BER}(\gamma)]^M$, and $\mathrm{BER}(\gamma)$ denotes bit-error rate at SNR γ. $f(\gamma)$ is a good approximation to the frame-success rate $[1-\mathrm{BER}(\gamma)]^M$ in our regime of interest,[2] and has been validated extensively in [15]–[17] and [20] for a variety of modulation schemes. The factor of 2 is included to ensure that $u_i(p_i)$ has the desirable behavior of vanishing at zero p_i. T_i could also be replaced with the Shannon capacity if the denominator of (1) is modified to include a fixed positive power cost due to processing, which plays the analogous role of the above factor of 2 to yield $u_i(0) = 0$. All analytical results in this work are true for any monotonically decreasing $\mathrm{BER}(\gamma)$. Note that in the above definition of the utility, we implicitly assume a busy source model, within which, nodes always have data to send.

1) User Metric in Multihop Networks: In the utility definition in (1) above, $T_i(p_i)$ denotes the point-to-point link throughput achieved over a wireless link. However, in a multihop network, the effective or net throughput between two nonneighboring nodes is the minimum of all link throughputs along that route. In order to distinguish between them, a link throughput from node i to its next node n_i in its route is denoted as T_{in_i} and the effective throughput from node i to the unique access point AP is denoted as $T_{ia}^{(\mathrm{eff})}$. In a multihop environment, the satisfaction of a user, its effective utility, should be measured in *effective* bits-per-Joule rather than just bits-per-Joule, i.e., $U_i(p_i) \triangleq T_{ia}^{(\mathrm{eff})}(p_i)/p_i$.

Incidentally, the aggregate bits-per-Joule, $\sum_i U_i(p_i)$, can be thought of as a system efficiency metric from a network view point.

[1]User channels are approximately orthogonal in, for example, an ultra-wideband system which is proposed for ad hoc and sensor networks, and networks consisting of low-power devices, which ad hoc networks tend to be.

[2]The deviation of $f(\gamma)$ from the frame-success rate is within 15% for SINR above a few decibels. See [17, Fig. 2].

B. Network Metric, Charge, and Reimbursement

In a pricing scheme involving both charging and reimbursing, a natural metric of network satisfaction is its revenue, the difference between the charges paid by all users and the reimbursements paid to forwarders.

The charge to a user is the product of the price per unit service and the amount of service provided. Since the network provides radio resources for each data link irrespective of the effective utility, the amount of service provided by the network to any user i is proportional to the amount of useful data user i sends over its radio link to its hop destination n_i (not its route destination AP) in a certain time frame. Hence, for a *fixed* time frame, the amount of service is proportional to the link throughput. A fixed time frame is actually implicit in the definition of our utility because two users enjoying identical bits-per-Joule can have different levels of satisfaction depending on the time it takes to achieve this level of bits-per-Joule. A fixed transaction-time frame must be assumed for our utility definition to be consistent. In this time frame, the network provides service T_{in_i} for the link between user i and its next hop n_i and charges each transmitting node the common unit price of λ. Consequently, for having a link throughput T_{in_i}, user i pays the network λT_{in_i}.

A potential forwarder j is reimbursed for the effective amount of data per unit time, $T_{ja}^{(\text{eff}-\text{for})}$, it forwards to the AP, which depends on its effective throughput to AP $T_{ja}^{(\text{eff})}$, the fraction $k_j (0 \leq k_j \leq 1)$ of this throughput user j is devoting for forwarding (with the remaining $(1-k_j)$ fraction for its own data), and the amount of data per time it is asked to forward. At any instant, the network offers a common *unit reimbursement* μ per unit of effective throughput a forwarder forwards for others. For the amount $T_{ja}^{(\text{eff}-\text{for})}$ forwarded, user j receives the reimbursement of $\mu T_{ja}^{(\text{eff}-\text{for})}$. The network revenue is, therefore

$$\rho \equiv \sum_{\text{all users } i} \lambda T_{in_i} - \sum_{\text{all forwarders } j} \mu T_{ja}^{(\text{eff}-\text{for})}. \quad (2)$$

C. Pricing: The Interaction Between Users and Network

After adjusting for network charges and reimbursements, the net satisfaction or the *net utility* U_i^{net} of a nonforwarder i is $U_i - \lambda T_{in_i}$, where λT_{in_i} is the payment by user i to the network (the AP in our case), while the net utility U_j^{net} of a forwarder j is $U_j - \lambda T_{jn_i} + \mu T_{ja}^{(\text{eff}-\text{for})}$, where λT_{jn_i} is the payment by user j to AP, and $\mu T_{ja}^{(\text{eff}-\text{for})}$ is the reimbursement from AP to user j. The conversion factor from price to bits-per-Joule, being constant, is absorbed into both λ and μ and, thus, payments and reimbursements are measured in bits-per-Joule rather than in monetary units.

In summary, users are charged by the network for the actual channel use, while they are reimbursed only for the useful data forwarded (i.e., the forwarded data that reaches the AP). Note that this modeling approach also avoids misbehaving nodes that falsely claim credit for forwarding. The actual implementation of such a scheme can be enabled by including secure packet headers that identify source nodes, their routes, and forwarding nodes on those routes that could be used by the AP to identify charges and reimbursements for each node.

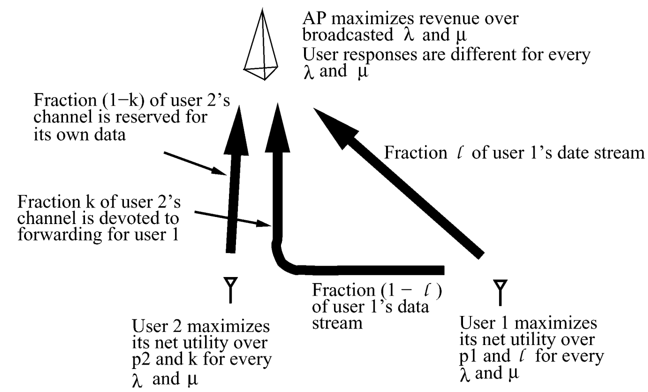

Fig. 2. Illustration of pricing-based self-organization for two users and one AP.

Note that *with* power control a multihop route to the AP has higher average end-to-end throughput than a direct link to it [2]. Therefore, there will be many network geometries with node placements for which the increased payments coming from higher effective throughputs as a result of multihop due to induced forwarding more than offsets the reimbursements required. For these node placements, higher throughputs coincides with higher net satisfactions, net utilities, and revenue, i.e., economic efficiencies, for each party (including the AP) of the network. We will examine the correlation between forwarding behavior, node placements, and individual economic benefits in our results sections.

III. Pricing-Based Self-Organization

Routing is inherently part of the self-organization problem and is intricately related to pricing. We first consider a two-user-one-AP network as illustrated in Fig. 2, and develop the pricing (with reimbursement) mechanism for it. Based on the insight gained, we will generalize to the case of a multinode network. We initialize the network prior to pricing by assigning a routing algorithm which connects each node to its nearest neighbor (including the AP)[3] towards the AP. Without loss of generality, let user 1 and user 2 be the nonforwarder and the potential forwarder, respectively. Both are trying to connect to the AP, but during the course of the pricing algorithm, the forwarding preference of user 2 and the destination preference of user 1 may change, i.e., the initial route assignment may be changed by pricing.

User 1 is allowed to split $(1-\ell)$, $\ell \in [0,1]$, fraction of its data stream onto a direct channel to the AP and the remaining ℓ fraction onto the forwarded channel to user 2. The ℓ fraction of user 1's data stream will be riding on the k, $k \in [0, 1/2]$, fraction of user 2's channel to AP that user 2 allocates for forwarding. User 2 uses the rest $(1-k)$ fraction of its channel for its own data stream. The rationale for $k \leq 1/2$ is discussed in the numerical results section.

For each pair λ and μ, users maximize their net utilities unilaterally over p_1 and ℓ, and p_2 and k, respectively. After they

[3]A node always connects directly to the AP if the AP is its nearest neighbor "towards" the AP. Consider the line passing through the node i and perpendicular to the line from node i to AP. It divides the plane into two halves, one "towards" the AP and the other "away from" the AP. A node is toward AP from the point of view of node i if it lies in the half plane toward the AP.

reach a Nash equilibrium, the AP evaluates its revenue ρ using the users' responses (the equilibrium values of p_1, ℓ, p_2, and k). The AP tries another pair of λ and μ for a potentially higher revenue, to which the users respond again. This process stops when AP has maximized its revenue over λ and μ.

A. User Optimizations: Noncooperative Power Control Game

Although in principle, any binary partitions of both the bandwidth and power budgets (W and p_1) for the purpose of splitting user 1's data stream is possible, we allow only a specific class of bandwidth partition for simplicity and the belief that any binary partition of radio resources will yield similar *qualitative* results. We choose to keep the value of the power per unit bandwidth constant at p_1/W on both channels, independent of the partition, which could be achieved via time or code division. As a result, the SNRs on the two channels are independent of the partition as well. The throughputs of the direct channel from user 1 to AP and the forwarded channel from user 1 to user 2 are $T_{1a}(p_1, W_{1a}) = W_{1a} f(h_{1a}p_1/N_0W)$ and $T_{12}(p_1, W_{12}) = W_{12} f(h_{12}p_1/N_0W)$, respectively, where W_{1a} and W_{12} are the partitions of W for the direct and forwarded channels with the respective path gains h_{1a} and h_{12}. The arguments of the efficiency functions $f(\cdot)$ are the two channels' SNRs. Based on these throughputs, user 1 is charged $\lambda T_{1a} + \lambda T_{12}$ when the unit price is λ.

In addition to T_{12}, the effective forwarded throughput user 1 enjoys is limited by the part of user 2's throughput $k T_{2a}$ devoted to forwarding. Therefore, the end-to-end throughput of user 1 to AP through user 2 is $\min\{T_{12}, k T_{2a}\}$ rather than simply T_{12}. Putting the utility and payment together, we have the optimization problem for user 1, Puser1

$$\begin{aligned} \max \quad & \frac{T_{1a}(p_1, W_{1a}) + \min\{T_{12}(p_1, W_{12}), kT_{2a}\}}{p_1} \\ & - \lambda[T_{1a}(p_1, W_{1a}) + T_{12}(p_1, W_{12})] \\ \text{subject to} \quad & 0 \le p_1 \le p^{\max} \quad \text{and} \quad W_{1a} + W_{12} = W \end{aligned} \tag{3}$$

where $p^{\max}$ is the maximum power constraint. The difficulty presented by the $\min\{\}$ can be eliminated by noticing that within the context of Puser1, $\min\{T_{12}(W_{12}, p_1), k T_{2a}\}$ is equivalent to $T_{12}(W_{12}, p_1)$ subject to $T_{12}(W_{12}, p_1) \le kT_{2a}$. Whenever $T_{12} > kT_{2a}, U_1^{\text{net}}$ can be increased by slightly decreasing T_{12} and p_1 via slight decreases in both W_{12} and p_1 that keep $T_{1a}(p_1, W_{1a})$ and $\min\{T_{12}(W_{12}, p_1),\ k T_{2a}\}$ stationary. The former can be kept fixed because it is increasing in p_1 but decreasing in W_{12}, while the latter can be kept constant due to the finite gap between T_{12} and $k T_{2a}$. This variation increases utility $(T_{1a} + \min\{T_{12}, kT_{2a}\})/p_1$ and decreases payment $\lambda[T_{1a} + T_{12}]$ and, hence, increases the net utility of user 1. We have just shown that the net utility can always be increased if $T_{12} > kT_{2a}$, i.e., in the context of Puser1, $T_{12} \le kT_{2a}$. Therefore, Puser1 can be reformulated as follows.

Theorem 3.1:

$$\text{Puser1: } \max\left(\frac{1}{p_1} - \lambda\right)[T_{1a}(p_1, W_{1a}) + T_{12}(p_1, W_{12})]$$
$$\text{s.t.} \begin{cases} 0 \le p_1 \le p^{\max}, W_{1a} + W_{12} = W, \\ T_{12}(W_{12}, p_1) \le kT_{2a} \end{cases}. \tag{4}$$

The trading off of $\min\{\}$ for the constraint $T_{12}(p_1, W_{12}) \le kT_{2a}$ makes the bottleneck imposed by user 2 onto user 1 explicit.

Given that user 2's forwarding preference is k, if the throughput from user 2 to AP is $T_{2a}(p_2) = W f(h_{2a}p_2/N_0W)$ when user 2 is expending power p_2, user 2 is charged $\lambda T_{2a}(p_2)$ by the AP at the unit price λ, despite the fact that its communication satisfaction is reduced to $(1-k)T_{2a}(p_2)/p_2$. The forwarding-incentive part of the current pricing mechanism is implemented through the reimbursement $\mu \min\{kT_{2a}(p_2), T_{12}\}$ to user 2 for its *net* forwarding which cannot exceed T_{12}, the amount user 1 is sending over for forwarding. Combining all the pieces, the optimization problem of user 2 is Puser2

$$\begin{aligned} \max \quad & \frac{(1-k)T_{2a}(p_2)}{p_2} - \lambda T_{2a}(p_2) \\ & + \mu \min\{kT_{2a}(p_2), T_{12}\} \\ \text{subject to} \quad & 0 \le p_2 \le p^{\max} \text{ and } 0 \le k \le 1/2. \end{aligned} \tag{5}$$

It can be observed readily that reducing k slightly when $kT_{2a}(p_2) > T_{12}$ increases user 2's net utility. Thus, similar to Puser1, Puser2 can be reformulated as

Theorem 3.2:

$$\text{Puser2: } \max \frac{(1-k)T_{2a}(p_2)}{p_2} - \lambda T_{2a}(p_2) + \mu k T_{2a}(p_2)$$
$$\text{s.t. } 0 \le p_2 \le p^{\max}, 0 \le k \le 1/2, kT_{2a}(p_2) \le T_{12}. \tag{6}$$

The constraint $k T_{2a}(p_2) \le T_{12}$ is a direct consequence of the fact that user 2 is not reimbursed for relaying more than user 1's forwarding request.

With the definitions $F(\xi) \triangleq f(\xi/N_0W), \forall\xi$, and $\ell \triangleq W_{12}/W$, we finally express our user optimizations as

$$\begin{aligned} &\text{Puser1}(\lambda, h_{1a}, h_{12})\text{: } \max U_1^{\text{net}}(p_1, \ell) \\ &\quad \equiv W\left(\frac{1}{p_1} - \lambda\right)[(1-\ell)F(h_{1a}p_1) + \ell F(h_{12}p_1)] \\ &\text{s.t.} \begin{cases} 0 \le p_1 \le p^{\max}, 0 \le \ell \le 1, \\ \ell F(h_{1a}p_1) \le k F(h_{2a}p_2) \end{cases} \end{aligned} \tag{7}$$

and

$$\begin{aligned} &\text{Puser2}(\lambda, \mu, h_{2a})\text{: } \max U_2^{\text{net}}(p_2, k) \\ &\quad \equiv W\left[(1-k)\left(\frac{1}{p_2} - \lambda\right) + k(\mu - \lambda)\right] F(h_{2a}p_2) \\ &\text{s.t.} \begin{cases} 0 \le p_2 \le p^{\max}, 0 \le k \le 1/2, \\ \ell F(h_{1a}p_1) \ge kF(h_{2a}p_2) \end{cases}. \end{aligned} \tag{8}$$

The simultaneous executions of both user problems constitutes a *noncooperative game* [21], implementing distributed power control at every AP broadcasted unit price λ and unit reimbursement μ. The notations Puser1$(\lambda, h_{1a}, h_{12})$ and Puser2(λ, μ, h_{2a}) emphasize the fact that this noncooperative game is a function of λ and μ.

In order to characterize the Nash equilibria (if any) of the above game, we first need the following preliminary characterizations of the solutions of Puser1 and Puser2. As $U_1^{\text{net}}(p_1, \ell)$ and $U_2^{\text{net}}(p_2, k)$ are linear in ℓ and k, respectively, they can only have maxima at $\ell = 0, 1$ and $k = 0, 1/2$, respectively. Each $U_1^{\text{net}}(p_1, \ell = 0)$ and $U_1^{\text{net}}(p_1, \ell = 1)$ has

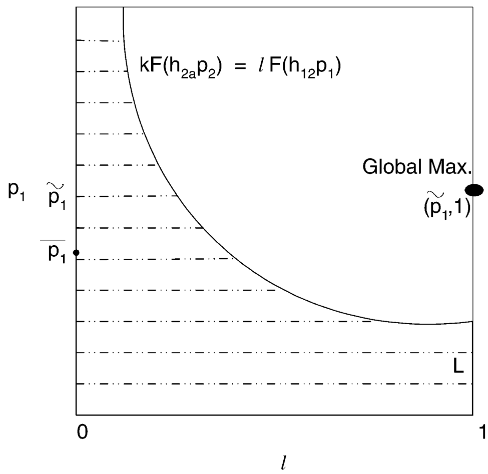

Fig. 3. Shaded region is the strategy space (allowed values of p_1 and ℓ) of user 1. The excluded region (unshaded) is due to the bottleneck constraint imposed by user 2, which gives rise to the equation of the constraint-saturation boundary shown.

precisely one maximum,[4] and likewise for $U_2^{\rm net}(p_2, k = 0)$ and $U_2^{\rm net}(p_2, k = 1/2)$. Denote the maximizers of $U_1^{\rm net}(p_1, 0)$ and $U_1^{\rm net}(p_1, 1)$ as $\bar{p}_1$ and $\tilde{p_1}$, respectively, and those of $U_2^{\rm net}(p_2, 0)$ and $U_2^{\rm net}(p_2, 1/2)$ as $\bar{p}_2$ and $\tilde{p_2}$, respectively. Note that $\bar{p}_1, \tilde{p_1}, \bar{p}_2$, and $\tilde{p_2}$ are all functions of λ and μ. We are now ready to give geometric criteria (criteria on link gain values) for forwarding solutions of the user problems. Suppose (p_{1*}, ℓ_*) and (p_{2*}, k_*) individually solve Puser1 and Puser2, respectively. (Note that they are not necessarily at the Nash equilibrium.)

Lemma 3.1: If $h_{12} > h_{1a}$ and $kF(h_{2a}p_2) \gneq 0$, then either $(p_{1*}, \ell_*) = (\tilde{p_1}, 1)$ when $F(h_{12}\tilde{p_1}) \leq kF(h_{2a}p_2)$, or $\ell_* F(h_{12}p_{1*}) = kF(h_{2a}p_2)$ when $F(h_{12}\tilde{p_1}) \geq kF(h_{2a}p_2)$. If $h_{12} < h_{1a}$ or $k\,F(h_{2a}p_2) = 0$, then $\ell_* = 0$. (We do not consider the $h_{12} = h_{1a}$ case, since ℓ_* is undetermined there, and the chance of that case is vanishing.)

Proof: $U_1^{\rm net}(p_1, \ell)$ being linear in ℓ dictates that $\ell_* = 0, 1$, or $\ell_* F(h_{12}p_{1*}) = c$, where $c \triangleq kF(h_{2a}p_2)$. The sizes of the coefficients of $(1 - \ell)$ and ℓ in (7) depend on the relative sizes of h_{12} and h_{1a}. When $h_{12} < h_{1a}$, the coefficient of the $(1 - \ell)$ term is larger ($F(\cdot)$ is strictly increasing), and $\ell_* = 0$; else, the coefficient of the ℓ term is larger, and $\ell_* \neq 0$. In the latter case, as $\ell F(h_{12}p_1)$ is increasing in both ℓ and p_1, if $1 \cdot F(h_{12}\tilde{p_1}) \leq c, (p_{1*}, \ell_*) = (\tilde{p_1}, 1)$; otherwise, the constrained maximum is either on $F(h_{12}\tilde{p_1}) = c$ or on the line segment L in Fig. 3, from $(p_1, \ell) = (0, 1)$ to the intersection of $\ell = 1$ and $F(h_{12}\tilde{p_1}) = c$. The highest value of $U_1^{\rm net}$ on this line segment must be at the intersection because $U_1^{\rm net}(p_1, \ell = 1)$ has a unique stationary point at the excluded point $(\tilde{p_1}, 1)$, which is also a maximum. □

The first part of the lemma states that if user 1 prefers to be forwarded through user 2 (due to $h_{12} > h_{1a}$) and user 2 is willing to forward and is imposing a nonzero bottleneck constraint, then user 1 either attains its global maximum net utility if the bottleneck allows it, or the best net utility given the constraint, both involving nonzero forwarding by user 2. If the user 2 chooses not to forward for user 1, then user 1 will choose a direct connection to the AP.

A similar argument with the fact that when $\mu > 1/\bar{p}_2, (\partial U_2^{\rm net}/\partial k)(\bar{p}_2, 0) = (\mu - 1/\bar{p}_2)F(h_{2a}\bar{p}_2) > 0$ rejects $k_* = 0$ yields.

Lemma 3.2: If $\mu > 1/\bar{p}_2$ and $\ell\,F(h_{12}p_1) \gneq 0$, then either $(p_{2*}, k_*) = (\tilde{p_2}, 1/2)$ when $(1/2)F(h_{2a}\tilde{p_2}) \leq \ell F(h_{12}p_1)$, or $k_* F(h_{2a}p_{2*}) = \ell F(h_{12}p_1)$ when $1/2F(h_{2a}\tilde{p_2}) \geq \ell F(h_{12}p_1)$. When $\mu < \lambda$ or $\ell\,F(h_{12}p_1) = 0, k_* = 0$. □

While user 1's destination preference is not directly affected by the level of reimbursement μ (but directly affected by its link gains), user 2's forwarding preference is. This lemma can be interpreted as: when μ is high enough (note that what level is high enough depends on network geometry because $\bar{p}_2$ depends on h_{2a}) and when user 1 does have forwarding request, then user 2 achieves a constrained net-utility maximum with nonzero level of forwarding, otherwise, user 2 chooses a nonforwarding net-utility maximum.

Nash Equilibria of Users: Given λ and μ, if the individual maximization attempts of users settle down, they must have reached a *Nash equilibrium* [21] with equilibrium strategy vector $(p_1^*, \ell^*, p_2^*, k^*)$ at which no user can increase its net utility by unilaterally changing its own strategy. That is

$$\begin{aligned} (p_1^*, \ell^*) &= \arg\max U_1^{\rm net}(p_1, \ell) \text{ s.t. } 0 \leq p_1 \leq p^{\max}, \\ &\quad 0 \leq \ell \leq 1, \quad \ell F(h_{12}p_1) \leq k^* F(h_{2a}p_2^*) \\ (p_2^*, k^*) &= \arg\max U_2^{\rm net}(p_2, k) \text{ s.t. } 0 \leq p_2 \leq p^{\max}, \\ &\quad 0 \leq k \leq 1/2, \quad \ell^* F(h_{12}p_1^*) \geq kF(h_{2a}p_2). \end{aligned} \quad (9)$$

If this game has multiple Nash equilibria for general λ and μ (and h_{1a}, h_{12}, and h_{2a}), we need to design an algorithm to reach the most *Pareto-superior* equilibrium. A utility vector $\mathbf{u}^b \triangleq (u_1^b, \ldots, u_N^b)$ is said to be Pareto superior to $\mathbf{u}^a \triangleq (u_1^a, \ldots, u_N^a)$ if $u_i^b \geq u_i^a, \forall i$. The rest of this subsection works toward characterizing the spectrum of all Nash equilibria and designing an algorithm to reach the most Pareto superior one.

Since an equilibrium strategy vector $(p_1^*, \ell^*, p_2^*, k^*)$ must satisfy the two opposing bottleneck constraints of Puser1 and Puser2 [(7)–(9)] $\ell^* F(h_{12}p_1^*) = k^* F(h_{2a}p_2^*)$. We call this the equilibrium constraint of the equilibrium and denote it c^*. We also refer to the equilibrium itself as c^*. The important points are that all equilibria can be parameterized by their equilibrium constraints and be *totally*[5] ordered by virtue of the following lemma.

Lemma 3.3: Given two Nash equilibria of the game with equilibrium constraints c_1^* and c_2^*, if $c_2^* \geq c_1^*$, then the equilibrium net-utility vector at c_2^* is Pareto superior to that at c_1^*. □

Proof: First note that if the noncooperative game has an equilibrium with equilibrium constraint c_1^*, then the game subject to the additional constraints (call them C_1) $\ell F(h_{12}p_1) \leq c_1^*$ and $kF(h_{2a}p_2) \leq c_1^*$ still has the same equilibrium. Let C_2 be the analogous constraints corresponding to c_2^*. Although C_1 and C_2 do not change the equilibria, they do restrict the strategy space (the values of possible p_1, ℓ, p_2, k) of the game differently. Since $c_1^* \leq c_2^*$ and $\ell\,F(h_{1a}p_1)$ is increasing in both ℓ and p_1 and $kF(h_{2a}p_2)$ is increasing in both k and p_2, the restriction by C_2 cannot be stronger than that by C_1. Therefore, the equilibrium

[4]Strictly speaking, $U_2^{\rm net}(p_2, k = 1)$ has a maximum only with the constraint $p_2 \leq 1$ imposed, but we always operate inside the strategy space [15].

[5]Note that unlike the real numbers, any two vectors cannot always be compared, for example, neither one of the utility vectors (1, 2) and (2, 0) is Pareto superior to the other. However, in our game consisting of Puser1 and Puser2, we can order all equilibria in the Pareto-superior sense.

net utilities achieved by the game with C_1 cannot be higher than the corresponding ones achieved by the game with C_2. □

We can see that as a result of the opposing bottleneck constraints of Puser1 and Puser2 and the fact that $\ell F(h_{1a}p_1)$ is increasing in both ℓ and p_1 and $kF(h_{2a}p_2)$ is increasing in both k and p_2, as the game continues, the strategy space cannot increase and, hence, the maximum values of the net utilities subject to the bottleneck constraints cannot increase. Further, since the unconstrained maximizers of U_1^{net} are $(\bar{p}_1, 0)$ and $(\tilde{p_1}, 1)$, and those of U_2^{net} are $(\bar{p}_2, 0)$ and $(\tilde{p_2}, 1/2)$, these four points define the bounds on Pareto superiority.

Lemma 3.4: The equilibrium constraint c^* of a Nash equilibrium satisfies $\bar{c} \leq c^* \leq \tilde{c}$, where

$$\tilde{c} \triangleq \min\{F(h_{12}\tilde{p_1}), (1/2)F(h_{2a}\tilde{p_2})\} \quad (10)$$
$$\bar{c} \triangleq \min\{0 \times F(h_{12}\bar{p}_1), 0 \times F(h_{2a}\bar{p}_2)\} = 0. \quad (11)$$

$\tilde{c}$ is the equilibrium constraint of the most Pareto superior *possible* equilibrium if it exists. $\bar{c}$ is the equilibrium constraint of the nonforwarding ($\ell^* = k^* = 0$) or the "zero" equilibrium which always exists. Therefore, a Nash equilibrium always exists, and must be the zero equilibrium if unique. We have found the spectrum of all possible Nash equilibria; the logical question now is whether all of this spectrum correspond to *actual* Nash equilibria. The following almost complete criterion (complete in the sense that it covers all values of h_{1a} and h_{12}, and almost all values of μ and λ) for the existence of Nash equilibria offers an affirmative answer.

Theorem 3.3: If $h_{12} > h_{1a}$ and $\mu > 1/\bar{p}_2(\lambda)$, then each value from $\bar{c}$ to $\tilde{c}$ corresponds to a Nash equilibrium. When $h_{12} < h_{1a}$ or $\mu < \lambda$, only the nonforwarding (zero) equilibrium exists. □

Proof: Assume $h_{12} > h_{1a}$ and $\mu > 1/\bar{p}_2$. Given an $x \in [\bar{c}, \tilde{c}]$, consider a game with extra constraints $\ell F(h_{12}p_1) \leq x$ and $kF(h_{2a}p_2) \leq x$. A Nash equilibrium with equilibrium constraint x exists by Lemmas 3.1 and 3.2. Now, lift the two extra constraints. The users will not change their strategies because each will be imposing a constraint with value x on the other. We have produced a Nash equilibrium with equilibrium constraint x. When $h_{12} < h_{1a}$, user 1 necessarily chooses $\ell = 0$ [see (7)], which forces user 2 to choose $k = 0$. When $\mu < \lambda$, user 2 necessarily chooses $k = 0$ [see (8)], which forces user 1 to choose $\ell = 0$. Hence, only the zero equilibrium is allowed in these two cases. □

Note that $\bar{p}_2$ depends on λ. As the gap between $1/\bar{p}_2$ and λ is small for practical values of M or λ [15], the second part of the theorem is almost a converse. The two parts of the criteria have a nice interpretation: h_{12} and h_{1a} controls the forwarding and destination preference of user 1, while the size of μ affects whether user 2 finds it worthwhile to forward.

Since the most Pareto-superior possible equilibrium corresponds to $\tilde{c}$, starting from the global optima of U_1^{net} and U_2^{net} guarantees reaching the most Pareto-superior Nash equilibrium (which could be Pareto-inferior to $\tilde{c}$ by Theorem 3.3).

Theorem 3.4: For the noncooperative power control game given by Puser1 and Puser2, starting from the initial strategy vector $(\tilde{p_1}, 1, \tilde{p_2}, 1/2)$, an iterative round-robin algorithm for this game converges to the most Pareto-superior Nash equilibrium. □

Proof: As was mentioned before Lemma 3.4, the game strategy space is nonincreasing as the game progresses. Initializing the game at $(\tilde{p_1}, 1, \tilde{p_2}, 1/2)$ guarantees the largest initial strategy space. The Nash equilibrium is reached when one user's constraint optimization does not reduces the strategy space. □

This theorem describes an algorithm for reaching the most Pareto-superior Nash equilibrium, which is used to implement the noncooperative game that produces our numerical results. Since feedback is required for the users to be aware of the bottleneck constraints imposed by the other user, this initial condition can be interpreted as the consequence of the delay of these feedbacks at the beginning of the noncooperative game.

To recap, Lemmas 3.1 and 3.2, characterizing the individual solutions to Puser1 and Puser2, are needed for the later theorems concerning Nash equilibria. Lemma 3.3 and Theorem 3.3 provide the range of possible Nash equilibria and how they are related to channel conditions (or network geometries) and the unit price and unit reimbursement. The second part of Theorem 3.3 states when the Nash equilibrium is unique. Finally, Theorem 3.4 describes how to reach the most Pareto-superior equilibrium.

For every unit price λ and unit reimbursement μ broadcasted by the access point, users react by engaging in a noncooperative power control game with each other. By taking into account their utilities, payments, and reimbursements, the users produce the most Pareto-superior equilibrium strategy vector $\mathbf{s}^*(\lambda, \mu) \triangleq (p_1^*(\lambda, \mu), \ell^*(\lambda, \mu), p_2^*(\lambda, \mu), k^*(\lambda, \mu))$. The user noncooperative game can be thought of as a function returning $\mathbf{s}^*(\lambda, \mu)$ for every λ and μ.

B. Access Point (Network) Optimization

For any unit price λ and unit reimbursement μ, the AP evaluates its revenue as

$$\rho(\lambda, \mu) = W\{\lambda[(1-\ell^*)F(h_{1a}p_1^*) + \ell^* F(h_{12}p_1^*) + F(h_{2a}p_2^*)] - \mu k^* F(h_{2a}p_2^*)\} \quad (12)$$

by knowing the users' responses[6] $\mathbf{s}^*(\lambda, \mu)$ to λ and μ. The AP searches over $\lambda \geq 0$ and $\mu \geq 0$ for its maximum revenue ρ^* at λ^* and μ^*

$$\text{Pnet}(h_{1a}, h_{12}, h_{2a}) \colon \max_{\lambda \geq 0, \mu \geq 0} \rho(\lambda, \mu). \quad (13)$$

Our self-organization problem is formulated as the joint optimization problem $\text{Pnet}(h_{1a}, h_{12}, h_{2a})$, which has similarities to a Stackelberg (leader–follower) game and is summarized in Fig. 4. The arguments of Pnet() remind us that this self-organization problem (and, hence, its solution) depends on the network geometry defined by the three path gains. At every iteration of the self-organization, the Stackelberg-leader like AP announces λ and μ. The two users engage in the noncooperative game consisting of $\text{Puser1}(\lambda, \mu)$ and $\text{Puser2}(\lambda, \mu)$ and produce their responses in the form of an equilibrium strategy vector $\mathbf{s}^*(\lambda, \mu) \equiv (p_1^*(\lambda, \mu), \ell^*(\lambda, \mu), p_2^*(\lambda, \mu), k^*(\lambda, \mu))$. The AP evaluates its revenue using the expression in (12). The AP then tries another pair of λ and μ for potentially higher revenue. The interaction between AP and users stops when the revenue is

[6]The λ and μ dependencies of ℓ^*, p_1^*, k^*, and p_2^* are not shown for notational simplicity.

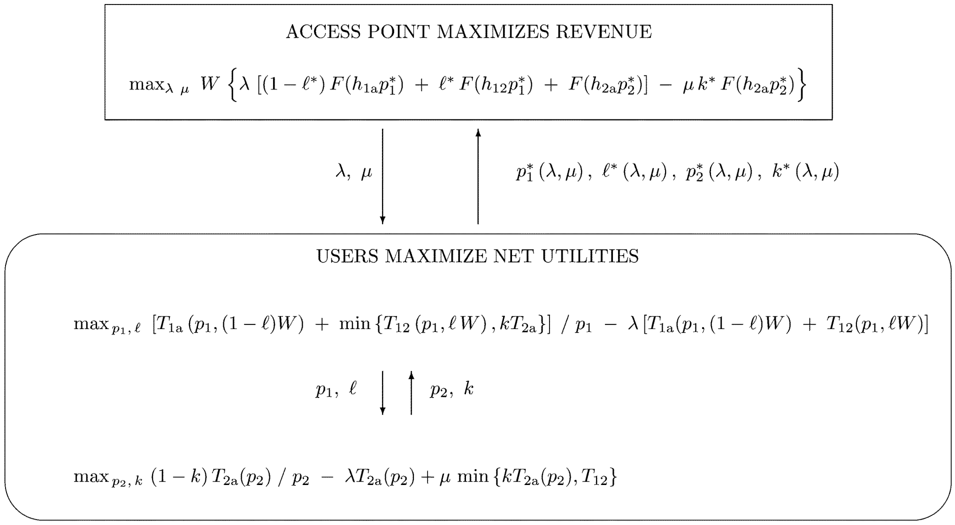

Fig. 4. Pricing-based self-organization as a joint optimization problem (for two users and one AP).

maximized. Here, we implicitly assume that the AP waits for the noncooperative game to converge (which takes only two iterations for two users) before announcing another pair of λ and μ.

The existence of maximum revenue can be established without solving Pnet.

Theorem 3.5: The revenue function $\rho(\lambda,\mu)$ has the following properties.

1) $\rho\to 0$ as $\lambda\to\infty$ for any μ.
2) For any finite λ, when $\mu\to\infty$, either $\rho\to-\infty$ or ρ is independent of μ.
3) $\lim_{\mu\to\infty}\lim_{\lambda\to\infty}\rho\ =\ 0$ and $\lim_{\lambda\to\infty}\lim_{\mu\to\infty}\rho\ =\ -\infty$ or 0
4) $\rho>0$ when $\mu=0$ and λ finite. □

Proof: First consider property 1). The proof in [15, Appendix C] indicates that payments and equilibrium transmit powers both vanish when $\lambda\ \to\ \infty$, because of bottleneck constraints, $k^*\ =\ 0$, leading to zero reimbursement and, hence, zero revenue. For property 2), fix a finite λ, consider increasing μ from zero to infinity. If user 2 is forwarding at zero μ, increasing μ can only increase its forwarding tendency. Boundless increase in reimbursement leads to $\rho\to-\infty$. If user 2 is not forwarding at $\mu=0$, and remains so even for arbitrarily large μ, then ρ is independent of μ (as $k^*\ =\ 0$), otherwise, $\rho\to-\infty$. Property 3) follows directly from property 1) and 2). Property 4) is evident from (12). □

Property 1) simply means users cannot afford exceedingly high prices. Property 2) means if the potential forwarder is actually forwarding, boundless increase in reimbursement bankrupts the AP. If the potential forwarder never forwards despite large potential incentive, then reimbursement is irrelevant to the revenue. Property 4) shows that positive revenue is attainable.

From the above, since arbitrarily large λ and μ are not desirable in terms of the revenue for AP (in the case of $\mu\to\infty$ but ρ independent of μ, there is no reason to set μ at ∞), we conclude the following.

Corollary 3.1: There exists a pair of finite λ^{**} and μ^{**} that maximizes the network revenue $\rho(\lambda,\mu)$, and the maximum revenue $\rho(\lambda^{**},\mu^{**})$ is finite and positive. The finiteness of maximum revenue comes from the boundedness of $F(\,\cdot\,)$.

IV. Numerical Results for Two Users and One AP

The solution of our pricing-based self-organization is ultimately a function of all the path gains $\mathbf{h}\triangleq(h_{1a},h_{12},h_{2a})$. Ideally, one would be able to find simple expressions for the unit price, unit reimbursement, and equilibrium user strategy vector, all at maximum access point revenue, as functions of $\mathbf{h}$. This goal so far seems unrealistic and we turn to numerical calculations instead.

A. When to Forward, and Is It Efficient?

Consider planar network geometries in which user 2 is fixed at the origin and the AP is at 5 m north of user 2, we observe the forwarding behavior at different locations of user 1. Enough samples of user 1 locations are taken to map out regions in which the system is forwarding or nonforwarding. Other parameters used in the simulations are $L=64, M=80, \mathrm{BER}(\gamma)=(1/2)\exp(-\gamma/2)$ [for noncoherent frequency shift keyed (FSK)], bandwidth $W=10^6$ Hz, noise variance $N_0W\ =\ 5\times10^{-15}$ Watts, and a path gain formula given by $h=0.1/d^2$, where d is the distance between the transmitter and receiver in meters.

Fig. 5 shows, in real- and path-gain spaces, the regions in which, under pricing, the network converges to forwarding and nonforwarding (only direct communications with the AP is possible) networks at maximum revenue. The levels of destination and forwarding preferences, ℓ^{**} and k^{**}, at maximum revenue for the horizontal origin-passing cut of Fig. 5(a) is shown in Fig. 6. We observe that these results are consistent with both our intuition about when forwarding is beneficial and Theorem 3.3.

When user 1 is far away from user 2, the channel from user 1 to AP and that from user 1 to user 2 are very similar, and we do not expect forwarding to be needed. This corresponds to the area outside of the closed curve in Fig. 5(a) and to the right side of Fig. 6. In the path-gain space, this regime corresponds to $h_{1a}\ \approx\ h_{12}\ \ll\ 1$, or the lower left corner of Fig. 5(b). As user 1 gets closer to user 2 (relative to the AP), i.e., $h_{1a}<h_{12}$, user 1 prefers forwarding, which, according to Figs. 5(a) and 6, is supported by pricing with reimbursement. The explanation

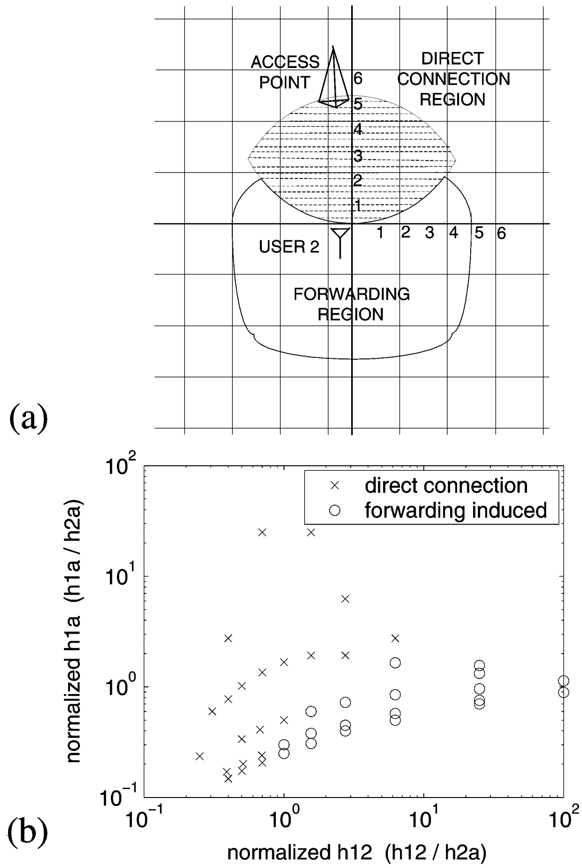

Fig. 5. (a) User 2 is located at the origin. When user 1 is within the closed curve (excluding the shaded region), the network converges to one in which user 1 forwards through user 2; otherwise, both users 1 and 2 have only direct connects to the access point. Note that since user 1 is always the nonforwarder and user 2 is always the potential forwarder, user 1 never locates inside the shaded sector because our initial routing would have reversed their roles and, hence, their indices. Equivalent data points for the role reversed cases are already present here. (b) Data used to construct (a) in the path-gain space. All path gains are normalized by h_{2a}.

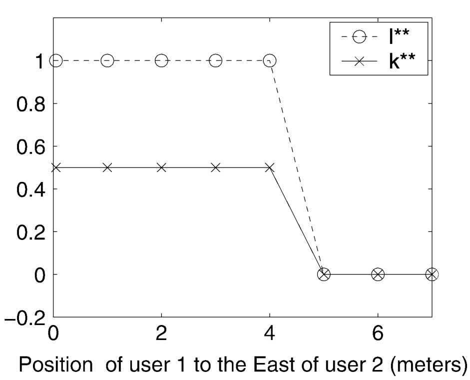

Fig. 6. Levels of destination and forwarding preferences ℓ^{**} and k^{**} of user 1 and user 2, respectively, at maximum network revenues (over λ and μ at all user 1 locations shown) along the horizontal cut of Fig. 5(a) that passes through the origin. Note that only data to the east of user 2 are shown, the data to the west of user 2 are symmetrical.

is that if h_{12} is large enough, the AP can derive much revenue from the strong willingness to pay by user 1 given such a good channel. AP, therefore, increases μ so that user 2 will be willing to increase its power and, hence, increase the throughput of the channel between it and the AP to allow user 1 to have a good net throughput. Note that this argument is only valid when both h_{12} and h_{1a} are not too small to enter into the last regime corresponding to the lower left portion of Fig. 5(b). The data points in that portion are actually outside of the plot in Fig. 5(a). Together with Theorem 3.3, our results suggest that only when $h_{12} > h_{1a}$ and both above certain unknown threshold, does the AP find it sufficiently profitable to increase μ enough to induce forwarding. Finally, when user 1 is closer to the AP than to user 2 ($h_{1a} > h_{12}$), forwarding ought not to occur, since it makes more sense for user 1 to communicate with the access point directly. This trend is clearly borne out by our numerical results [above the $h_{12} = h_{1a}$ line in Fig. 5(b)] and is unambiguously predicted by Theorem 3.3.

Finally, we compare our current forwarding system to the corresponding nonforwarding system with zero forwarding incentive, i.e., with μ set identically to zero. When $\mu \equiv 0$, user 2 chooses $k \equiv 0$ necessarily, user 1 then chooses $\ell \equiv 0$ necessarily, and the revenue simplifies to $\lambda W\,[F(h_{1a}p_1^*) + F(h_{2a}p_2^*)]$. The data at maximum revenues over λ and μ of these two systems, at each user 1 location on the horizontal origin-passing line in Fig. 5(a), is shown in Fig. 7. The revenue and U_1^{net} are much higher (note the log scales) in the forwarding system than in the nonforwarding system. U_2^{net} is generally higher for the forwarding system except at a few data points. The reason for this is that the incentive mechanism proposed here is dominated by the network's desire to maximize its revenue. As a result, there are instances where some of the users may not benefit from the incentives provided. Every scheme to incentivize forwarding starts benefiting the parties at different network geometries and we generally cannot expect all parties to begin to benefit from a forwarding scheme at the same geometry. The point here is that incentivizing forwarding *does* benefit all parties (often significantly) at most geometries.

More importantly, the reimbursement mechanism increases the system-wide communication efficiency metric aggregate bits-per-Joule $\sum_i (T_{ia}^{(\text{eff})}/p_i)$ (see Fig. 8), at maximum AP revenue, despite the fact that users are not cooperating in reaching their equilibrium strategies and the AP does not have direct concern for user utilities.

V. Multinode and Single AP

It is easy to appreciate the likely intractability that will result from a complete generalization of our simple network considered so far. For instance, a full generalization requires the possibility for a node to use all other nodes as potential forwarders for its data. Indeed, allowing this particular option effectively requires pricing to *derive* routing. However, we have seen in the two-users-one-AP case that the nonforwarder almost always sends packets either exclusively to the AP or exclusively to the forwarding node at maximum revenue, i.e., $\ell^{**} = 1$ or 0 in almost all cases (Fig. 6). We draw on this observation and stipulate that even in the multinode setting, a node i either sends all its data to another node (its "next node" denoted as n_i) or to the unique access point AP, corresponding to $\ell_i = 1$ or 0, respectively. A node still has the option to share $k_i \in [0, 1/2]$ fraction of its throughput to the AP with the incoming stream

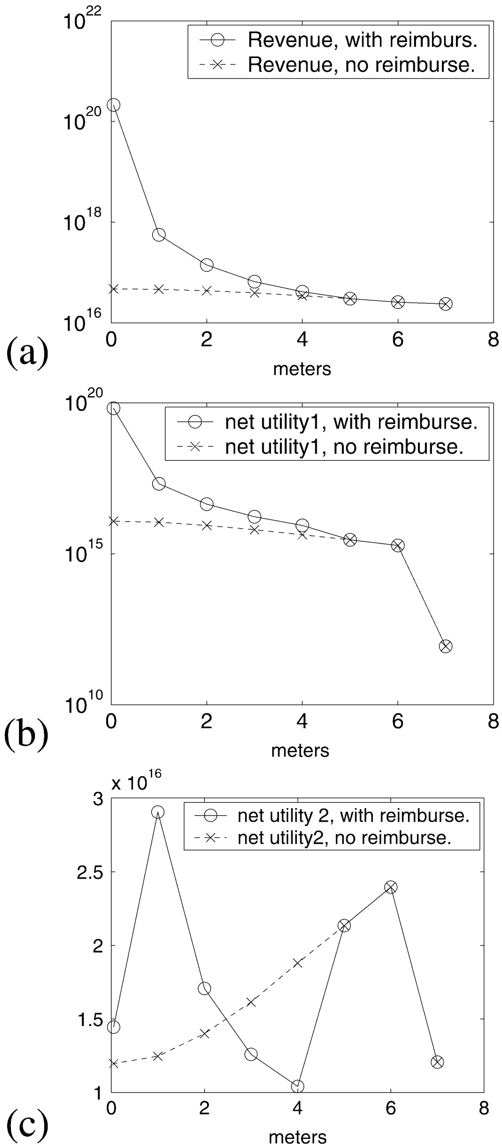

Fig. 7. (a) Revenue ρ, (b) net utility of user 1 $U_1^{\rm net}$, and (c) net utility of user 2 $U_2^{\rm net}$, (at maximum revenues over λ and μ for every user 1 positions shown) for both pricing mechanisms with and without reimbursements versus the position of user 1 to the east of user 2. Note that only eastward data are shown because the data to the west of user 2 are symmetrical.

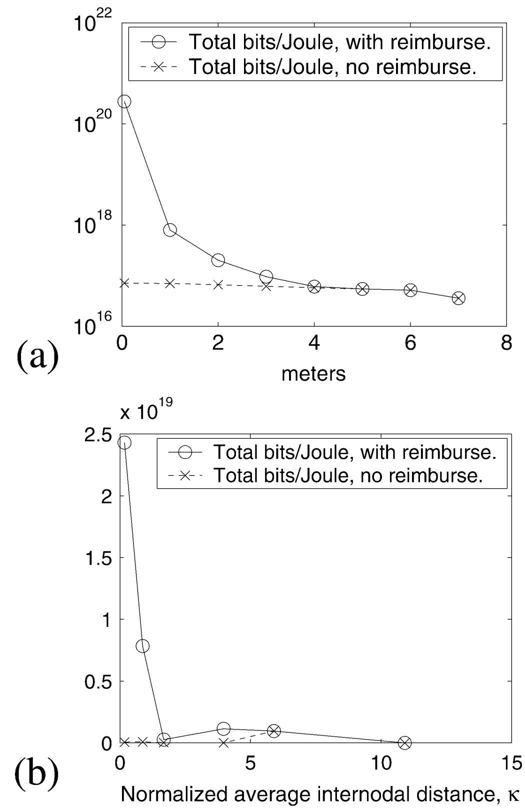

Fig. 8. Comparison of aggregate bits-per-Joule (which is also the aggregate utility), $\sum_i(T_{ia}^{(\rm eff)}/p_i)$, at maximum AP revenue in the forwarding (with reimbursement, where the network maximizes revenue over μ) and nonforwarding (without reimbursement, where μ is always zero) systems. (a) Two-user-one-AP case. Data to the west of user 2 are symmetrical. (b) Six-user-one-AP case, using a random subset of experiments presented in Fig. 10. Whenever the forwarding and nonforwarding systems yield identical aggregate bits-per-Joule, the network in the forwarding system converges to an architecture that does not induce forwarding, in which every node connects to the AP directly. The difference in aggregate bits-per-Joule between the forwarding and nonforwarding systems is roughly proportional to the difference in revenue between them (result not shown).

$T_i^{\rm IC}$ (could be from multiple nodes). Our simpler generalization requires an initial routing assignment before the pricing algorithm can be completely specified, i.e., the implementation of pricing is initial-routes dependent. However, after the pricing algorithm converges at maximum AP revenue, the routes will likely be modified depending on whether they are supported by the converged destination and forwarding preferences ℓ_i^{**} and k_i^{**}, respectively, $\forall i$.

Typically, the throughput of the link between node i and $n_i, T_{in_i}(h_{in_i}p_i)$, is higher than the net multihop throughput from node i to the AP because of the bottleneck $T_{in_i}^{\rm BN}$ from node n_i to the AP, which in turn depends on the degrees of cooperation and link qualities of the nodes downstream. For example, if data from node 1 routes through nodes 2, 3, and 4 to the AP, then $T_{12}^{\rm BN} = \min\{k_2T_{23},\ k_2k_3T_{34},\ k_2k_3k_4T_{4a}\}$. With the same rules for charging and reimbursing, the optimization performed by node i is

$$\begin{aligned}\text{Puser}_i\text{:}\ \max\ & \frac{(1-k_i)\min\left\{T_{in_i}\left(h_{in_i}p_i\right), T_{in_i}^{\rm BN}\right\}}{p_i}\\ &-\lambda T_{in_i}\left(h_{in_i}p_i\right)\\ &+\mu\min\left\{T_i^{\rm IC}, k_i\min\left\{T_{in_i}\left(h_{in_i}p_i\right), T_{in_i}^{\rm BN}\right\}\right\}\\ \text{s.t. } & 0\le p_i\le p^{\max},\quad 0\le k_i\le 1/2,\quad \ell_i=0,1. \qquad (14)\end{aligned}$$

Theorem 5.1: Puser_i simplifies to

$$\begin{aligned}&\max U_i^{\rm net}(p_i,\ell_i,k_i)\equiv\left(\frac{1-k_i}{p_i}-\lambda+\mu k_i\right)T_{in_i}\left(h_{in_i}p_i\right)\\ &\text{s.t. } 0\le p_i\le p^{\max},\ 0\le k_i\le 1/2, \ell_i=0,1,\\ &T_{in_i}\left(h_{in_i}p_i\right)\le T_{in_i}^{\rm BN}, k_i\,T_{in_i}\left(h_{in_i}p_i\right)\le T_i^{\rm IC}. \qquad (15)\end{aligned}$$

The proof of this theorem is similar to those of Theorems 3.1 and 3.2 in the two-node-one-AP network. Specialization of Puser_i to a source node and a node directly connected to the AP (nodes with $\ell_i = 0$) is achieved by setting $T_i^{\rm IC} \equiv 0$ (which forces $k_i \equiv 0$) and $T_{ia}^{\rm BN} \equiv \infty$, respectively. With the equilibrium strategies p_i^*, ℓ_i^*, and k_i^*, for all the users, the revenue can be expressed as

$$\rho(\lambda, \mu) = \sum_{\text{all users } i} (\lambda - \mu k_i^*) T_{in_i}(h_{in_i} p_i^*). \qquad (16)$$

Note that since $T_{in_i}(h_{in_i} p_i^*)$ satisfies all the constraints in Puser_i, it is both the throughput of the link between node i and n_i and the effective throughput from node i to the AP. Because of the similarities to the two-user-one-AP case, the revenue function $\rho(\lambda, \mu)$ satisfies all the properties listed in Theorem 3.5 even in the multinode setting. A corollary very similar to Corollary 3.1, thus, follows.

Corollary 5.1: There exists a pair of finite λ^{**} and μ^{**} that maximizes the network revenue $\rho(\lambda, \mu)$, and the maximum revenue $\rho(\lambda^{**}, \mu^{**})$ is finite and positive (for finite number of nodes). □

The network is initialized by a heuristic routing algorithm which connects each node to its closest neighbor toward the access point. If all nodes find efficient routes to the AP (e.g., no direct connections to the AP when too far away from AP) through pricing despite the selfish potential forwarders, then pricing has enabled efficient self-organization by providing enough forwarding incentive.

We present numerical experiments for networks with up to six nodes on a quarter plane and one AP at the origin. The node positions are randomly generated with a uniform distribution on the quarter sector with 12-m radius. One such network is shown in Fig. 9(a). Its initial routes and its final routes after pricing are shown in Fig. 9(b) and (c), respectively. Each six-node-one-AP simulation like this becomes one data point in our result shown in Fig. 10. A random sample of these experiments is represented in Fig. 8(b).

We choose the number of forwarded nodes in a network to be a measure of the level of cooperation among nodes, where, a node is considered forwarded if a noticeable part of another node's effective throughput is devoted to forwarding its data. Recall that in the two-user-one-AP case, the forwarding tendency of user 2 was reduced when the internodal distance between it and user 1 was large. Motivated by this observation, we define the "clustering" metric

$$\kappa \triangleq \frac{\text{Average internodal distance}}{\text{Shortest distance between the AP and any other node}}$$

whose denominator can be thought of as a normalization scale for internodal distances.

Fig. 10 empirically suggests that the degree of forwarding at maximum AP revenue is strongly dependent on κ. At large κ (nodes clustering around the AP), nodes are closer to the AP than to each other and they prefer direct connections. This is the basis of the same phenomenon in the two-user network described by Lemma 3.1. At small κ (node cluster is far away from the AP), nodes prefer to forward through each other. While only a six-node-one-AP situation is shown here, our extensive experiments with other numbers of nodes also support the same observation. Further, our results (not shown here) indicate that user payments are proportional to their path gains to AP; therefore, if the last hop is not too long, the revenue potentials of the short forwarding hops can offset the required reimbursements for the network to induce forwarding. If the last hop is too long, the node making the last hop is likely to expend little power, making the last hop a severe bottleneck, which in turn makes reimbursements too expensive for the network. This likely explains the decline of forwarding as κ decreases in the small κ regime.

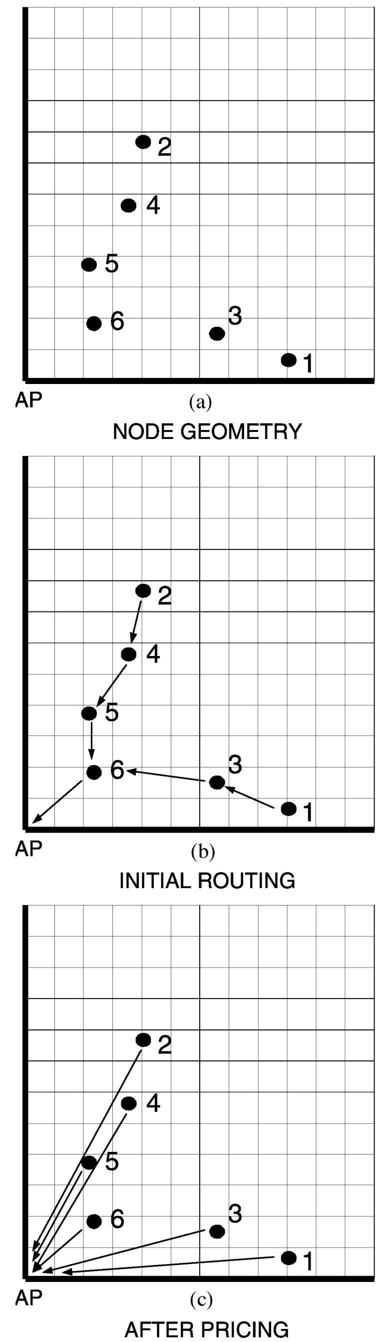

Fig. 9. (a) One network geometry we simulated which resulted in one of the data points in Fig. 10. (b) Initial routes where nodes are connected to its nearest neighbor toward the access point AP (see the relevant footnote 3 on p. 153). (c) Final routes after pricing converges to optimum AP revenue. Note how routing and pricing interact with each other. Pricing implementation depends on initial routing, which in turn can be changed by pricing.

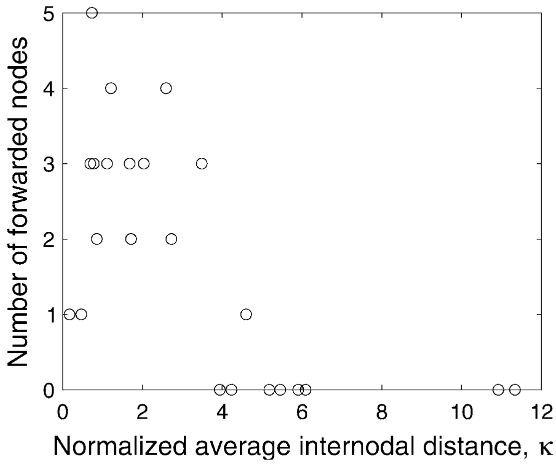

Fig. 10. Dependence of forwarding behavior on κ, the average internodal distance (excluding those involving the AP) normalized by the shortest distance between the AP and any other node. Network tends to forward for small κ (node cluster is far away from the AP) and not to forward at large κ (nodes clustering around the AP).

Finally, similar to the two-user-one-AP case, data in Fig. 8(b) shows that the system with reimbursement mechanism (AP maximizes its revenue over μ) achieves higher aggregate bits-per-Joule than its counter part without reimbursement mechanism ($\mu \equiv 0$).

VI. Conclusion

We have designed a pricing-based joint user-and-network centric incentive mechanism that induces forwarding (and, hence, cooperation) among selfish users by compensating the real and opportunity costs of the forwarders (these costs are captured by their net utility expressions). For the two-node-one-AP network, with a systematic understanding of Nash equilibria, we designed game implementations to achieve the most Pareto superior one. We have shown the network geometric dependence (effect of physical wireless channels) of pricing for forwarding and the superior benefit it brings for all members of the network compared with pricing without forwarding incentive. For the multinode network, forwarding is induced when the normalized average internodal distance κ is small enough, i.e., when the nodes in the network are organized into tight clusters that are sufficiently distant from the access point. The results presented here apply in the context of a static network and provide interesting insights into similar approaches for time-varying mobile networks.

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