Multiradio Channel Allocation in Multihop Wireless Networks

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Abstract—Channel allocation was extensively investigated in the framework of cellular networks, but it was rarely studied in the wireless ad hoc networks, especially in the multihop networks. In this paper, we study the competitive multiradio multichannel allocation problem in multihop wireless networks in detail. We first analyze that the static noncooperative game and Nash equilibrium (NE) channel allocation scheme are not suitable for the multihop wireless networks. Thus, we model the channel allocation problem as a *hybrid game* involving both cooperative game and noncooperative game. Within a communication session, it is cooperative; and among sessions, it is noncooperative. We propose the min-max coalition-proof Nash equilibrium (MMCPNE) channel allocation scheme in the game, which aims to maximize the achieved data rates of communication sessions. We analyze the existence of MMCPNE and prove the necessary conditions for MMCPNE. Furthermore, we propose several algorithms that enable the selfish players to converge to MMCPNE. Simulation results show that MMCPNE outperforms NE and coalition-proof Nash equilibrium (CPNE) schemes in terms of the achieved data rates of multihop sessions and the throughput of whole networks due to cooperation gain.

Index Terms—Multiradio, channel allocation, game theory, Nash equilibria.

1 Introduction

Wireless communication system is often assigned a certain range of communication medium (e.g., frequency band). Usually, this medium is shared by different users through multiple access techniques. Frequency Division Multiple Access (FDMA), which enables more than one users to share a given frequency band, is one of the extensively used techniques in wireless networks [3], [4]. In FDMA, the total available bandwidth is divided permanently into a number of distinct subbands named as *channels*. Commonly, we refer to the assignment of radio transceivers to these channels as the *channel allocation* problem. An efficient channel allocation is essential for the design of wireless networks.

In this paper, we present a game-theoretic analysis of fixed channel allocation strategies of devices that use multiple radios in the multihop wireless networks. Static noncooperative game is a novel approach to solve the channel allocation problem in single-hop networks, and Nash equilibrium (NE) provides an efficient criterion to evaluate a given channel allocation (e.g., in [5]). In multihop networks, however, noncooperative game results in low achieved data rate for multihop sessions and low throughput for whole networks due to the reasons mentioned in Section 4. Hence, we introduce a *hybrid game* involving both cooperative game and noncooperative game into our system in which the players within a communication session are cooperative, and among sessions, they are noncooperative.

We first define the min-max coalition-proof Nash equilibrium (MMCPNE) in this hybrid game, which is aiming to achieve the maximal data rate of all sessions (including single-hop sessions and multihop sessions). We also define three other equilibria schemes that approximate to MMCPNE, named as minimal coalition-proof Nash equilibrium (MCPNE), average coalition-proof Nash equilibrium (ACPNE), and I coalition-proof Nash equilibrium (ICPNE), respectively. Then, we study the existence of MMCPNE in this game and our main result, Theorem 2, shows the necessary conditions for the existence of MMCPNE. Furthermore, we propose the MMCP algorithm which enables the selfish players to converge to MMCPNE from an arbitrary initial configuration and the DCP-x algorithms which enable the players converge to approximated MMCPNE states (e.g., MCPNE, ACPNE, and ICPNE). Finally, we present the simulation results of the proposed algorithms, which show that MMCPNE outperforms NE and coalition-proof Nash equilibrium (CPNE) channel allocation schemes in terms of the achieved data rates of multihop sessions and the throughput of whole networks due to cooperative gain.

The paper is organized as follows: In Section 2, we present related work on channel allocation and channel access in wireless networks. In Section 3, we introduce the system model which contains multihop sessions. In Section 4, we introduce the game-theoretic description of competitive channel allocation problem in multihop wireless networks. In Section 5, we provide a comprehensive analysis of NE and MMCPNE in the channel allocation game. Additionally, we propose several algorithms to reach the exact and approximated MMCPNE state in Section 6. In Section 7, we study their convergence properties and

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present the simulation results of previous algorithms. Finally, we conclude in Section 8.

2 Related Work

There has been a considerable amount of research on channel allocation in wireless networks, especially in cellular networks. Three major categories of channel allocation schemes are always used in cellular networks: fixed channel allocation (FCA), dynamic channel allocation (DCA), and hybrid channel allocation (HCA) which is a combination of both FCA and DCA techniques.

In FCA schemes, a set of channels is permanently allocated to each cell in the network. In general, graph coloring/labeling technique (e.g., in [6]) provides an efficient way to solve the problems of fixed channel allocation. FCA method can achieve satisfactory performance under a heavy traffic load; however, it cannot adapt to the change of traffic conditions or user distributions. To overcome the inflexibility of FCA, many researchers propose dynamic channel allocation methods (e.g., as presented in [7] and [1]). In DCA schemes, in contrast, there is no constant relationship between the cells and their respective channels. All channels are potentially available to all cells and are assigned dynamically to cells as new calls arrive in. Because of its dynamic property, the DCA method can adapt to the change of traffic demand. However, when the traffic load is heavy, DCA method performs worse than FCA due to some cost brought by adaptation. Hybrid channel allocation schemes (e.g., in [8]) are the combination of both FCA and DCA techniques. In HCA schemes, the total number of available channels are divided into fixed and dynamic sets. The fixed set contains a number of nominal channels that are assigned to the cells as in the FCA schemes, whereas the dynamic set is shared by all users in the system to increase flexibility.

Recently, channel allocation problem is becoming a focus of research again due to the appearance of new communication technologies, e.g., wireless local area networks (WLANs), wireless mesh networks (WMNs, e.g., as present in [9] and [10]), and wireless sensor networks (WSNs, e.g., as present in [11] and [12]). Using weighted graph coloring method, Mishra et al. propose a channel allocation method for WLANs in [13]. In WMNs, many researchers have considered devices using multiple radios. Equipping multiple with radios in the devices in WMNs, especially the devices acting as wireless routers, can improve the capacity by transmitting over multiple radios simultaneously using orthogonal channels. In the multiradio communication context, channel allocation and access are also considered as the vital topics. By joint considering the channel assignment and routing problem, Alicherry et al. propose an algorithm to optimize the overall throughput of WMNs in [14].

In the above cited work, the authors make the assumption that the devices cooperate with the purpose of the achievement of high system performance. However, this assumption might not hold for the following two reasons. In one hand, players are usually selfish who would like to maximize their own performance without considering the other players' objective. In the other hand, the full cooperation of arbitrary devices is difficult to achieve due to the transmission distance limitation and transmission interference of neighboring devices.

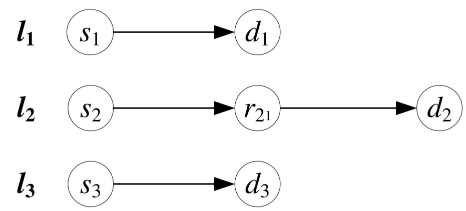

Fig. 1. An example of three communication sessions.

Game theory provides a straightforward tool to study channel allocation problems in competitive wireless networks. As far as know, game theory has been applied to the CSMA/CA protocol [15], [16], to the Aloha protocol [17] and the peer-to-peer (P2P) system [18], [19], [20]. Furthermore, on the basis of graph coloring, Halldorsson et al. use game theory to solve a fixed channel allocation problem in [21]. Unfortunately, their model does not apply to multiradio devices. In wireless ad hoc networks (WANETs), using a static noncooperative game, Felegyhazi et al. analyze the channel allocation strategies of devices that use multiple radios in [5]. However, their results can be only applied to single-hop wireless networks without considering multihop networks. In [22], we have studied the multiradio channel allocation problem in *2-hop* wireless networks. In this paper, we extend the results to the wireless networks with arbitrary hops.

3 System Model and Game Formulation

3.1 System Model

We assume that the available frequency band is divided into M orthogonal channels of the same bandwidth using the FDMA method (e.g., 24 orthogonal channels in case of the IEEE 802.11a protocol). We denote the set of available orthogonal channels by $\mathbf{C} = \{c_1, c_2, \dots, c_M\}$.

In our model, we assume that there exist L communication *sessions*,[1] including multihop and single-hop sessions. We denote the xth session by l_x and the set of all sessions by $\mathbf{L} = \{l_1, \dots, l_L\}$. We further assume that each user participates in only one session, either being a ending user or a relaying user, and thus, we can divide all users into L *disjoint groups*. We denote the xth group by $\mathbf{g}_x$ and the set of all groups by $\mathbf{G} = \{\mathbf{g}_1, \dots, \mathbf{g}_L\}$. Note that each member of $\mathbf{G}$ corresponds to each member of $\mathbf{L}$. We denote the sender, destination, and the ith relaying user in group $\mathbf{g}_x$ (or session l_x) by s_x, d_x, and r_{x_i}, respectively, and we further denote the set of all senders, destinations, and relaying users in *all groups* by $\mathbf{S}$, $\mathbf{D}$, and $\mathbf{R}$, respectively. Fig. 1 presents an example of three communication sessions, where

$$L = 3, \mathbf{L} = \{l_1, l_2, l_3\}, \mathbf{G} = \{\{s_1, d_1\}, \{s_2, r_{2_1}, d_2\}, \{s_3, d_3\}\},$$
$$\mathbf{S} = \{s_1, s_2, s_3\}, \mathbf{D} = \{d_1, d_2, d_3\},$$

and $\mathbf{R} = \{r_{2_1}\}$.

1. In this paper, a session refers to an end-to-end flow (a layer 4 connection).

We assume that all sessions reside in a *single collision domain*, which means that each session will interfere the transmission of all other sessions if they are using the same channel. Note, however, that the users within a session may reside in different collision domains, e.g., in a multihop session.

We assume that each user owns a device equipped with two independent sets of radio transceivers, denoted by Ψ_t and Ψ_r, which used to transmit and receive the data packets, respectively. Each transceivers set contains $K < |\mathbf{C}|$ radio transceivers, all having the same communication capabilities.[2] We assume that the communication between two users is bidirectional and they always have some packets to exchange. Due to the bidirectional communication, transmitter's Ψ_t and receiver's Ψ_r are able to coordinate, and thus, to select the same channels to communicate.

We assume that there is a mechanism that enables the multiple radios within a transceivers set to communicate simultaneously by using orthogonal channels (as it is implemented in [23], for example). We further assume that the total available bandwidth on channel c is shared *equally* among the radios using that channel. This fair rate allocation is achieved, for example, by using a reservation-based TDMA schedule on a given channel. In [24], a similar result was reported by Bianchi for the CSMA/CA protocol. Even if the radio transmitters are controlled by selfish users in the CSMA/CA protocol, they can achieve this fair sharing as shown in [25].

We denote the total available bandwidth on channel c by R_c and the number of radios deployed on channel c by k_c.[3] In theory, R_c is independent of k_c for a TDMA protocol and for the CSMA/CA protocol using optimal backoff window values [24]. In practice, however, due to packet collision caused by nonoptimal backoff window in the CSMA/CA protocol implementation (e.g., in the 802.11 standard) or due to communication overhead induced by control or harmony signal in the TDMA protocol implementation, R_c becomes a decreasing function of k_c for $k_c > 0$. In our model, we assume that an ideal TDMA protocol or optimal CSMA/CA protocal is used, and thus, R_c is independent of k_c. We further assume that all channels have the same bandwidth, i.e., $R_b = R_c,\ \forall b \neq c$. Note that our simulation shows similar results when R_c is a slowly decreasing function of k_c.

To facilitate the reading, we list the major notations used in our model and analysis in Table 1.

TABLE 1
Notations

$\mathbf{L}$	The set of communication sessions (sessions)
$\mathbf{G}$	The set of player groups, each corresponding to a communication session
$\mathbf{Q}$	The set of coalitions, each corresponding to a communication session
$\mathbf{S}, \mathbf{R}, \mathbf{D}$	The set of all senders, relaying users and destinations, respectively
$\mathbf{U}$	The set of players in game, i.e., $\mathbf{U} = \mathbf{S} \bigcup \mathbf{R}$
Ψ_t,Ψ_r	The sets of transceiver radios in each device, each contains K radios
$\mathbf{C}^+$,$\mathbf{C}^-$	The set of channels choosed by relatively more players and less players, respectively
δ^+,δ^-	The number of radios deployed on the channel in $\mathbf{C}^+$ and $\mathbf{C}^-$, respectively
$k_{i,c}$	The number of radios of player i deployed on channel c
k_b, k_c	The total number of radios deployed on channel b and c, respectively, i.e., $k_c = \sum_i k_{i,c}$
k_i	The total number of channels used by player i, i.e., $k_i = \sum_c k_{i,c}$
k_i^+,k_i^-	The number of radios of player i deployed on $\mathbf{C}^+$ and $\mathbf{C}^-$, respectively
$R_{i,c}$	The bandwidth occupied by player i on channel c
R_i	The total bandwidth (utility) occupied by player i
R_i^e	The end-to-end rate (payoff) of player i
$\delta_{b,c}$	The difference of radios umber between channel b and c

3.2 Game Formulation

We refer to each communication link as a selfish *player*. A communication *link* is defined as a direct connection of two users. It is obvious that each N-hop session contains N links and each link contains K pairs of radios. In the example of Fig. 1, there are four communication links, i.e., $s_1 \leftrightarrow d_1$, $s_2 \leftrightarrow r_{2_1}$, $r_{2_1} \leftrightarrow d_2$, and $s_3 \leftrightarrow d_3$, where $i \leftrightarrow j$ denotes a direct connection of users i and j. For the simplicity of presentation, we rewrite player as the user at left-hand side of the link. Thus, the *players set*, denoted by $\mathbf{U}$, can be defined as the set of all senders and relaying users, i.e., $\mathbf{U} = \mathbf{S} \bigcup \mathbf{R}$. Recall the example in Fig. 1, we have $\mathbf{U} = \{s_1, s_2, s_3, r_{2_1}\}$.

We further define *coalition* as the set of players participating in the same session. We denote the xth coalition by σ_x and the set of all coalitions by $\mathbf{Q}$. Note that each member of $\mathbf{Q}$ corresponds to each member of $\mathbf{L}$. In fact, $\mathbf{Q}$ is a partition of $\mathbf{U}$, i.e., $\bigcup_{x=1}^{L} \sigma_x = \mathbf{U}$ and $\sigma_x \bigcap \sigma_y = \emptyset,\ \forall x \neq y$. In the example of Fig. 1, we have $\mathbf{Q} = \{\{s_1\}, \{s_2, r_{2_1}\}, \{s_3\}\}$.

We denote the number of radios of player i using channel c by $k_{i,c}$ for every $c \in \mathbf{C}$. We further denote the set of channels used by player i by $\mathbf{C}_i$. For the sake of suppressing coradios interference in device, we assume that the different radios within a transceiver set cannot use the same channel, i.e., $k_{i,c} \in \{0, 1\}$ for arbitrary player $i \in \mathbf{U}$ and channel $c \in \mathbf{C}$. Each player's *strategy* consists of the number of radios on each of the channels. Hence, we define the strategy of player i as its channel allocation vector:

$$\mathbf{x}_i = \left(k_{i,1}, k_{i,2}, \dots, k_{i,|\mathbf{C}|}\right). \tag{1}$$

The strategy matrix, denoted by $\mathbb{X}$, is defined by all players' strategy vectors:

$$\mathbb{X} = \begin{pmatrix} \mathbf{x}_1 \\ \dots \\ \mathbf{x}_{|\mathbf{U}|} \end{pmatrix}. \tag{2}$$

For the simplicity of presentation, we denote strategy matrix of players set $\mathbf{u} \subseteq \mathbf{U}$ by $\mathbb{X}_{\mathbf{u}}$. We further denote $\mathbb{X}_{-i}$ and $\mathbb{X}_{-\mathbf{u}}$ as the strategy matrix except for the strategy of player i and except for the strategies of players set $\mathbf{u} \subseteq \mathbf{U}$, respectively. Accordingly, strategy matrix $\mathbb{X}$ can be written as $(\mathbf{x}_i, \mathbb{X}_{-i})$ or $(\mathbb{X}_{\mathbf{u}}, \mathbb{X}_{-\mathbf{u}})$.

Fig. 2 presents an example of channel allocation strategy for the system in Fig. 1 with four available channels ($|\mathbf{C}| = 4$) and three radios in each transceiver set (K = 3).

2. In this paper, $|\mathbf{X}|$ is defined as the cardinality of set $\mathbf{X}$. Thus, we can easily find that $|\mathbf{C}| = M$.

3. In fact, R_c can be seen as the maximal aggregate data rate of all radios deployed on channel c.

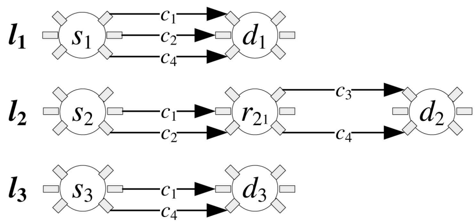

Fig. 2. An example of channel allocation strategy, where $|\mathbf{C}| = 4, |\mathbf{L}| = 3$, and $K = 3$.

The tubers on the left and right of node denote the radios of Ψ_r and Ψ_t, respectively. The number on each radios pair denotes the channel used by this radio. In other words, player s_1 chooses channels c_1, c_2, and c_4, player s_2 chooses channels c_1 and c_2, and so on. Accordingly, the strategies of all players can be easily written as $\mathbf{x}_{s_1} = (1\ 1\ 0\ 1)$, $\mathbf{x}_{s_2} = (1\ 1\ 0\ 0)$, $\mathbf{x}_{s_3} = (1\ 0\ 0\ 1)$, and $\mathbf{x}_{r_{2_1}} = (0\ 0\ 1\ 1)$.

The total number of channels used by player i can be written as $k_i = \sum_{c \in \mathbf{C}} k_{i,c}$, and $k_i \leq K$ obviously. Similarly, the total number of radios using a particular channel c can be written as $k_c = \sum_{i \in \mathbf{U}} k_{i,c}$. In Fig. 2, regarding the number of channels per player, we have $k_{s_1} = 3$ and $k_{s_2} = k_{s_3} = k_{r_{2_1}} = 2$, and regarding the number of radios per channel, we have $k_{c_1} = k_{c_4} = 3$, $k_{c_2} = 2$, and $k_{c_3} = 1$.

As mentioned previously, the total available bandwidth on channel c (i.e., R_c) is shared *equally* among the radios deployed on this channel. We denote $R_{i,c}$ as the available bandwidth occupied by player i on channel c and can write $R_{i,c}$ as

$$R_{i,c} = \frac{k_{i,c}}{k_c} \cdot R_c, \quad \forall i \in \mathbf{U},\ c \in \mathbf{C}. \tag{3}$$

It is easy to see that $R_{i,c} > 0$ for all $c \in \mathbf{C}$, where $k_{i,c} > 0$, if the players do not cheat at the MAC layer (as opposed to the model, for example, in [15]). From (3), we can also find that the higher the number of radios in a given channel is, the lower the bandwidth per radio is.

We define the *utility* of player i as the total available bandwidth occupied by i. In fact, any player's utility is equivalent to its *link rate*. We denote the utility of player i by R_i and can write R_i as

$$R_i = \sum_{c \in \mathbf{C}} R_{i,c}, \quad \forall i \in \mathbf{U}. \tag{4}$$

We define the *payoff* of player i as the achieved data rate of i. In single-hop sessions, the utility of any player reflects its achieved data rate. In multihop sessions, however, the utility of any player may not reflect its achieved data rate, since the actual data rate is bounded by the minimal link rate of the players within the session. Thus, we introduce the concept of *end-to-end rate*, which exactly reflects its achieved data rate in both single-hop session and multihop session. The end-to-end rate of player i, denoted by R_i^e, is defined as the minimal link rate of players within the same session and we can write R_i^e as

$$R_i^e = \min_{u \in \mathbf{g}_x} R_u, \quad \forall i \in \mathbf{U}, \tag{5}$$

where $\mathbf{g}_x$ is the session player i belongs to.[4] For single-hop session, $R_i^e = R_i$ since there is only one player i in the session. For multihop session, the players within a session have the same end-to-end rate.

In the example of Fig. 2, assume that all channels have normalized bandwidth, i.e., $R_c = 1$, $\forall c \in \mathbf{C}$, we can easily obtain the normalized link rates: $R_{s_1} = \frac{1}{3} + \frac{1}{2} + \frac{1}{3} = 1.17$, $R_{s_2} = \frac{1}{3} + \frac{1}{2} = 0.83$, $R_{s_3} = \frac{1}{3} + \frac{1}{3} = 0.67$, and $R_{r_{2_1}} = 1 + \frac{1}{3} = 1.33$. Accordingly, we can obtain the normalized end-to-end rates: $R_{s_1}^e = 1.17$, $R_{s_2}^e = R_{r_{2_1}}^e = \min\{R_{s_2}, R_{r_{2_1}}\} = 0.83$, and $R_{s_3}^e = 0.67$.

We refer to each player as a *rational* and *self-interested* player, who will always choose strategy that maximizes its payoff. Thus, we can formulate the multiradio channel allocation problem as a *static game*, which corresponds to a fixed channel allocation among the players.

4 Nash Equilibria

4.1 Noncooperative Game NE

In single-hop networks, the payoff of player i is equivalent to its utility R_i and the multiradio channel allocation problem can be formulated as a static noncooperative game [5]. In order to study the strategic interaction of the players in such a game, we first introduce the concepts of Nash equilibrium [2], [27].

Definition 1 (NE). *The strategy matrix* $\mathbb{X}^* = \{\mathbf{x}_1^*, \ldots, \mathbf{x}_{|\mathbf{U}|}^*\}$ *defines a Nash Equilibrium, if for every player* $i \in \mathbf{U}$, *we have*

$$R_i\big(\mathbf{x}_i^*, \mathbb{X}_{-i}^*\big) \geq R_i\big(\mathbf{x}_i', \mathbb{X}_{-i}^*\big), \tag{6}$$

for every strategy $\mathbf{x}_i'$, *where* $R_i(\mathbb{X})$ *denotes the utility of player* i *in strategy matrix* $\mathbb{X}$.

The definition of NE expresses the resistance to the deviation of a single player in noncooperative game. In other words, *in an NE, none of the players can unilaterally change its strategy to increase its payoff*. An NE solution may be inefficient from the system point of view. We characterize the efficiency of the solution by the concept of Pareto optimality.

Definition 2 (Pareto Optimality). *The strategy matrix* $\mathbb{X}^{op}$ *is Pareto-optimal if there* does not exist *any strategy* $\mathbb{X}'$ *such that the following set of conditions is true:*

$$R_i(\mathbb{X}') \geq R_i(\mathbb{X}^{op}), \quad \forall i \in \mathbf{U}, \tag{7}$$

with strict inequality for at least one player i.

This means that in a Pareto-optimal channel allocation $\mathbb{X}^{op}$, one cannot improve the payoff of any player without decreasing the payoff of at least one other player.

4.2 Cooperative Game CPNE

It is worth noting that noncooperative game is *not* suitable for multihop networks due to the following two reasons. On one hand, the payoff of any player in multihop session is not equivalent to its utility. In fact, the payoff (achieved data rate) of player i is not only determined by the utility itself, but also by the utilities of other players within the

4. Strictly speaking, i is the player in group $\mathbf{g}_x - \{d_x\}$ since we do not consider the destinations as player.

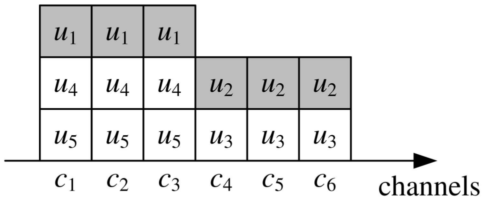

Fig. 3. An example of CPNE channel allocation, where u_1 and u_2 formulate a coalition.

same session. On the other hand, it is possible that the players in the same session cooperatively choose their strategies for the purpose of high payoff.

Hence, we formulate the problem in multihop networks as a *hybrid game* involving both cooperative game and noncooperative game. In detail, the players within a coalition (session) are cooperative, and among coalitions, they are noncooperative. In order to study the strategic interaction of the coalitions in cooperative game, we introduce the concept of classical coalition-proof Nash equilibrium [28].

Definition 3 (CPNE). *The strategy matrix* $\mathbb{X}^{cp}$ *defines a coalition-proof Nash Equilibrium, if for every coalition* $\sigma_x \in \mathbf{Q}$, *we have*

$$R_i\left(\mathbb{X}^{cp}_{\sigma_x}, \mathbb{X}^{cp}_{-\sigma_x}\right) \geq R_i\left(\mathbb{X}'_{\sigma_x}, \mathbb{X}^{cp}_{-\sigma_x}\right), \quad \forall i \in \sigma_x \tag{8}$$

for every strategy set $\mathbb{X}'_{\sigma_x}$.

This means that no coalition can deviate from $\mathbb{X}^{cp}$ such that the utility of at least one of its members increases and the utilities of other members do not decrease.

It is notable that classical CPNE in Definition 3 is *not* strictly suitable for multihop networks. In detail, for each coalition, CPNE does not allow an increment of a utility if such an increment leads to a decrease in other members' utility, even though such an increment is great. Thus, the utilities of players within a coalition may be imbalance, which will lead to poor performance in terms of achieved date rate. We show this phenomena as an example of CPNE in Fig. 3, where $|\mathbf{C}| = 6$, $|\mathbf{U}| = 5$, $K = 3$, and players u_1, u_2 formulate a coalition σ_x. The number in each box denotes the player who chooses this channel. We can easily find $k_{c_1} = k_{c_2} = k_{c_3} = 3$ and $k_{c_4} = k_{c_5} = k_{c_6} = 2$, and thus, $R_{u_1} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1.0$, $R_{u_2} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 1.5$, and $R^e_{u_1} = R^e_{u_2} = \min\{R_{u_1}, R_{u_2}\} = 1$. Now, suppose we do $(u_1, c_1) \leftrightarrows (u_2, c_6)$,[5] the utility of u_2 (i.e., R_{u_2}) decreases to 1.33, while the achieved data rates $R^e_{u_1}$ and $R^e_{u_2}$ increase to 1.17. In other words, we can increase the payoff by sacrificing the utility of any member within the coalition.

To overcome the shortcoming of utility imbalance and payoff inefficiency in classical CPNE, we define a novel coalition-proof Nash equilibrium, named as MMCPNE, in which players choose strategies so as to improve the minimal utility of players within the coalition. We generalize the notion of MMCPNE as follows:

5. Note that $(i, c_m) \leftrightarrows (j, c_n)$ means exchanging the radio of i in channel c_m and the radio of j in channel c_n. In other words, player i moves its radio which deployed on channel c_m to c_n and player j moves its radio which deployed on channel c_n to c_m.

Definition 4 (MMCPNE). *The strategy matrix* $\mathbb{X}^{mm}$ *defines a novel coalition-proof Nash Equilibrium, if for every coalition* $\sigma_x \in \mathbf{Q}$, *we have*

$$\min_{i \in \sigma_x} R_i\left(\mathbb{X}^{mm}_{\sigma_x}, \mathbb{X}^{mm}_{-\sigma_x}\right) \geq \min_{i \in \sigma_x} R_i\left(\mathbb{X}'_{\sigma_x}, \mathbb{X}^{mm}_{-\sigma_x}\right) \tag{9}$$

for every strategy set $\mathbb{X}'_{\sigma_x}$.

In fact, MMCPNE can be seen as a special coalition-proof Nash equilibrium with a judiciously designed objective function, i.e., the end-to-end rate. In other words, in an MMCPNE, none of the coalition can deviate from $\mathbb{X}^{mm}$ such that the minimal utility of its members increases. MMCPNE channel allocation is always higher efficient compared with CPNE, which can be seen from our analysis in Section 5. Recall the example in Fig. 3, if we do $(u_1, c_1) \leftrightarrows (u_2, c_4)$, we obtain the MMCPNE channel allocation and the achieved data rates of u_1 and u_2 increase to 1.17.

Unfortunately, we find that it is difficult to find such an MMCPNE (or CPNE) strategy since we must jointly search the strategy in the strategies set of $|\sigma_x|$ players. The computation of achieving MMCPNE (or CPNE) increases exponentially with the size of coalition, typically, $O(\bar{\omega}^{|\sigma_x|})$, where $\bar{\omega}$ is the expectation of ω, i.e., the number of moves for a single player finding its best response strategy. In the worst case, a player must try all possible strategies to find its best response strategy, i.e., $\omega = \binom{|\mathbf{C}|}{K} = \frac{M \cdot (M-1) \cdot \cdots \cdot (M-K+1)}{K \cdot (K-1) \cdot \cdots \cdot 1}$, where M is the number of channels and K is the number of radios in each transceivers set.

To reduce the large computation in finding MMCPNE, we introduce three approximated solutions, denoted by MCPNE, ACPNE, and ICPNE. The definitions of MCPNE, ACPNE, and ICPNE are given below.

Definition 5 (MCPNE). *The strategy matrix* $\mathbb{X}^m$ *defines a special coalition-proof Nash Equilibrium, if for every player* $i \in \mathbf{U}$, *we have*

$$\min_{u \in \sigma_x} R_u\left(\mathbf{x}^m_i, \mathbb{X}^m_{-i}\right) \geq \min_{u \in \sigma_x} R_u\left(\mathbf{x}'_i, \mathbb{X}^m_{-i}\right) \tag{10}$$

for every strategy $\mathbf{x}'_i$, *where* σ_x *is the coalition player* i *belongs to, i.e.,* $i \in \sigma_x$

Definition 6 (ACPNE). *The strategy matrix* $\mathbb{X}^a$ *defines a special coalition-proof Nash Equilibrium, if for every player* $i \in \mathbf{U}$, *we have*

$$\min_{u \in \sigma_x} R_u\left(\mathbf{x}^a_i, \mathbb{X}^a_{-i}\right) > \min_{u \in \sigma_x} R_u\left(\mathbf{x}'_i, \mathbb{X}^a_{-i}\right) \tag{11}$$

or

$$\begin{cases} \min\limits_{u \in \sigma_x} R_u\left(\mathbf{x}^a_i, \mathbb{X}^a_{-i}\right) = \min\limits_{u \in \sigma_x} R_u\left(\mathbf{x}'_i, \mathbb{X}^a_{-i}\right) \\ \sum\limits_{u \in \sigma_x} R_u\left(\mathbf{x}^a_i, \mathbb{X}^a_{-i}\right) \geq \sum\limits_{u \in \sigma_x} R_u\left(\mathbf{x}'_i, \mathbb{X}^a_{-i}\right) \end{cases} \tag{12}$$

for every strategy $\mathbf{x}'_i$.

Definition 7 (ICPNE). *The strategy matrix* $\mathbb{X}^s$ *defines a special coalition-proof Nash Equilibrium, if for every player* $i \in \mathbf{U}$, *we have*

$$\min_{u \in \sigma_x} R_u\left(\mathbf{x}^s_i, \mathbb{X}^s_{-i}\right) > \min_{u \in \sigma_x} R_u\left(\mathbf{x}'_i, \mathbb{X}^s_{-i}\right) \tag{13}$$

	$\mathbf{x}_i^1$	$\mathbf{x}_i^2$	$\mathbf{x}_i^3$	$\mathbf{x}_i^4$	$\mathbf{x}_i^5$	$\mathbf{x}_i^6$
R_i	2	1.5	1	1	0.5	0.5
R_j	1	0.5	3	2.5	5	0.5

Fig. 4. The resulting utility for player i choosing $\mathbf{x}_i^n$.

or

$$\begin{cases} \min_{u\in\sigma_x} R_u\left(\mathbf{x}_i^s, \mathbb{X}_{-i}^s\right) = \min_{u\in\sigma_x} R_u\left(\mathbf{x}_i', \mathbb{X}_{-i}^s\right) \\ R_i\left(\mathbf{x}_i^s, \mathbb{X}_{-i}^s\right) \geq R_i\left(\mathbf{x}_i', \mathbb{X}_{-i}^s\right) \end{cases} \tag{14}$$

for every strategy $\mathbf{x}_i'$.

Obviously, players within a coalition can select their strategies *independently* to achieve the above three approximated MMCPNE situations, and thus, the computations increase linearly to the size of coalition, i.e., $O(\bar{\omega}\cdot|\sigma_x|)$. Strictly speaking, MCPNE (or ACPNE, ICPNE) is noncooperative Nash equilibrium with a judiciously designed objective function, rather than coalition-proof Nash equilibrium of cooperative game.

We show the difference of MCPNE, ACPNE, and ICPNE by an example of two-player coalition $\sigma_x = \{i, j\}$. Without loss of generality, we assume that there are six strategies for player i, i.e., $\mathbf{x}_i^n, n = 1, \ldots, 6$, and the resulting utility for selecting one of the strategies is shown in Fig. 4. For MCPNE, each player would like to choose the strategy which maximizes the minimal utility of players within the coalition, and thus, the best strategies for player i are $\mathbf{x}_i^1$, $\mathbf{x}_i^3$, and $\mathbf{x}_i^4$. While for ACPNE, each player would like to choose the strategies which maximize the minimal utility and further maximize the average utility of players within the coalition. Similarly, for ICPNE, each player would like to choose the strategies which maximize the minimal utility and further maximize its own utility. Thus, for ACPNE and ICPNE, the best strategies of player i are $\mathbf{x}_i^3$ and $\mathbf{x}_i^1$, respectively. It is easy to see that the set of ACPNE (or ICPNE) schemes is the subset of MCPNE schemes.

In order to provide an intuitionistic impression, we show the previous channel allocation schemes by properties in Fig. 5. The fact of MMCPNE being a subset of MCPNE can be proved by contradiction as follows: Assume that there exists an MMCPNE strategy matrix $\mathbb{X}^{mm}$ which is not an MCPNE, then according to Definition 5, there exists at least one player (say i) who can improve the minimal utility of its coalition (say σ_x) by changing its strategy, which implies that there exists a coalition, i.e., σ_x, that can improve the minimal utility of its members by changing its member i's strategy. According to Definition 4, $\mathbb{X}^{mm}$ cannot be an MMCPNE, which leads to a contradiction. Thus, we declare that an MMCPNE must be an MCPNE.

5 Existance of MMCPNE

In this section, we study the existence of Nash equilibria and min-max coalition-proof Nash equilibria in the single collision domain channel allocation game.

It is straightforward to see that if the total number of radios (i.e., $|\mathbf{U}|\cdot K$) is smaller than or equal to the number of channels, then a flat channel allocation, in which the number of radios per channel does not exceed one, is a Nash equilibrium.

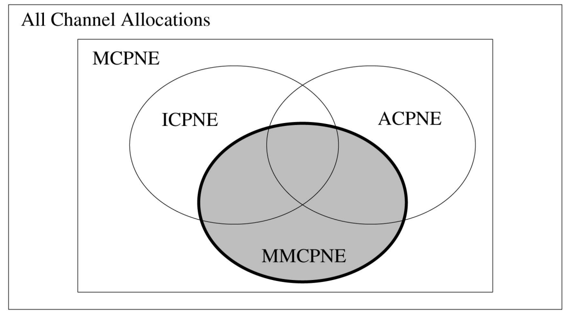

Fig. 5. Summary of channel allocations with different properties.

Fact 1. If $|\mathbf{U}|\cdot K \leq |\mathbf{C}|$, then any channel allocation, in which $k_c \leq 1$, $\forall c \in \mathbf{C}$, is a Pareto-optimal NE.

For the remainder of the paper, we assume that $|\mathbf{U}|\cdot K > |\mathbf{C}|$; hence, the players have a conflict during the channel allocation process.

5.1 NE in Multihop Networks

We first retrospect the work done by Felegyhazi et al. [5]. The authors study in detail the problem of competitive multiradio channel allocation in *single-hop* networks, i.e., $\mathbf{R} = \emptyset$ or $\mathbf{U} = \mathbf{S}$, and propose the conditions for Nash equilibria as the following theorem.

Theorem 1. *Assume that* $|\mathbf{S}|\cdot K > |\mathbf{C}|$. *Then, a channel allocation* $\mathbb{X}^*$ *is an NE if the following conditions hold:*[6]

- $k_{i,c} \leq 1$,
- $k_i = k, \forall i \in \mathbf{S}, \forall c \in \mathbf{C}$, *and*
- $\delta_{b,c} \leq 1, \forall b, c \in \mathbf{C}$,

where $\delta_{b,c} = k_b - k_c$ *denotes the difference of radios number between channels* b *and* c.

The first condition in Theorem 1 shows that a player cannot assign two radios with the same channel due to the coradios interference in device, and the second condition shows that a selfish player would like to use all of his radios in order to maximize his total bandwidth. The first two conditions provide the necessary conditions for NE from the aspect of individual players. The third condition shows that the whole system will achieve load-balancing over the channels in an NE. The third condition provides the necessary condition for NE from the aspect of whole system. For a detailed proof of Theorem 1, we refer the reader to [5].

We divide the channels in NE into two sets: $\mathbf{C}^+$, which contains the channels selected by relatively more players, and $\mathbf{C}^-$, which contains the channels selected by relatively less players. We denote the number of radios deployed on the channel in $\mathbf{C}^+$ and $\mathbf{C}^-$ by δ^+ and δ^-, respectively, and obviously that $\delta^+ = \delta^- + 1$.[7] In the example of Fig. 3, $\mathbf{C}^+ = \{c_1, c_2, c_3\}$, $\mathbf{C}^- = \{c_4, c_5, c_6\}$, $\delta^+ = 3$, and $\delta^- = 2$.

6. Note that we omit the second type of Nash equilibria proposed by Mark Felegyhazi in which the different radios of one player may deploy on the same channel.

7. Note that the equation holds only when $|\mathbf{C}^-| > 0$; otherwise, $\mathbf{C}^- = \emptyset$ and δ^- is meaningless.

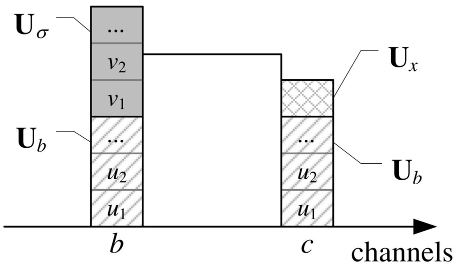

Fig. 6. An example of MMCPNE channel allocation corresponding to Proposition 1.

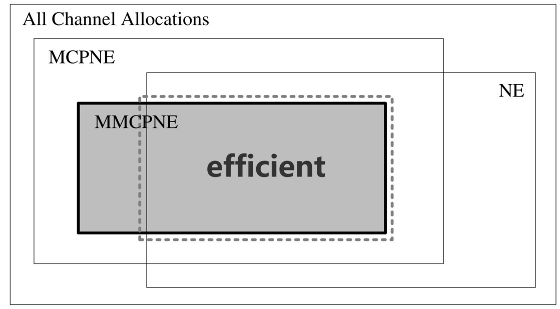

Fig. 7. Summary of MMCPNE and NE channel allocations with different properties.

Note, however, that Nash equilibrium is *not* suitable for the multihop wireless networks as mentioned previously. Recall the example in Fig. 3, according to Theorem 1, we can easily find that it is an NE (also CPNE) channel allocation, in which $R^e_{u_1} = R^e_{u_2} = \min\{R_{u_1}, R_{u_2}\} = 1.0, R^e_{u_3} = 1.5, R^e_{u_4} = 1.0$, and $R^e_{u_5} = 1.0$. If we do $(u_1, c_1) \leftrightarrows (u_2, c_4)$, we obtain an MMCPNE channel allocation in which the payoff of u_1 and u_2 increases to 1.17. Thus, we show that *NE channel allocation may be payoff inefficient in multihop networks*.

In the following, we will study the existence of MMCPNE. In general, there exist multiple MMCPNE channel allocation schemes in the system. We divide all MMCPNE schemes into two types: MMCPNE-1, which satisfy Theorem 1, and MMCPNE-2, which do not satisfy Theorem 1. In other words, each MMCPNE-1 channel allocation scheme is not only an MMCPNE, but also an NE. We find that the multiplayer coalitions (i.e., multihop sessions) occupy more bandwidth in MMCPNE-1 states compared with in MMCPNE-2 states. We show this property as the following proposition.

Proposition 1. *Assume that there exists an MMCPNE channel allocation $\mathbb{X}$ with* high *coalition utility*[8] *(for the multiplayer coalitions), then $\mathbb{X}$ is a Nash equilibrium, i.e., the conditions of Theorem 1 hold.*

Proof. It is straightforward to see that the first two conditions in Theorem 1 always hold in MMCPNE due to the coradios interference in device and the selfish nature of players. We validate the third condition in Theorem 1 by contradiction. Assume that there exists two channels b and c such that $\delta_{b,c} \geq 2$ in MMCPNE strategy $\mathbb{X}$. Without loss of generality, we assume that there exists a multiplayer coalition σ_x and some individual players (i.e., single-player coalitions) in the system. As shown in Fig. 6, we denote the set of individual players in channel b by $\mathbf{U}_b = \{u_1, u_2, \dots\}$, i.e., $k_{i,b} = 1, \forall i \in \mathbf{U}_b$. It is obvious that $k_{i,c} = 1, \ \forall i \in \mathbf{U}_b$; otherwise, player i can improve its payoff by moving its radio from channel b to c. Thus, we can easily find that $|\mathbf{U}_b| \leq k_c$. We denote the set of remainder players in channel b by $\mathbf{U}_\sigma = \{v_1, v_2, \dots\}$, and obviously that $\mathbf{U}_\sigma \subseteq \sigma_x$. Similarly, we denote $\mathbf{U}_x$ as the set of remainder players in channel c excluding the players in $\mathbf{U}_b$, i.e., $\mathbf{U}_x = \mathbf{U}_c - \mathbf{U}_b$, where $\mathbf{U}_c$ is the set of all players in channel c. We can write the total bandwidth (i.e., utility) of $\mathbf{U}_b$ and $\mathbf{U}_x$ on channels b and c as

$$P_1 = \frac{|\mathbf{U}_b|}{k_b} + \frac{|\mathbf{U}_b| + |\mathbf{U}_x|}{k_c}.$$

It is easy to see that $|\mathbf{U}_\sigma| - |\mathbf{U}_x| = \delta_{b,c} \geq 2$, i.e., there exist at least two players i and j such that $i \in \mathbf{U}_\sigma, \ j \in \mathbf{U}_\sigma$ and $i \notin \mathbf{U}_x, \ j \notin \mathbf{U}_x$. Now suppose that player i (or j) moves its radio from channel b to c, we can write the total bandwidth of $\mathbf{U}_b$ and $\mathbf{U}_x$ on channels b and c as

$$P_2 = \frac{|\mathbf{U}_b|}{k_b - 1} + \frac{|\mathbf{U}_b| + |\mathbf{U}_x|}{k_c + 1}.$$

Note that $k_b - k_c = \delta_{b,c} \geq 2$ and we can easily find that $P_1 > P_2$, i.e., the total bandwidth of $\mathbf{U}_b$ and $\mathbf{U}_x$ occupied decreases. Thus, the bandwidth of $\mathbf{U}_\sigma$ increases since the total available bandwidth is constant, which contradict to the assumption that $\mathbb{X}$ is an MMCPNE with high coalition utility. □

It is obvious that the coalition with high total bandwidth is likely to achieve high data rate. The value of Proposition 1 is that it provides a method to choose the MMCPNE with the high coalition utility, i.e., MMCPNE-1.[9] Fig. 7 shows the MMCPNE and NE channel allocations by properties. We refer to MMCPNE-1 channel allocations as the coalition-efficient MMCPNE, and will focus on MMCPNE-1 for the remainder of the paper.

We study the MMCPNE in two types of multihop networks: the *short-path* networks in which each session contains at most 2 hops, and the *long-path* networks in which at least one session contains more than 2 hops.

5.2 MMCPNE in 2-Hop Networks

In this section, we study the MMCPNE in short-path networks in which each session contains at most 2 hops, i.e., each coalition contains at most two players. It is easy to see that all players in 2-hop networks reside in a single collision domain.

Although none of the players can *unilaterally* change its strategy to increase its payoff in NE, it is possible that a player changes its strategy to improve the utility of another player he is in a coalition with, e.g., u_1 and u_2 in Fig. 3. As the utilities of all players increase by the help of others, the

8. Coalition utility is defined as the sum of all members' utilities in the coalition. High coalition utility is defined as the fact that the coalition cannot improve its utility by unilaterally changing its members' strategies.

9. It is notable that the bandwidth occupied by individual players does not decrease in MMCPNE-1 compared with those in NE state, since an MMCPNE-1 state is also an NE.

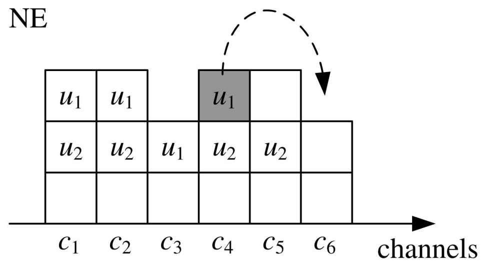

Fig. 8. An example of NE channel allocation corresponding to Lemma 2.

payoff of players increases undoubtedly. Players within a coalition can help each other in two ways. The first possibility is that a player *relocates* its radios to improve the utility of other when two players share any channels. This property is expressed as the following lemma:

Lemma 2. *Assume that there exists a coalition $\sigma_x = \{u_1, u_2\}$ and $R_{u_1} \neq R_{u_2}$ in an NE channel allocation $\mathbb{X}$. If there exist two channels $c_1 \in \mathbf{C}^+$ and $c_2 \in \mathbf{C}^-$ such that $k_{u_1,c_1} = k_{u_2,c_1} = 1$ and $k_{u_1,c_2} = k_{u_2,c_2} = 0$, then $\mathbb{X}$ is not MMCPNE.*

Proof. Without loss of generality, we assume that $R_{u_1} > R_{u_2}$ in the NE channel allocation $\mathbb{X}$, and thus, we can write the payoff of u_1 and u_2 as $R^e_{u_1}(\mathbb{X}) = R^e_{u_2}(\mathbb{X}) = \min\{R_{u_1}, R_{u_2}\} = R_{u_2}$. Suppose that u_1 moves its radio on channel c_1 to c_2, the utility of u_1 does not change, whereas the utility of u_2 change to $R_{u_2} + \frac{1}{\delta^-} - \frac{1}{\delta^+}$. We denote the new channel allocation by $\mathbb{X}'$ and have $R^e_{u_1}(\mathbb{X}') = R^e_{u_2}(\mathbb{X}') = \min\{R_{u_1}, R_{u_2} + \frac{1}{\delta^-} - \frac{1}{\delta^+}\} > R_{u_2}$ since $\delta^+ = \delta^- + 1$. So, we declare that $\mathbb{X}$ is not MMCPNE. □

An example of any NE channel allocation corresponding to Lemma 2 is shown in Fig. 8, where $|\mathbf{C}| = 6$, $K = 4$, and u_1, u_2 formulate a coalition. Note that empty boxes in Fig. 8 denote the unconcerned players who choose the channels. According to Lemma 2, it cannot be an MMCPNE, since we can increase the payoff of u_1 and u_2 by moving u_1 from channel c_4 to c_6.

In some cases, the assumption of unequal utilities of two players, i.e., $R_{u_1} \neq R_{u_2}$, might not hold. In other words, it is possible that $R_{u_1} = R_{u_2}$. In such cases, Lemma 2 may no longer hold. Thus, we show another necessary condition as given below.

Lemma 3. *If there exists a coalition $\sigma_x = \{u_1, u_2\}$ and multiple channels $\{c_1, c_2, \ldots\} \in \mathbf{C}^+$ and $\{b_1, b_2, \ldots\} \in \mathbf{C}^-$ such that $k_{u_1,c_i} = k_{u_2,c_i} = 1$, $i = 1, 2, \ldots,$ whereas $k_{u_1,b_i} = k_{u_2,b_i} = 0$, $i = 1, 2, \ldots,$ in an NE channel allocation $\mathbb{X}$, then $\mathbb{X}$ is not MMCPNE.*

Proof. Suppose that u_1 moves its radio on channel c_1 to b_1 and u_2 moves its radio on channel c_2 to b_2, it is easy to see that the utilities of both players increase, and thus, the payoff of u_1 and u_2 increases. □

The second possibility for players within a coalition helping each other is that they mutually *exchange* some radios with each other. We show this necessary condition as the following lemma.

Lemma 4. *Assume that there exists a coalition $\sigma_x = \{u_1, u_2\}$ and $R_{u_2} - R_{u_1} > (\frac{1}{\delta^-} - \frac{1}{\delta^+})$ in an NE channel allocation $\mathbb{X}$. If there exists two channels $c_1 \in \mathbf{C}^+$ and $c_2 \in \mathbf{C}^-$ such that $k_{u_1,c_1} = 1$ and $k_{u_2,c_1} = 0$, whereas $k_{u_1,c_2} = 0$ and $k_{u_2,c_2} = 1$, then $\mathbb{X}$ is not MMCPNE.*

Proof. We can easily write the payoff of u_1 and u_2 as $R^e_{u_1}(\mathbb{X}) = R^e_{u_2}(\mathbb{X}) = \min\{R_{u_1}, R_{u_2}\} = R_{u_1}$. Suppose that u_1 and u_2 exchange their radios in channels c_1 and c_2, i.e., $(u_1, c_1) \leftrightarrows (u_2, c_2)$, the utilities of u_1 and u_2 change to $R_{u_1} + \frac{1}{\delta^-} - \frac{1}{\delta^+}$ and $R_{u_2} - \frac{1}{\delta^-} + \frac{1}{\delta^+}$, respectively. We can easily find that $R_{u_1} + \frac{1}{\delta^-} - \frac{1}{\delta^+} > R_{u_1}$ since $\delta^+ = \delta^- + 1$. Using the conditions in the lemma, we can also find that $R_{u_2} - \frac{1}{\delta^-} + \frac{1}{\delta^+} > R_{u_1}$. We denote the new channel allocation by $\mathbb{X}'$ and have $R^e_{u_1}(\mathbb{X}') = R^e_{u_2}(\mathbb{X}') = \min\{R_{u_1} + \frac{1}{\delta^-} - \frac{1}{\delta^+}, R_{u_2} - \frac{1}{\delta^-} + \frac{1}{\delta^+}\} > R_{u_1}$. So, we declare that $\mathbb{X}$ is not MMCPNE. □

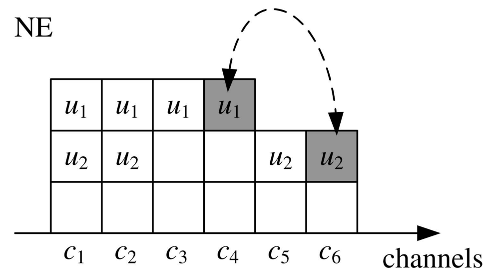

Fig. 9. An example of an NE channel allocation corresponding to Lemma 4.

We show an example of any NE channel allocation corresponding to Lemma 4 in Fig. 9, where $|\mathbf{C}| = 6$, $K = 4$, and u_1, u_2 formulate a coalition. According to Lemma 4, it cannot be an MMCPNE, since we can increase the payoff of u_1 and u_2 by exchanging their radios in channels c_4 and c_6. Note that after such exchanging, the utility of u_2 decreases from 1.67 to 1.5, while the payoff of u_2 increases from 1.33 to 1.5.

As mentioned above, all channels in NE can be divided into two sets: $\mathbf{C}^+$ and $\mathbf{C}^-$. We denote k_i^+ as the number of channels in $\mathbf{C}^+$ chosen by player i and k_i^- as the number of channels in $\mathbf{C}^-$ chosen by player i. It is easy to see that $k_i^+ + k_i^- = K$, $\forall i \in \mathbf{U}$. In Fig. 8, we have $k^+_{u_1} = 3$, $k^-_{u_1} = 1$, $k^+_{u_2} = 4$, and $k^-_{u_2} = 0$. In Fig. 9, we have $k^+_{u_1} = 4$, $k^-_{u_1} = 0$, $k^+_{u_2} = 2$, and $k^-_{u_2} = 2$. Now we can extend Lemma 4 to a more general situation.

Lemma 5. *Assume that there exists a coalition $co_i = \{u_1, u_2\}$ and $|k^+_{u_1} - k^+_{u_2}| > 1$ in an NE channel allocation $\mathbb{X}$, then $\mathbb{X}$ is not MMCPNE.*

Proof. Without loss of generality, we assume that $k^+_{u_1} > k^+_{u_2}$. As mentioned above, any player cannot use multiple radios in the same channel; thus, there exists at least one channel $c_1 \in \mathbf{C}^+$ such that $k_{u_1,c_1} = 1$ and $k_{u_2,c_1} = 0$. Similarly, there exists at least one channel $c_2 \in \mathbf{C}^-$ such that $k_{u_1,c_2} = 0$ and $k_{u_2,c_2} = 1$. Furthermore, we can write the utilities of two players as $R_{u_1} = \frac{k^+_{u_1}}{\delta^+} + \frac{k^-_{u_1}}{\delta^-}$ and $R_{u_2} = \frac{k^+_{u_2}}{\delta^+} + \frac{k^-_{u_2}}{\delta^-}$, respectively. Note that $k_i^+ + k_i^- = K$; thus, we can write the utility difference of two players as

$$R_{u_2} - R_{u_1} = \frac{k^+_{u_2} - k^+_{u_1}}{\delta^+} + \frac{k^-_{u_2} - k^-_{u_1}}{\delta^-} = \left(k^+_{u_1} - k^+_{u_2}\right)\left(\frac{1}{\delta^-} - \frac{1}{\delta^+}\right). \quad (15)$$

Using the conditions of the lemma, we can find that $R_{u_2} - R_{u_1} > (\frac{1}{\delta^-} - \frac{1}{\delta^+})$. Hence, the two conditions of Lemma 4 hold, and we can achieve the proof directly from Lemma 4. □

From (15), we can easily find that $|k^+_{u_1} - k^+_{u_2}| > 1$ if the condition $|R_{u_2} - R_{u_1}| > (\frac{1}{\delta^-} - \frac{1}{\delta^+})$ holds. Thus, we can immediately release some restrictions in Lemma 4. We express this property as the following corollary:

Corollary 6. *If there exists a coalition $\sigma_x = \{u_1, u_2\}$ and $|R_{u_2} - R_{u_1}| > (\frac{1}{\delta^-} - \frac{1}{\delta^+})$ in an NE channel allocation $\mathbb{X}$, then $\mathbb{X}$ is not MMCPNE.*

It is notable that Lemma 2 and Lemma 3 are also available for CPNE state, whereas the other lemmas are exclusively used in MMCPNE. Based on the previous lemmas, we prove the necessary conditions that enable a given NE allocation to be MMCPNE and present it as the following theorem:

Theorem 2. *Assume that there exists a coalition $\sigma_x = \{u_1, u_2\}$ and $R_{u_1} \geq R_{u_2}$ in an NE channel allocation $\mathbb{X}$, if $\mathbb{X}$ is MMCPNE, the following conditions hold:*

- $R_{u_1} - R_{u_2} \leq (\frac{1}{\delta^-} - \frac{1}{\delta^+})$.
- *Case 1: If $R_{u_1} \neq R_{u_2}$, then there* does not exist *two channels $b \in \mathbf{C}^+$ and $c \in \mathbf{C}^-$ such that $k_{u_1,b} = k_{u_2,b} = 1$, whereas $k_{u_1,c} = k_{u_2,c} = 0$.*
- *Case 2: If $R_{u_1} = R_{u_2}$, then there* does not exist *four channels $\{b_1, b_2\} \in \mathbf{C}^+$ and $\{c_1, c_2\} \in \mathbf{C}^-$ such that $k_{u_1,b_i} = k_{u_2,b_i} = 1$, $i = 1, 2$, whereas $k_{u_1,c_i} = k_{u_2,c_i} = 0, i = 1, 2$.*

We could not prove that the conditions in Theorem 2 are sufficient to enable an NE channel allocation to be MMCPNE, neither could we find a counterexample, where the conditions hold and the NE channel allocation is not MMCPNE.

5.3 MMCPNE in N-Hop Networks

In this section, we study the MMCPNE in long-path networks in which at least one session contains $N > 2$ hops. We will show that through the judiciously designing of scheduling scheme, the existence of MMCPNE in N-hop networks can be transformed into the problem in 2-hop networks.

Without loss of generality, we consider the 3-hop communication session as shown in Fig. 10, where s communicates with d by the relaying of r_1 and r_2. We can easily find that players s, r_1 and r_1, r_2 reside in a collision domain. It is notable that players s and r_2 do *not* reside in a collision domain if we schedule players s and r_2 as the following way: 1) in odd time slot, player s transmits the data packets to r_1 and player r_2 receives the data packets from d, i.e., $s \to r_1$ and $r_2 \leftarrow d$; 2) in even time slot, player s receives the data packets from r_1 and player r_2 transmits the data packets to d, i.e., $s \leftarrow r_1$ and $r_2 \to d$.[10] It is easy to see

10. Note that in such scheduling scheme, the volume of forward traffic, i.e., traffic from sender to destination, must be same as the volume of backward traffic, i.e., traffic from destination to sender.

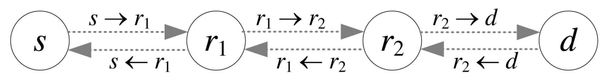

Fig. 10. An example of a 3-hop communication session.

that in such scheduling scheme, players s and r_2 can communicate on the same channel without interference.

Similarly, for the arbitrary N-hop session, where s communicates with d by the relaying of $r_1, r_2, \ldots$, if we use the above scheduling scheme, players $s, r_2, r_4, \ldots$ and players $r_1, r_3, \ldots$ can communicate on the same channel without interference.

We define a new 2-hop networks, denoted by *concomitant networks*, if we replace all multihop sessions in the original N-hop networks by virtual 2-hop sessions. Each multihop session l in the original networks corresponds to a 2-hop session l' in the concomitant networks. We find that the existence of MMCPNE in N-hop networks can be transformed into the problem in 2-hop networks. In detail, if $\mathbb{X}$ is an MMCPNE channel allocation of the concomitant networks, then $\mathbb{X}^{mm}$ is an MMCPNE of the original N-hop networks as we define $\mathbb{X}^{mm}$ as following: 1) for any single-hop session, the individual player holds the strategy in $\mathbb{X}$ and 2) for any multihop session l, we assign players $s, r_2, r_4, \ldots$ with the strategy of the first player in l' and assign players $r_1, r_3, \ldots$ with the strategy of the second player in l'. It is easy to see that $\mathbb{X}^{mm}$ is an MMCPNE channel allocation of the original N-hop networks as we use the scheduling scheme mentioned above.

6 Convergence to MMCPNE

In this section, we propose a distributed *MMCP Algorithm* to enable the selfish players to converge to MMCPNE from an arbitrary initial configuration. We also propose a distributed *DCP Algorithm* to enable the players to converge to an approximated MMCPNE situation, e.g., MCPNE, ACPNE, or ICPNE.

We define our algorithms (MMCP and DCP) as *round-based* distributed algorithms that work as follows: First, we assume that there exists a random radio assignment of the players over the channels. After the initial channel assignment, each coalition (or each player) detects the set of players on each of the channels $c \in \mathbf{C}$ and tries to improve his payoff by reorganizing his radios. Unfortunately, this procedure might result in a continuous reallocation of the radios for all players [5].

To avoid the unstable channel allocations caused by simultaneously moving of different players, we use the technique of backoff mechanism well known in the IEEE 802.11 medium access technology similarly as [5]. We can notice that using the backoff mechanism, the players play a game in an almost sequential order. We denote the backoff window by W and each coalition chooses a random initial value for his backoff counter with uniform probability from the set $\{1, \ldots, W\}$.

6.1 MMCP Algorithm

We divide the MMCP algorithm into two stages. In the first stage, each coalition moves its radios to achieve high utility.

Thus, we call this stage as *intersession competition* stage. In the second state, players in the same session mutually adjust their radios to achieve higher data rate. We call this stage as *intrasession improvement* stage.

We present the pseudocode of MMCP algorithm in Tables A and B. Part I is the algorithm used in the *intersession competition* stage. In this stage, for both single-hop sessions and multihop sessions, the players move their radios to occupy as more bandwidth as possible. In fact, the system state will converge to an approximated NE in the intersession competition stage.

Part II is the algorithm used in the *intrasession improvement* stage. In this stage, players in the same session mutually adjust their radios to achieve higher payoff. For single-hop session, the player does nothing in this stage. For multihop session, the players adjust their radios according to Lemmas 2-5 to improve their payoff. It is notable that the two stages are time overlapping.

A. MMCP Algorithm - Part I

```
a) Inter-session Competition Stage
random channel allocation
while time is not over do
    get the current channel allocation
    for $i = 1$ to $|\mathbf{Q}|$ do
        if backoff counter is 0 then
            for $n = 1$ to $|\sigma_i|$ do
                $u =$ the $n_{th}$ player in $\sigma_i$
                for $j = 1$ to $K$ do
                    $b =$ the channel radio $j$ deployed on
                    $\Omega := \{c | c \in \mathbf{C}, k_{u,c} = 0, k_b - k_c > 1\}$
                    move radio $j$ from $b$ to $a$ where
                    $a = \arg\min_{c \in \Omega} k_c$
                end for
            end for
            include part II if $|\Omega| = 0,\ \forall j, n$
            reset the backoff counter to a new value from
            $\{1, \ldots, W\}$
        else
            decrease the backoff counter value by one
        end if
    end for
end while
```

B. MMCP Algorithm - Part II

```
b) Intra-session Improvement Stage
if $|\sigma_i|$ is 1 then          //single-hop session
    $u$ does nothing
else                               //multihop session
    $u =$ the first player in $\sigma_i, v =$ the second player
    in $\sigma_i$
    if $|k_u^+ - k_v^+| > 1$ then
        players adjust their radios according to Lemma 4
    elseif $|k_u^+ - k_v^+| > 0$ then
        players adjust their radios according to Lemma 2
    elseif $|k_u^+ - k_v^+| = 0$ then
        players adjust their radios according to Lemma 3
    end if
end if
```

From Tables A and B, we can easily obtain the complexity of MMCP algorithm. In intersession competition stage, each player searches the candidate channel set (i.e., Ω) to find the best channel for each of its radios. Note that the size of candidate channel set Ω is upbounded by $M - K$, and thus, the computation in each round is $O(\omega \cdot |\sigma_x|)$, where ω is upbounded by $K \cdot (M - K)$. In intrasession improvement stage, we find that for any multiplayer coalition $\sigma_x = \{u, v\}$, the computation in each round is $O(K \cdot |\sigma_x|)$ if $|k_u^+ - k_v^+| > 1$, and $O(M \cdot |\sigma_x|)$ if $|k_u^+ - k_v^+| \le 1$. This is due to the fact that in the former case, each player $i \in \sigma_x$ needs to search the set $\mathbf{C}_i$ to find a candidate channel for exchanging, while for the hander case, each player needs to search the set $\mathbf{C}$ to find the best strategy.

6.2 DCP Algorithm

In order to reduce the large computation of achieving MMCPNE due to the mutual operation of $|\sigma_x|$ players, we propose a distributed low complexity algorithm, denoted by *DCP Algorithm*, to enable the selfish players to converge to an approximated MMCPNE situation. By transforming the mutual operation of $|\sigma_x|$ players into multiple independent operations of the players, DCP algorithm efficiently reduces the computational complexity, specifically, from exponentially increasing with $|\sigma_x|$ to linear increasing with $|\sigma_x|$.

We denote the DCP algorithm derived from Definition 5 by *DCP-M Algorithm*. In other words, DCP-M algorithm enables the players to converge to MCPNE from an arbitrary initial configuration. We present the pseudocode of DCP-M Algorithm in Table C.

C. DCP-M Algorithm

```
random channel allocation
while not in an MCPNE do
    get the current channel allocation
    for $i = 1$ to $|\mathbf{U}|$ do
        if backoff counter is 0 then
            $u =$ the $i_{th}$ player in $\mathbf{U}, \sigma =$ the coalition
            $u$ belongs to
            //reorganize the radios of $u$ according to Def.5:
            for $j = 1$ to $K$ do
                $b =$ the channel radio $j$ deployed on
                $\Omega := \{c | c \in \mathbf{C}, k_{u,c} = 0\}$
                for $m = 1$ to $|\Omega|$ do
                    $c =$ the $m_{th}$ channel in $\Omega$
                    $\mu_c = \min_{v \in \sigma}(R_v)$ suppose radio $j$ moves
                    to $c$
                end for
                move the radio $j$ from $b$ to $a$ where
                $a = \arg\max_{c \in \Omega} \mu_c$
            end for
            reset the backoff counter to a new value from
            $\{1, \ldots, W\}$
        else
            decrease the backoff counter value by one
        end if
    end for
end while
```

D. DCP-A Algorithm (Part)

```
//reorganize the radios of u according to Def.6:
for j = 1 to K do
    B = the channel used by radio j
    Ω := {c|c ∈ C, k_{u,c} = 0}
    for m = 1 to |Ω| do
        c = the m_th channel in Ω
        μ_c = min_{v∈σ}(R_v) and γ_c = Σ_{v∈σ}(R_v) suppose
        radio j move to c
    end for
    move the radio j from b to a where
    a = arg max_{c∈Φ} γ_c where Φ := {c|c ∈ Ω,
    μ_c = max_{x∈Ω} μ_x}
end for
```

E. DCP-I Algorithm (Part)

```
//reorganize the radios of u according to Def.7:
for j = 1 to K do
    B = the channel used by radio j
    Ω := {c|c ∈ C, k_{u,c} = 0}
    for m = 1 to |Ω| do
        c = the m_th channel in Ω
        μ_c = min_{v∈σ}(R_v) and λ_c = R_u suppose j move
        to c
    end for
    move the radio j from b to a where
    a = arg max_{c∈Φ} λ_c
    where Φ := {c|c ∈ Ω, μ_c = max_{x∈Ω} μ_x}
end for
```

Similarly, we denote the DCP algorithm derived from Definitions 6 and 7 by *DCP-A Algorithm* and *DCP-I Algorithm*, respectively. The processes of DCP-A and DCP-I algorithms are similar as DCP-M algorithm except the rules of reorganizing the radios, i.e., lines 7-16 in DCP-M algorithm. We do not present the detail pseudocode of DCP-A and DCP-I algorithms due to space limitations. We show the key codes of DCP-A and DCP-I in Tables D and E, respectively.

Similar to MMCP algorithm, we can obtain the complexity of DCP algorithm. For any multiplayer coalition σ_x, each player $i \in \sigma_x$ needs to search the unused channels set (i.e., Ω) so as to find the best channel for each of its radios. Thus, the computation in each round is $O(\omega \cdot |\sigma_x|)$, where $\omega = K \cdot (M - K)$. It is easy to see that the computation of DCP and MMCP algorithms in each round is approximately the same. Note, however, that the number of rounds for (DCP alrogithm) achieving MCPNE (or ACPNE, ICPNE) is significantly less than the number of rounds for MMCP algorithm achieving MMCPNE.

7 Simulation Results

7.1 Simulation Setup

We implemented the previous algorithms in MATLAB and with a special focus on wireless IEEE 802.11a protocol. According to [26], for multiradio device, the adjacent 802.11a channels interfere in practical communication (although they are theoretically orthogonal), but nonadjacent channels do not interfere. Thus, we choose eight nonadjacent channels, each with 20 MHz bandwidth, as a default value for $\mathbf{C}$.

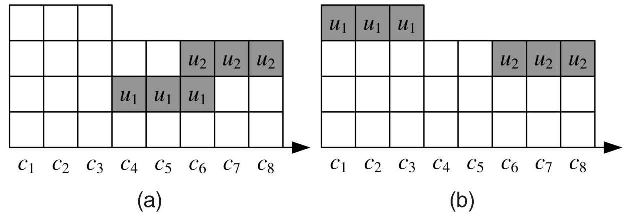

Fig. 11. An example for (a) a best case and (b) a worst case of NE channel allocation in terms of the payoff of coalition $\sigma_x = \{u_1, u_2\}$, where $|\mathbf{C}| = 8$, $|\mathbf{U}| = 9$, and $K = 3$.

We assume that the simulation networks contain one multihop session and some single-hop sessions, as shown in Fig. 1, and we denote the coalition corresponding to the multihop session by σ_x. Note, however, that our algorithms show similar results for system which contains several multihop sessions.

We assume that the duration of one round in the updating algorithm is 10 ms. This duration of one round corresponds roughly to the time needed for all these devices to transmit one MAC layer packet, i.e., the time that the devices can learn about other devices in the channel. We run each simulation for 600 rounds, which corresponds to 6 s according to the assumption above.[11] Each average value is the result of 1,000 simulation runs.

7.2 Performance Measures

We will focus on the performance of the multihop session (i.e., the coalition σ_x) in our simulation since there is no cooperative gain in the single-hop session.

Suppose that there exists an NE channel allocation, let us first highlight the best case and worst case in terms of payoff of the players in coalition σ_x. The best case is that all members in the coalition allocate as many radios as possible to the channel belongs to $\mathbf{C}^-$, and the worst case is that there exists at least one member who allocates as many radios as possible to the channel belongs to $\mathbf{C}^+$. In Fig. 11, we present an example for 1) a best case and 2) a worst case of NE channel allocation, where $|\mathbf{C}| = 8$, $|\mathbf{U}| = 9$, $K = 3$, and players u_1 and u_2 formulate the coalition σ_x.

We introduce three criterions, i.e., coalition utility, coalition efficiency, and coalition usage factor, to evaluate the performance of sessions in the system and present the concepts of them as follows:

- *Coalition utility*: The coalition utility of any coalition σ is defined as the ratio of the total bandwidth (i.e., utility) σ occupied to the average bandwidth per user, denoted by

$$\varphi_\sigma = \frac{\sum_{u \in \sigma}(R_u)}{|\mathbf{C}|/|\mathbf{U}|}.$$

- *Coalition usage factor*: The coalition usage factor of any coalition σ is defined as the ratio of the achieved date rate (i.e., payoff) to the total bandwidth σ occupied, denoted by

11. We find that 600 rounds are enough for players converge from an arbitrary initial configuration to a stable situation.

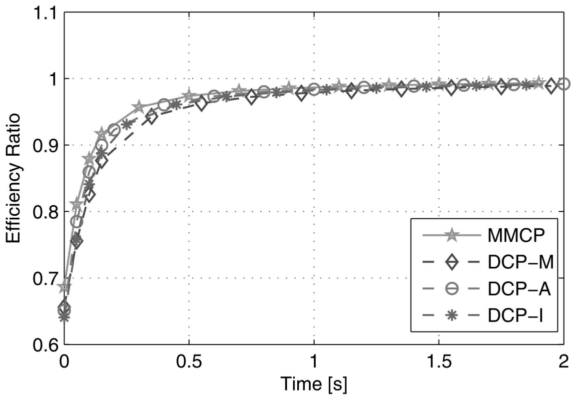

Fig. 12. Efficiency ratio versus time using $W = 15, |\mathbf{C}| = 8, |\mathbf{U}| = 5$, $K = 4$, and $\sigma_x = \{u_1, u_2\}$.

$$\tau_\sigma = \frac{\min_{u \in \sigma}(R_u)}{\sum_{u \in \sigma}(R_u)}.$$

- *Coalition efficiency*: The coalition efficiency of any coalition σ is defined as the ratio of the achieved date rate (i.e., payoff) to the average bandwidth per user, denoted by $\phi_\sigma = \frac{\min_{u \in \sigma}(R_u)}{|\mathbf{C}|/|\mathbf{U}|}$.

Recall the example in Fig. 11, it is easy to see that $R_{u_1} = R_{u_2} = 1.0$ for case in Fig. 11a, and $R_{u_1} = 0.75$, $R_{u_2} = 1.0$ for case in Fig. 11b. Thus, we have $\varphi_{\sigma_x} = 2.25$, $\tau_{\sigma_x} = 0.5$, and $\phi_{\sigma_x} = 1.125$ for case in Fig. 11a and $\varphi_{\sigma_x} = 1.97$, $\tau_{\sigma_x} = 0.43$, and $\phi_{\sigma_x} = 0.844$ for case in Fig. 11b.

The criterion of coalition utility reflects the ability of any coalition to occupy the system bandwidth. The criterion of coalition usage factor reflects the usage ratio of total bandwidth any coalition occupied. The criterion of coalition efficiency reflects the ability of any coalition to achieve a given data rate (i.e., payoff). In fact, the coalition efficiency is equal to the product of coalition utility and usage factor. Note that for single-hop session, coalition utility is equivalent to coalition efficiency and coalition usage factor is always equal to 1.

It is easy to see that: 1) $\tau_{\sigma_x} \le \frac{1}{|\sigma_x|}$ and 2) If $\tau_{\sigma_x} \ne \frac{1}{|\sigma_x|}$, the total bandwidth σ_x occupied is not fully used, i.e., any bandwidth wasted. In Fig. 11b, we have $\tau_{\sigma_x} = 0.43 < 0.5$, which implies that some players in the coalition waste any bandwidth. This can be validated by the fact that the achieved data rate (payoff) of u_2 is smaller than the bandwidth u_2 occupied. From Lemma 5 in Section 5, players in the same coalition tend to achieve the same utility in an MMCPNE channel allocation, and thus, τ_{σ_x} is close to its upbound, i.e., $\frac{1}{|\sigma_x|}$.

Furthermore, we define *average coalition utility* as the average of coalition utility per round over a long period of time. Similarly, we define *average coalition usage factor* and *average coalition efficiency* as the average of coalition usage factor and efficiency, respectively.

7.3 Convergence of MMCP and DCP Algorithms

We first investigate the convergence of the proposed algorithm. We introduce the notion of *Efficiency Ratio* defined in [5] to validate whether a channel allocation is equilibrium state. Due to space limitation, we do not present the definition in detail.

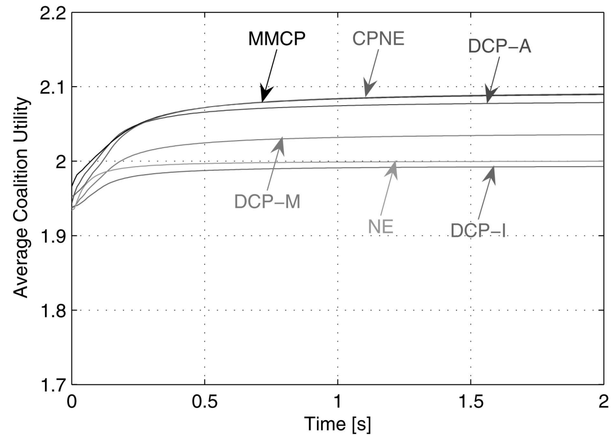

Fig. 13. Average coalition utility versus time using $W = 15, |\mathbf{C}| = 8$, $|\mathbf{U}| = 5, K = 4$, and $\sigma_x = \{u_1, u_2\}$.

We present the simulation results of *efficiency ratio* for MMCP and DCP algorithms in Fig. 12, where $W = 15$, $|\mathbf{C}| = 8$, $|\mathbf{U}| = 5$, $K = 4$, and players u_1 and u_2 formulate the coalition σ_x.[12] From Fig. 12, we can easily find that MMCP, DCP-M, DCP-A, and DCP-I algorithms all converge to equilibrium state. Furthermore, according to [5], a particular channel allocation is NE if its efficiency ratio converge to one. Thus, we can also find that our algorithms all converge to the Nash equilibrium state.

7.4 Performance of Multihop Session

We investigate the performance of the multihop session σ_x in different equilibrium states. We use the measures defined in Section 7.2, i.e., coalition utility, usage factor, and efficiency, to evaluate the performance of the multihop session. Note that the NE algorithm is an algorithm which enables the players converge to any Nash equilibrium [5]. The CPNE algorithm is an algorithm which enables the players converge to conventional CPNE state. We do not present the pseudocodes of NE and CPNE due to space limitation.

We present the simulation results of *coalition utility* of σ_x in Fig. 13, from which we find that MMCP, CPNE, and DCP-A algorithms show higher coalition utility compared with other algorithms, specifically, the curves of MMCP and CPNE are almost overlapped. In other words, the coalition σ_x tends to occupy more bandwidth in the state of MMCPNE, CPNE, and ACPNE. This phenomenon in MMCPNE (or CPNE) can be seen as the results of Lemmas 2 and 3. In detail, players in the coalition increase the utility by moving their radios according to Lemma 2 or Lemma 3. In ACPNE, this phenomenon is caused by the fact that all players in the coalition are willing to improve the total bandwidth.[13] The coalition utility of CPNE is a little higher than DCP-A due to the cooperation gain.

As mentioned previously, the criterion of coalition utility cannot reflect the ability of a coalition to achieve a given data rate. In other words, the algorithm with high coalition

12. Note that we present the results of the first 2 second since the curves tend to be steady in the latter time.

13. Accurately speaking, in ACPNE, all players in the coalition try their best to improve the total bandwidth under the restriction of no harming the minimal utility of the members.

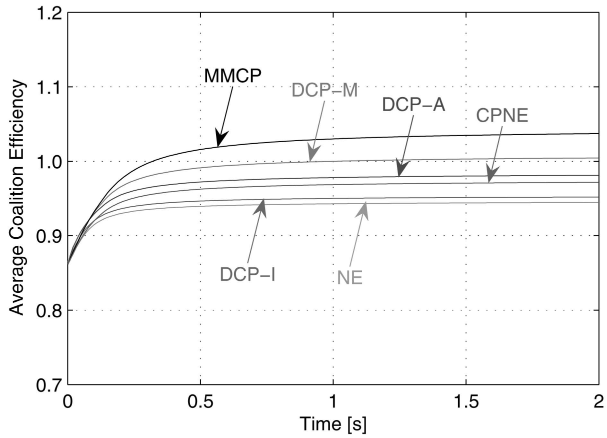

Fig. 14. Average coalition efficiency versus time using $W = 15$, $|\mathbf{C}| = 8$, $|\mathbf{U}| = 5$, $K = 4$, and $\sigma_x = \{u_1, u_2\}$.

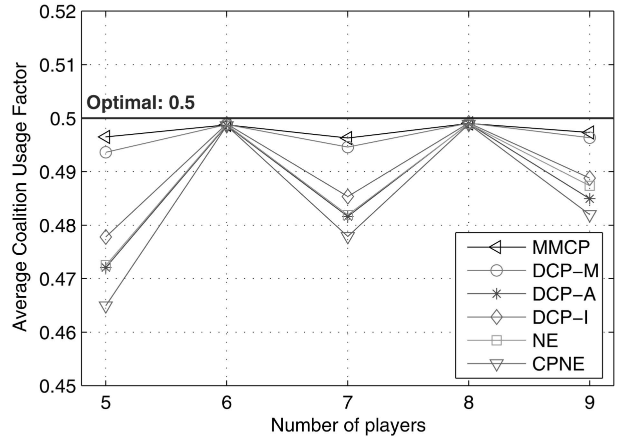

Fig. 16. Average coalition usage factor versus player using $W = 15$, $|\mathbf{C}| = 8$, $K = 4$, and $\sigma_x = \{u_1, u_2\}$.

utility cannot ensure high payoff, i.e., the achieved data rate. We show this phenomenon by CPNE and DCP-A algorithms in Fig. 14.

In Fig. 14, we present the simulation results of *coalition efficiency* of σ_x, which exactly reflects the achieved data rate of the coalition. We can observe that CPNE and DCP-A algorithms converge with low coalition efficiency (i.e., low payoff or low data rate) although it shows high coalition utility in Fig. 13. However, MMCP algorithm still shows the highest performance in terms of coalition efficiency. The NE and DCP-I algorithms show the lowest performance in terms of both coalition utility and efficiency in the multihop networks.

To measure the usage ratio of total bandwidth σ_x occupied, we present the simulation results of *coalition usage factor* of co_x in Fig. 15. We find that MMCP algorithm converges to the upbound of coalition usage factor, i.e., $\tau_{\sigma_x} \approx \frac{1}{|\sigma_x|} = 0.5$. Thus, we consider MMCPNE as an *efficient* channel allocation scheme. CPNE and DCP-A algorithms show low coalition usage factor due to the fact that they occupy high total bandwidth; however, the available bandwidth is low. It is well coincident with former analysis. We can also find that NE algorithm shows low

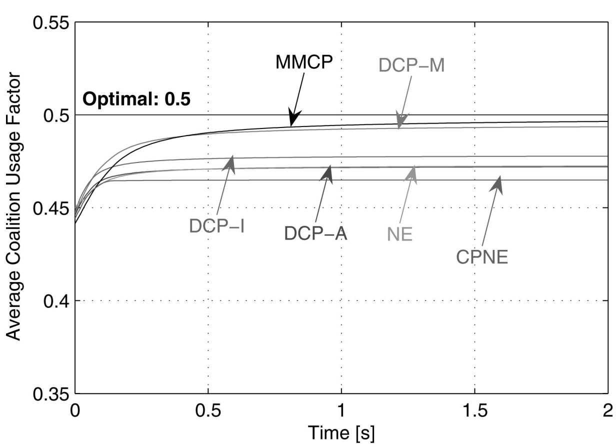

Fig. 15. Average coalition usage factor versus time using $W = 15$, $|\mathbf{C}| = 8$, $|\mathbf{U}| = 5$, $K = 4$, and $\sigma_x = \{u_1, u_2\}$.

performance in terms of coalition usage factor, specifically, the performance of NE is very closely to that of DCP-A. Similarly, we consider CPNE, ACPNE, and NE as *inefficient* channel allocation schemes.

In Figs. 13, 14, and 15, we find that DCP-M and MMCP algorithms show tiny performance difference, e.g., less than 1 percent in terms of coalition usage factor and 5 percent in terms of coalition efficiency. The DCP-A and MMCP algorithms show large performance difference in terms of the coalition usage factor due to the large total bandwidth DCP-A occupied and low bandwidth it availably used. The DCP-I and MMCP algorithms show large performance difference in terms of the coalition efficiency due to the low total bandwidth DCP-I occupied.

We investigate the impaction of player number on coalition usage factor and present our simulation results in Fig. 16. We can see that MMCP algorithm keeps the system in a state of high coalition usage factor, whereas CPNE, NE, and DCP-A algorithms show low usage factor in most case.

It is interesting that all algorithms converge to the same value, i.e., the upbound of the coalition usage factor, when total players number $|\mathbf{U}| = 6$ or 8. We explain this phenomenon as follows: When radio number $K = 4$ and player number $|\mathbf{U}| = 6$, all radios are equally distributed in $|\mathbf{C}| = 8$ channel in NE state. Each channel is shared by the same number of radios, i.e., $k_c = 3,\ \forall c \in \mathbf{C}$, and thus, $\mathbf{C}^+ = \mathbf{C}$ and $\mathbf{C}^- = \emptyset$. It is easy to see that the NE channel allocation is also MMCPNE (or MCPNE, ACPNE, ICPNE, CPNE) in this case. Thus, all algorithms converge to the same value of coalition utility factor. The conclusion is well proved by the simulation results when $|\mathbf{U}| = 4$ and $K = \{2, 4, 6\}$. Due to the space limitation, we do not present the simulation results in such cases.

7.5 Performance of Networks

We investigate the performance of the whole networks in different equilibrium states. To evaluate the performance of the networks, we introduce the concept of *networks throughput* defined as

$$\Theta = \sum_{l \in \mathbf{L}} (\mathcal{R}_l \times \mathcal{N}_l),$$

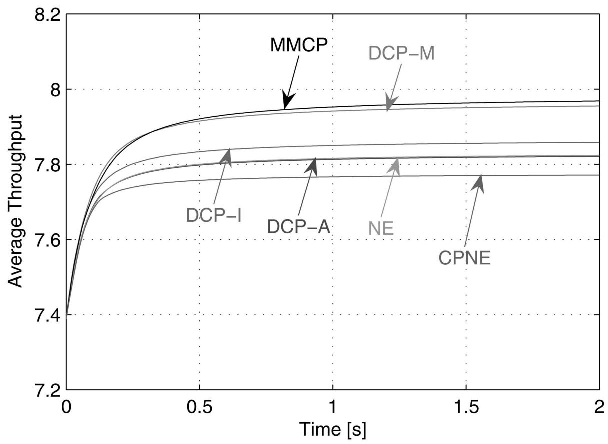

Fig. 17. Average throughput versus time using $W = 15$, $|\mathbf{C}| = 8$, $|\mathbf{U}| = 5$, $K = 4$, and $\sigma_x = \{u_1, u_2\}$.

where $\mathcal{R}_l$ is the achieved data rate of session l and $\mathcal{N}_l$ is the number of hops of session l.

We present the simulation results of networks throughput in Fig. 17, from which we find that MMCP and DCP-M algorithms show higher networks throughput compared with other algorithms. It is worth noting that the curves of throughput in Fig. 17 are very similar to the curves of coalition usage factor in Fig. 15. This phenomenon can be explained by the fact that the higher coalition usage factor is, the less bandwidth multihop session wastes, and thus, the higher networks throughput is.

7.6 Discuss

From the simulation results, we list the performance evaluations for the different equilibrium states in Table 2, where "L," "M," and "H" are the abbreviations for low, middle, and high, respectively. We can observe that the proposed MMCPNE ensures not only high achieved data rate for the multihop sessions, but also high throughput for the whole networks. The NE proposed in [5] shows low performance in all terms in multihop networks. Furthermore, we find that MCPNE can be seen as a feasible approximation of MMCPNE, while ACPNE and ICPNE show poor performance in terms of coalition efficiency and usage factor, respectively, compared with MMCPNE.

For both MMCP and DCP algorithms, a practical issue is *how each player obtains the channel allocation in the form of matrix* $\mathbb{X}$ (e.g., line 3 in Table A or Table C). In fact, this can be achieved by having an extra radio per device for scanning all of the channels, or by having an extra common channel for all players periodically broadcasting their strategies. For the former case, it will introduce additional complexity for device due to the hardware of extra radio and the ability of scanning broad spectra, while for the hander case, it will decrease the spectral efficiency due to the common channel for broadcasting strategy.

For MMCP algorithm, another practical issue is *how players within a coalition adjust their strategies in a cooperative manner* (e.g., lines 6, 8, and 10 in Table B). This can be achieved by defining a center node (e.g., sender) for each session and running the MMCP algorithm in the center node. After obtaining the best strategies for all players within the session, the center node propagates the best strategies in the session. Such a propagation will introduce additional overhead in communications. Note, however, that this overhead is very small compared with the data rate. Specifically, the strategies propagating traffic for session σ_x are $|\mathbf{C}| \cdot (|\sigma_x| - 1)$ bits per round, since each player's strategy defined in (1) can be expressed as a $|\mathbf{C}|$ bits number.

TABLE 2
Performance Evaluations for Different Equilibrium States

	Multi-hop Sessions			Networks
	Utili.	Usage	Effic.	Throughput
NE	L	L	L	L
CPNE	H	L	L	L
MCPNE	M	H	M	H
ACPNE	H	L	L	L
ICPNE	L	L	L	L
MMCPNE	H	H	H	H

8 Conclusion

In this paper, we have studied the problem of competitive channel allocation among devices which use multiple radios in the multihop networks. We first analyze that NE and CPNE channel allocation schemes are not suitable for the multihop networks due to the poor performance of achieved data rate of the multihop sessions. Then, we propose a novel coalition-proof Nash equilibrium, denoted by MMCPNE, to ensure the multihop sessions to achieve high data rate without worsening the performance of single-hop sessions. We investigate the existence of MMCPNE and propose the necessary conditions for the existence of MMCPNE. Finally, we provide several algorithms to achieve the exact and approximated MMCPNE states. We study their convergence properties theoretically. Simulation results show that MMCPNE outperforms CPNE and NE schemes in terms of the achieved data rates of multihop sessions and the throughput of whole networks due to cooperation gain.

Acknowledgments

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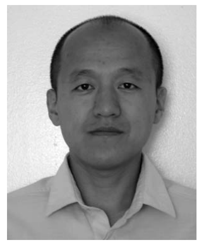

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