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# Modeling the Dynamics of Coalition Formation Games for Cooperative Spectrum Sharing in an Interference Channel

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**Abstract**

Although establishing cooperation in a wireless network is a dynamic process, most game theoretic coalition formation models proposed in the literature are static. We analyze a dynamic coalition formation game based on a Markovian model for the spectrum sharing problem in an interference channel. Our model is dynamic in the sense that distributed transmitter-receiver pairs, with partial channel knowledge, reach stable coalition structures (CSs) through a time-evolving sequence of steps. Depending on an interference environment, we show that the game process either converges to the absorbing state of the grand coalition or to the absorbing state of internal and external stability. We also show that, due to myopic links, it is possible that the core of the game is non-empty, but links cannot form the grand coalition to utilize the core rate allocations. We then formulate a condition for the formation of the stable grand coalition. Using simulation we show that coalition formation yields significant gains in terms of average rates per link for different network sizes. We also show average maximum coalition sizes for different distances between the transmitters and their own receivers. Finally, we analyze the mean and variance of the time for the game to reach the stable coalition structures.

**Index Terms**

Coalition formation, game theory, ad hoc wireless networks, network dynamics, resource allocation.

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# I. INTRODUCTION

In wireless interference channels, distributed nodes operating in the same spectrum band act as independent and autonomous agents and can use their limited power and spectrum to either compete or cooperate with one another [1], [2]. In certain interference environments network nodes can optimize their achievable data rates through mutual cooperation [2]. Cooperative game theory has been used to analyze fair and efficient sharing of available spectrum in [3]. A resource allocation scheme for delay-sensitive services based on a Nash bargaining model is analyzed in [4]. In [5], it is shown that in a wireless multiuser network, when nodes play rationally, i.e., to maximize their payoffs, and are allowed to cooperate, cooperation among all nodes is desirable only when all nodes experience similar signal-to-noise ratio (SNR). Nodes with arbitrary SNR values may achieve maximum gains by cooperating only with the nodes that are causing high interference to them. In such scenarios, it is important to analyze cooperative interactions between small groups of nodes, as well as cooperative interactions among all network nodes. It is useful to model these scenarios as a game in which the possibility of cooperative interactions between subsets of players, i.e., coalitions, is analyzed. While strategies based on non-cooperative game theory are mainly concerned with individual utility, coalition game theory deals with maximizing the entire system payoff while satisfying individual rational demands of the network nodes [5], [6]. These features make coalition game theory suitable for analyzing cooperative interactions among wireless nodes in distributed scenarios.

In coalition games, a coalition is a set of autonomous agents or players which may cooperate in order to increase their individual gains. The autonomous formation of coalitions is an important form of interaction in multi-agent systems, such as ad hoc wireless networks, networks of autonomous robots, etc , because many applications require agents to cooperate to fulfill tasks or to improve the efficiency of network resource usage. For instance, in [7], the problem of task allocation among a number of autonomous agents such as Unmanned Aerial Vehicles (UAVs) is modeled as a coalition formation game. Coalition game theory has also been applied to the field of Distributed Artificial Intelligence, in most cases to find efficient solutions to the allocation of tasks to group of agents [8]. In ad hoc networks, radios interact with one another to establish a network topology, enabling communications between any pair of nodes either through a direct link or a multi-hop path. In [5], coalition game theory was used to analyze the stability of the grand coalition in a cooperative wireless network. A coalition game approach to provide incentives to selfish nodes in wireless packet-forwarding networks was utilized in [6]. In a coalition game, the coalition of all the cooperating players is called the grand coalition. An individual player is called a singleton coalition [9].

In this paper, we formulate a dynamic coalition formation game to analyze the spectrum sharing problem among $N$ wireless links, co-existing in an interference channel of bandwidth $W$. The transmitters on each of the $N$ links have the same power constraint $P$. The strategic decision for any link consists of an allocation of power $P$ across the available bandwidth to maximize the individual data rate. Coalitions are formed if some rational demands of wireless links are satisfied, with coalition participants agreeing to share (allocate power over) bandwidth $W$ according to certain ratios; otherwise, each link will form a singleton coalition by allocating power $P$ over the entire spectrum band. A key question this paper tries to address, is how to coordinate distributed wireless links with partial channel knowledge to form coalitions to maximize their data rates, while also taking into account the overhead in message exchange needed for coalition formation.

Most traditional approaches to coalition formation analyze stable coalition structures. Stable coalition structures correspond to the equilibrium state in which players do not have incentives to leave already established coalitions. These approaches fail to specify *how* the players arrive at equilibrium. Establishing cooperation in a wireless network is a dynamic process and three important questions must be addressed: 1) How are the coalitions formed?; 2) How do players arrive at equilibrium?; and 3) What is the long term behavior of the coalition formation process? As cooperation among nodes may involve some short term costs, long term behavior can provide more detailed insight into the true benefits of coalition formation.

The dynamics of coalition formation for economic behavior and artificial intelligence is the topic of [10], [11]. Individual adaptation rules are defined as a finite Markov chain in [7] to analyze the dynamic model of coalition formation for trade and finance. The main difference between that model and ours is that [10] assumes that each player can observe all the other players' demands at any time $t$, while the players in our model learn the demands of the other players through coalitional interactions. The authors in [10] also assume that no single player can do better on her own than as a member of any coalition; our model does not make such an assumption. Moreover, in contrast with [10], we analyze our Markovian model to evaluate how long it will take the players to establish stable coalition structures. We model coalition structures as a sequence of random variables describing the state of the system, and the transition mechanism between coalition structures is modeled as a Markov chain.

A Bayesian reinforcement learning model is presented in [11] to analyze agent interaction in uncertain environments and to evaluate the long term impact of agent coalition technologies on e-commerce. In [12], the dynamics of coalition formation for networks of autonomic agents are presented using graph theory. While [12] investigates network formation under the fundamental assumption that nodes in networks always want to cooperate with one another, our model does not make such an assumption.

Our main contribution is to study the spectrum sharing problem as a dynamic coalition formation game in which links self-organize to reach stable coalition structures through a time-evolving sequence of steps. The proposed game accounts for the case of partial channel knowledge, where a specific link is aware of the aggregate strength of interfering signals but has no knowledge of the level of interference it inflicts on the other receivers in the network. We model stable coalition structures as the absorbing states of a Markov process. Depending on the interference environment, our coalition formation game model either converges to the absorbing state of the grand coalition, or it converges to the absorbing state of any coalition structure satisfying the condition of internal and external stability (IES). Internal stability means that no coalition member has an incentive to leave its coalition to become a singleton, and external stability means that existing coalitions within a coalition structure have no incentive to merge. Using simulations we analyze the average data rate per link for different network sizes, when all links act independently, when links can form coalitions but there is no cost to coalition formation, and when links can form coalitions but incur some cost in the coalition formation process. Our results show that the proposed coalition formation solution yields significant gains in terms of average data rates per link for different network sizes. We show that the coalitional interference game with $N > 2$ is a game with positive externalities, i.e., players outside any coalition $\mathbf{S}$ do not lose by any player $i$ joining $\mathbf{S}$ [13], and has partition function form (PFF). We convert this PFF of the game into characteristic function form (CFF) to analyze the coalition formation process. PFF expresses a game in which coalition values are dependent on the outside coalition behavior, and CFF a game in which coalition values are independent of outside coalition behavior. We also show that for certain interference environments the core of the game is non-empty, but myopic links cannot form the stable grand coalition to utilize the core rate allocations. We then formulate a condition for the formation of the stable grand coalition. The core of a game is the set of feasible allocations that cannot be blocked by any coalition of players. We demonstrate that the condition of superadditivity for the formation of the stable grand coalition reduces the region of optimal coalition structures, as compared to our formulated condition. Superadditivity implies that forming a coalition never harms, i.e., the value or worth of two disjoint coalitions is at least as large when they form a coalition as when they remain apart. Simulation results are presented to show stable coalition structure regions for various network sizes.

The rest of the paper is organized as follows. Section II presents a coalition formation game theoretic framework for an interference channel. Section III presents the system setup, while Section IV introduces a Markovian model of dynamic coalition formation. Section V presents simulation results and an analysis of the proposed dynamic game model, and Section VI concludes the paper.

## II. Coalition game theory and interference channel

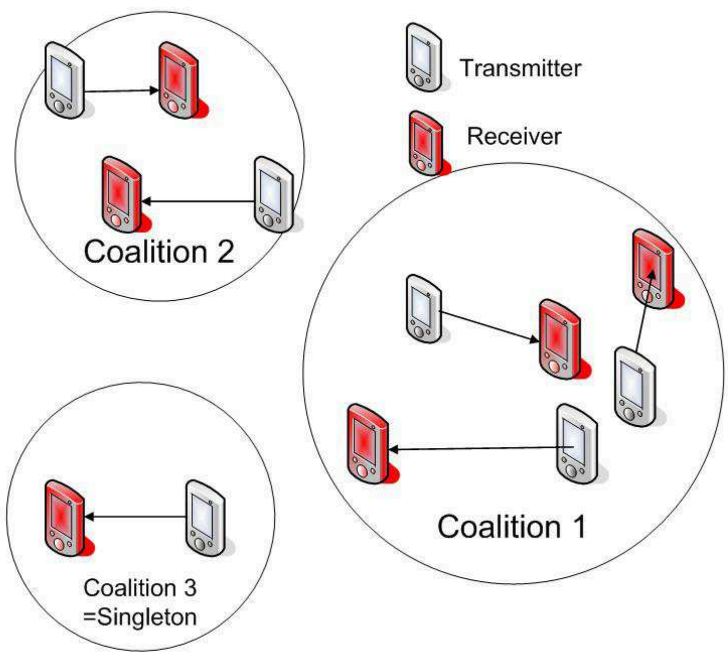



Fig. 1. A simple example of coalition formation for distributed transmitter/receiver pairs in a wireless interference channel.

Coalition game theory provides useful tools to decide which group of players will cooperate with each other to achieve their goals efficiently [9]. Therefore, to analyze cooperative interactions between distributive spectrum sharing transmitter/receiver pairs, we model the distributed cooperative spectrum sharing problem as a coalition formation game among $N$ transmitter/receiver pairs. Our game is specified by the set $\mathbf{N}$ of players, where $\mathbf{N} = \{1, 2, ....., N\}$. A coalition, $\mathbf{S}$, in our game is defined to be a non-empty subset of $\mathbf{N}$. The set $\mathbf{N}$ is also a coalition, called the grand coalition. In Fig. 1 we provide a simple example of coalition formation for distributed transmitter/receiver pairs in a wireless interference channel.

The most common form of a coalition game is the characteristic function form. In the characteristic function form (CFF) of coalition games, utilities achieved by the players in a coalition are unaffected by those outside it.

*Definition 1:* The $N$-player coalitional game in *characteristic function form* (CFF) is given by the pair $(\mathbf{N}, v)$, where $\mathbf{N}$ is the set of players and $v$ is a real-valued function, called the value of the game, defined on the subsets of $\mathbf{N}$, with $v(\emptyset) = 0$.

The quantity $v(\mathbf{S})$ in the CFF game is a real number associated with each subset of $\mathbf{S} \subset \mathbf{N}$, which may be considered as the value, or worth, of a coalition when its members group together as a unit. Similarly, the quantity $v_{i,i\in\mathbf{S}}(\mathbf{S})$ in the CFF game is a real number for each link $i \in \mathbf{S}$, which represents the payoff obtained by link $i$ from coalition $\mathbf{S}$. To model all $N$ link coalitions, we define coalition structures as follows:

*Definition 2:* A *coalition structure* (CS) is a partition of $\mathbf{N}$ into exhaustive and disjoint subsets, where each subset is a coalition. The set of all possible CSs is denoted as $\mathbf{C}$, where $\mathbf{C} = \{\mathbf{C}_1, \mathbf{C}_2, \ldots, \mathbf{C}_{|\mathbf{C}|}\}$.

The cardinality of $\mathbf{C}$, i.e., the number of all possible CSs (excluding empty coalitions) for $N$ links is given by the Bell number

$$\begin{aligned} &B_0 = 1 \\ &B_N = \sum_{k=0}^{N-1} \binom{N-1}{k} B_k, \text{ for } N \geq 1. \end{aligned} \tag{1}$$

For instance, $\mathbf{C}$ for $N = 3$ links is given as:

$$\mathbf{C} = \Big\{ \{\{1\},\{2\},\{3\}\}, \{\{1,2\},\{3\}\}, \{\{1,3\},\{2\}\}, \{\{1\},\{2,3\}\}, \{\{1,2,3\}\} \Big\}.$$

We define worth or value $V$ of a CS as the sum of values of coalitions that are elements of that CS, i.e., $V(\mathbf{C}_j) = \sum_{\mathbf{S}_i \in \mathbf{C}_j} v(\mathbf{S}_i)$, for $j = 1, 2, \ldots, |\mathbf{C}|$.

The study of coalition formation in wireless networks has previously focused on cohesive games, [5], [14], i.e., games where the value of the grand coalition formed by the set of all users $\mathbf{N}$ is at least as large as the sum of the values of any partition of $\mathbf{N}$. The authors in [5], [14], also assume that there is no cost to the coalition formation process. In such coalition games, coalition structure generation is trivial because the wireless nodes always benefit by forming the grand coalition.

However, many coalition game models of wireless node cooperation are not cohesive (see, for example, [15]), because in wireless networks there is some cost to the coalition formation process itself or because in the grand coalition there is no resource reuse, which is sometimes beneficial in wireless networks. For instance, coalition formation in wireless networks may require wireless links to exchange coalition messages, which may be costly. In coalition games that are not cohesive, some coalitions gain by merging, while others do not. In such network scenarios, the network welfare maximizing coalition structure varies. The aim in such scenarios is to maximize the social welfare of $\mathbf{N}$ by finding a coalition structure: $\mathbf{C}_j^* = \arg \max_{\mathbf{C}_j \in \mathbf{C}} V(\mathbf{C}_j)$ [16]. For such scenarios, we say that any coalition has reached individual stability or equilibrium if it is internally and externally stable.

*Definition 3:* *Internal stability* means that no link has an incentive to leave its coalition to become a singleton (individual non-cooperative link), i.e., $v_{i,i \in \mathbf{S_1}}(\mathbf{S_1}) \geq v(\{i\})$, $\forall i \in \mathbf{S_1}$, and *external stability* means that no other coalition has an incentive to join coalition $\mathbf{S_1}$, i.e., $v(\mathbf{S_2}) > v(\mathbf{S_1} \cup \mathbf{S_2}) - v(\mathbf{S_1})$, $\forall \mathbf{S_2} \subseteq \mathbf{S_1}^c$, where $\mathbf{S_1}^c$ represents the complement of $\mathbf{S_1}$.

If each coalition in a CS is IES, then the CS is called IES stable; this is also known as a multi-coalition equilibrium. For instance, if $N = 4$ links are participating in a coalition game, then the CS $\{\{1,2\},\{3\},\{4\}\}$ is IES stable CS if all the coalitions, i.e., $\{1,2\},\{3\}$ and $\{4\}$, satisfy the IES requirements. The CS that consists of all singleton coalitions, i.e., $\{\{1\},\{2\},\{3\},....,\{N\}\}$ is stable if each singleton coalition satisfies $v(\{i\}) > v_{i,i \in \mathbf{S}}(\mathbf{S})$, $\forall i$, $\forall \mathbf{S} = \{i\} \cup \{j\}$, $i \neq j$. This is known as the Nash Equilibrium with no cooperation. We define *minimum rational payoff* for an individual link in the coalition formation game as:

*Definition 4:* A payoff is a *minimum rational payoff* for link $i$ in the coalition $\mathbf{S}$, if a payoff partition gives $i$ at least as much value as it receives before forming the coalition $\mathbf{S}$. The *minimum rational payoff* for link $i$ in coalition $\mathbf{S}$ is denoted by $d_i(\mathbf{S})$, $i \in \mathbf{S}$.

## III. System setup and interference channel with externalities

We consider a network consisting of N transmitter/receiver pairs. Each transmitter/receiver pair is referred to as a link. Links operate in a wireless channel with bandwidth $W$. We assume that each wireless link treats multiuser interference as noise and no interference cancelation techniques are employed. The transmission strategy for each link is simply its power allocation. Links are aware of one another's presence, for instance by overhearing one another's transmissions. The received signal for link $i$ is given by:

$$Y_i = \sum_{j=1}^{N} c_{ji} X_j + z_i, \; i \in \mathbf{N}, \tag{2}$$

where $X_j$ and $Y_i$ are input and output signals, respectively. The noise processes are independent and identically distributed (i.i.d.). $h_{ji} = \mid c_{ji} \mid^2$ is the channel gain between the transmitter of link $j$ and the receiver of link $i$. We assume that in a given scenario the direct channel gains, i.e., $h_{ii} = \mid c_{ii} \mid^2$, are the same for all the links and $h_{ji} = \kappa / d_{ji}^{\alpha}$ with $\kappa$ being the path loss constant, $\alpha$ the path loss exponent and $d_{ji}$ the distance between the transmitter of link $j$ and the receiver of link $i$ with $j \neq i$. The transmitter of any link $i$ has an average power constraint $P$. We assume that at the start of any coalition formation, each link knows its respective direct channel gains and the aggregate interference caused by all others but has no knowledge of the level of interference it inflicts on the other receivers in the network. A link will form a singleton coalition by allocating power uniformly over the entire band $W$.

We quantify the payoff obtained by player $i$ as the Shannon capacity of the $i$th link. Assuming uniform power allocations over the entire band $W$ from the other players, the value $v(\{i\})$ of a singleton coalition, i.e., the rate payoff achieved by link $i$ when acting alone, can be expressed as:

$$v(\{i\}) = R_i = \frac{W}{2} log_2(1 + \frac{h_{ii} P_i}{W N_0 + \sum_{j, j \neq i} h_{ji} P_j}), \tag{3}$$

We can rewrite Eq. 3 in terms of SNR of link $i$ as:

$$v(\{i\}) = R_i = \frac{W}{2} log_2(1 + \frac{h_{ii} SNR_i}{1 + \sum_{j, j \neq i} h_{ji} SNR_j}), \tag{4}$$

where $SNR_i = \frac{P_i}{W N_0}$ [18]. The value $v(\mathbf{S})$ of a coalition $\mathbf{S}$, i.e., the rate payoff achieved by all links in $\mathbf{S}$, can be expressed as:

$$v(\mathbf{S}) = \sum_{i \in \mathbf{S}} R_i = \sum_{i \in \mathbf{S}} \frac{W}{2} \eta_i log_2(1 + \frac{(\frac{h_{ii} SNR_i}{\eta_i})}{1 + \sum_{j \in \mathbf{S}^c} h_{ji} SNR_j}), \ \mathbf{S} \subset \mathbf{N}, \tag{5}$$

where $\mathbf{S}^c$ is the complement of $\mathbf{S}$ in $\mathbf{N}$, $0 \leq \eta_i \leq 1$ is the fraction of the band that link $i$ uses and $\sum_{i, i \in \mathbf{S}} \eta_i \leq 1$. Finally, the value $v(\mathbf{N})$ of a grand coalition is:

$$v(\mathbf{N}) = \sum_{i \in \mathbf{N}} R_i = \sum_{i \in \mathbf{N}} \frac{W}{2} \eta_i log_2(1 + \frac{h_{ii} SNR_i}{\eta_i}), \tag{6}$$

with $\sum_{i, i \in \mathbf{N}} \eta_i \leq 1$. We now consider the cases of a 2-player game and an $N$-player game.

*Case 1: $N = 2$ links*

When two links transmit with an average power constraint $P$ in the spectrum band $W$, it is a good strategy to form the grand coalition whenever:

$$\begin{aligned} \frac{W}{2} log_2(1+\frac{h_{ii}SNR_i}{1+h_{ji}SNR_j}) &\leq \frac{W}{2}\eta_i log_2(1+\frac{h_{ii}SNR_i}{\eta_i}) \\ &= \frac{W}{2} log_2(1+\frac{h_{ii}SNR_i}{\eta_i})^{\eta_i} \end{aligned} \tag{7}$$

or

$$h_{ji}SNR_j \geq \frac{h_{ii}SNR_i}{(1+(\frac{h_{ii}SNR_i}{\eta_i}))^{\eta_i}-1} - 1, \text{ where } i \neq j. \tag{8}$$

*Case 2: $N$ links*

For $N$ links there are $2^N$ possible coalitions. As compared to the singleton coalitions, links will benefit from the formation of the grand coalition as long as:

$$\sum_{j,j\neq i} h_{ji}SNR_j \geq \frac{h_{ii}SNR_i}{(1+(\frac{h_{ii}SNR_i}{\eta_i}))^{\eta_i}-1} - 1, \; i, \; j \in \mathbf{N}. \tag{9}$$

It is also possible that some links form coalitions $\mathbf{S} \subset \mathbf{N}$. Links will benefit from the formation of the coalition $\mathbf{S}$ as long as:

$$\sum_{j,j\neq i} h_{ji}SNR_j \geq \frac{h_{ii}SNR_i}{(1+(\frac{(\frac{h_{ii}SNR_i}{\eta_i})}{1+\sum_{k\in\mathbf{S}^c} h_{ki}SNR_k}))^{\eta_i}-1} - 1, \; i, \; j \in \mathbf{S}, \tag{10}$$

where $\sum_{k\in\mathbf{S}^c} h_{ki}SNR_k$ is the perceived interference from the links in $\mathbf{S}^c$. We next introduce some definitions related to the concept of externalities.

*Definition 5:* A *coalition game with externalities* is a game in which the utility that a group of players can achieve through cooperation depends on what other coalitions form.

In CFF games, any coalition $\mathbf{S} \subset \mathbf{N}$ generates a value $v(\mathbf{S})$ and this value is independent of what other players not in $\mathbf{S}$ do. In PFF games externalities are allowed, and these externalities are captured by presenting the real valued function $v$ as a function of a coalition $\mathbf{S}$ and a partition $\rho$, i.e., any coalition $\mathbf{S} \subset \mathbf{N}$ generates a value $v(\mathbf{S};\rho)$ where $\rho$ is a partition of $\mathbf{N}$ with $\mathbf{S} \in \rho$. We define $v(\emptyset;\rho) = 0$ for all partitions $\rho$ of $\mathbf{N}$.

*Definition 6:* A coalition game is said to have *positive externalities* if for any mutually disjoint coalitions $\mathbf{S_1}, \mathbf{S_2}, \mathbf{S_3} \subset \mathbf{N}$:

$$v(\mathbf{S_3};\{\mathbf{S_1} \cup \mathbf{S_2}, \mathbf{S_3}\}) \geq v(\mathbf{S_3};\{\mathbf{S_1}, \mathbf{S_2}, \mathbf{S_3}\}). \tag{11}$$

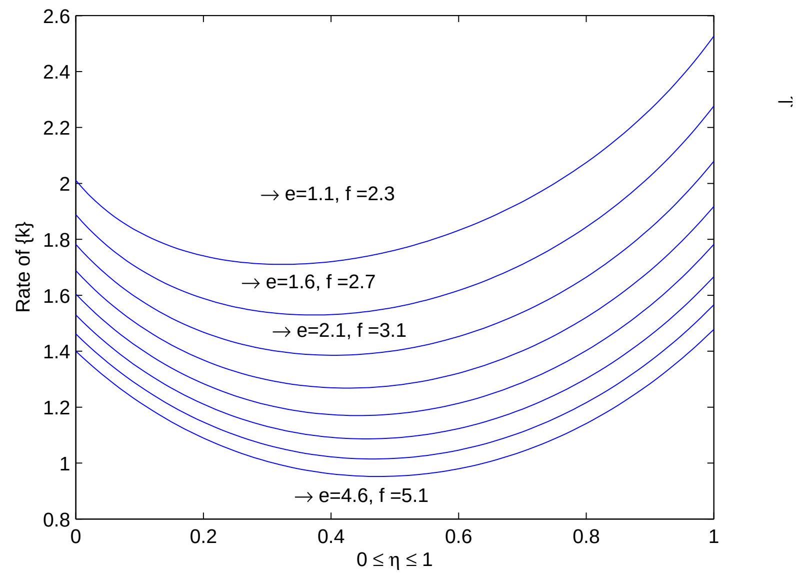



Fig. 2. Convexity of rate function for singleton coalition $\{k\}$ with respect to $\eta$ for different values of $e = h_{ik}SNR_i$ and $f = h_{jk}SNR_j$ with $SNR_i = SNR_j = 10$dB, when $\{k\}$ remains a singleton coalition and $\{i\} \cup \{j\}$ is formed.

In other words, the coalition value $v(\mathbf{S_3})$ either remains the same or increases whenever $\mathbf{S_1} \cup \mathbf{S_2}$ is formed as compared to when $\mathbf{S_1}$ and $\mathbf{S_2}$ prefer to remain apart.

*Theorem 3.1:* The proposed coalition formation game in an interference channel is a game with positive externalities.

*Proof:* Consider any three singleton coalitions $\{i\}$, $\{j\}$ and $\{k\}$ such that $\{i\} \cap \{j\} \cap \{k\} = \varnothing$. To maximize their rate payoffs, each singleton coalition either allocates its constrained power over the entire band $W$, or two of them decide to form $\{i\} \cup \{j\}$ by allocating the same power across disjoint sub bands in certain ratios $\eta$ and $(1-\eta)$, of the spectrum band $W$, subject to the *minimum rational payoff* condition. Then, using the Shannon capacity equation for an interference channel, coalitions values of $\{k\}$ are:

$$\begin{aligned} v(\{k\};\{\{i\}\cup\{j\},\{k\}\}) = & \frac{W}{2}\eta log_2(1+\frac{h_{kk}SNR_k}{1+h_{ik}\frac{SNR_i}{\eta}})+ \\ & \frac{W}{2}(1-\eta)log_2(1+\frac{h_{kk}SNR_k}{1+h_{jk}\frac{SNR_j}{(1-\eta)}}); \end{aligned} \quad (12)$$

$$v(\{k\};\{\{i\},\{j\},\{k\}\}) = \frac{W}{2}log_2(1+\frac{h_{kk}SNR_k}{1+h_{jk}SNR_j+h_{ik}SNR_i}). \quad (13)$$

By using notation $e = h_{ik}SNR_i,\ f = h_{jk}SNR_j$, for inequality (11) to hold, we need to prove that:

$$\eta log_2(1+\frac{h_{kk}SNR_k}{1+\frac{e}{\eta}})+(1-\eta)log_2(1+\frac{h_{kk}SNR_k}{1+\frac{f}{(1-\eta)}}) \geq log_2(1+\frac{h_{kk}SNR_k}{1+(e+f)}). \tag{14}$$

It can be seen that inequality (14) becomes equality whenever $\{i\}\cup\{j\}$ share the spectrum in the ratio $\eta = \frac{e}{e+f}$ and $1-\eta = \frac{f}{e+f}$. To prove inequality (14), we need to show that the point $\eta = \frac{e}{e+f}$ is the minimum point for the left hand side of inequality (14). If we vary $0 \leq \eta \leq 1$ in the left hand side of inequality (14), with $SNR_i = SNR_j$, keeping the direct channel gain $h_{kk}$ normalized to one and cross channel gains $h_{ik}$ and $h_{jk}$ taking values such that $0 \leq \sum_m h_{mk} \leq 1$, $m \in \{i, j\}$, it can be seen from Fig. 2 that the left hand side of inequality (14) is convex with respect to $\eta$. By taking the first and second derivatives it can be verified that:

$$\frac{d}{d\eta}\left[\eta log\big(1+\frac{h_{kk}SNR_k}{1+\frac{e}{\eta}}\big)+(1-\eta)log_2\big(1+\frac{h_{kk}SNR_k}{1+\frac{f}{(1-\eta)}}\big)\right]_{\eta=\frac{e}{e+f}} = 0 \tag{15}$$

and

$$\frac{d^2}{d\eta^2}\left[\eta log\big(1+\frac{h_{kk}SNR_k}{1+\frac{e}{\eta}}\big)+(1-\eta)log_2\big(1+\frac{h_{kk}SNR_k}{1+\frac{f}{(1-\eta)}}\big)\right]_{\eta=\frac{e}{e+f}} > 0. \tag{16}$$

Eq. (15) and inequality (16) show that $\eta = \frac{e}{e+f}$ corresponds to the minimum point for the left hand side of inequality (14). Hence, whenever the coalition $\{i\}\cup\{j\}$ is formed, the outside coalition is either indifferent to it or it gains from the coalition formation. Therefore, the game has positive externalities. ∎

Coalition games with externalities have PFF and are difficult to analyze [5]. To overcome this problem and to avoid any impact of externalities on the coalition formation, we assign a characteristic function to our coalition game. The usual way to assign a characteristic function is to define $v(\mathbf{S})$ for each $\mathbf{S} \subset \mathbf{N}$ in a way that assumes that the players within each coalition $\mathbf{S}$ cooperate to act together as a unit and the players in $\mathbf{S}^c$ allocate their power over the entire band to compete with $\mathbf{S}$ [9]. The value $v(\mathbf{S})$ may be called the safety level of coalition $\mathbf{S}$. It represents the total payoff that coalition $\mathbf{S}$ can guarantee for itself, even if the only objective of the members of $\mathbf{S}^c$ is to keep the sum of the payoffs to players in $\mathbf{S}$ at a minimum. The value $v(\mathbf{S})$ is a lower bound to the payoff that $\mathbf{S}$ should receive because it assumes that the players in $\mathbf{S}^c$ ignore what possible payoffs they might receive as a result of their actions. For a coalition game in an interference channel, this characteristic function form conversion means that each coalition $\mathbf{S}$ determines its value $v(\mathbf{S}) = \sum_{i\in\mathbf{S}} R_i$, where $\mathbf{S} \subset \mathbf{N}$, by using Eq. 5.

We assume that links are rational and myopic, i.e., links are maximizing their payoffs, conditional on feasibility, and care only about their current payoffs. The decisions by the links to form coalitions are made unanimously, i.e., a coalition is formed only if it is acceptable to everyone involved. Hence, CS changes are decided by the members of a coalition acting together as a unit. Any singleton coalition can decide on its own, as it is an individual coalition of a single link.

A group of links or a coalition can deviate only if all links within the coalition are at least as well off as a result of the proposed deviation. In other words, once links decide to form a coalition they enter a binding agreement and hence cannot unilaterally deviate on their own [9].

## IV. A Markovian Model of Coalition Formation Game

In this section we introduce a dynamic model of distributed coalition formation game for the spectrum sharing problem in an interference channel where multiple users coexist and interfere with one another. In the proposed dynamic coalition formation game a time-evolving sequence of steps is used by links to reach self-organizing stable spectrum sharing CSs. CSs in the dynamic coalition formation game are modeled as a sequence of random variables describing the state of the system, and the mechanism of transitions between CSs is represented by a Markov chain. To incorporate slow changes in the interference environment, the coalition game after reaching equilibrium restarts after some time $T$. This time $T$ may be assigned according to variations in the interference environment. We assume that during one round of coalition formation the interference environment does not change.

The value $v(\mathbf{S})$ of the coalition formation game is the sum of the rates achieved by the member links in the coalition $\mathbf{S}$, as explained in the previous section. Coalition formation in wireless networks requires a decentralized negotiation process, which in general is costly. This cost represents the overhead in message exchange needed for coalition formation, and it can be modeled as directly proportional to coalition size. In this sense, there is no cost to "form" the singleton coalition. A cost function satisfying the above assumption is given as:

$$C(|\mathbf{S}|) = \phi \cdot (|\mathbf{S}| - 1), \tag{17}$$

where $\phi$ is a proportionality constant. Therefore, the net value of a coalition $\mathbf{S}$ considering the cost for coalition formation is given as:

$$\hat{v}(\mathbf{S}) = v(\mathbf{S}) - \big(\phi \cdot (|\mathbf{S}| - 1)\big). \tag{18}$$

### A. A Coalition Formation Game Model

The coalition formation game involves four steps. At the beginning of each round of the coalition formation game, the network is composed of all singleton coalitions, i.e., non-cooperative links. The four steps of the coalition formation game are summarized as follows:

1) *Initialization*: Each individual link computes the aggregate interference caused by all other links.

2) *Coalition formation proposal*: a) At each time slot, each coalition, with probability $p$, proposes a new CS. In this process, one of the links in the coalition acts on behalf of the coalition. (In the case of singleton coalitions, each singleton coalition, i.e., link, individually proposes a new CS with some probability $p$; when two or more links form a coalition $\mathbf{S_1}$, then any link within $\mathbf{S_1}$ is selected as a coalition head to propose a new CS with some probability $p$ on behalf of that coalition.) b) The evolution from one CS to the next can only occur through the merging of two existing coalitions. For instance, any coalition head of any existing coalition $\mathbf{S_1}$ may propose to merge with another coalition $\mathbf{S_2}$, forming $\mathbf{S_1} \cup \mathbf{S_2} = \mathbf{S}$.

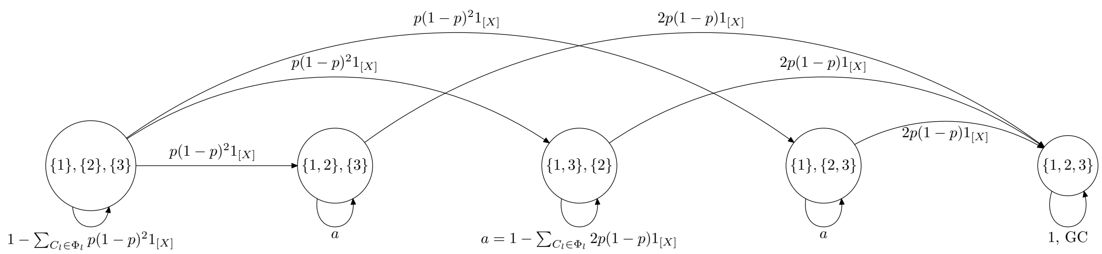
3) *Coalition formation decision*: Coalition formation only occurs when all links in the new coalition are at least as well off through the merge as they were before it, i.e., the new coalition satisfies the *minimum rational payoff* condition. Due to this coalition formation condition, whenever links agree to form $\mathbf{S}$ the new coalition is internally stable, i.e., no link has an incentive to become a singleton (an individual non-cooperative link).

4) *Payoff distribution*: The total value of the newly formed coalition is arbitrarily partitioned among the links by dividing the spectrum in $\eta_1, \eta_2, \ldots, \eta_{|\mathbf{S}|}$, ratios of the spectrum band $W$, subject to the *minimum rational payoff* condition, where $|\mathbf{S}|$ is the cardinality of set $\mathbf{S}$, with $\sum_{k=1}^{|\mathbf{S}|} \eta_k = 1$ . The total value may also be partitioned according to some fairness criteria, for instance proportional fairness [15]. The study of the distribution of these payoffs is beyond the scope of this paper.

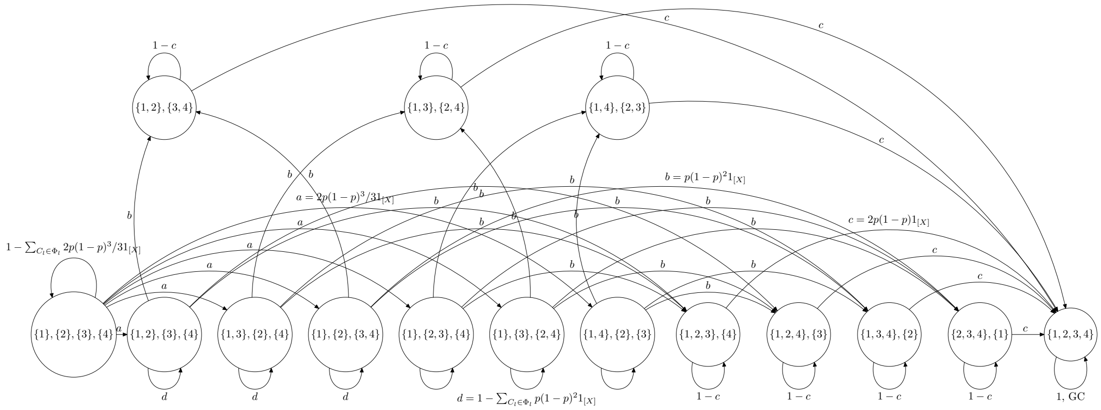
The four steps of the coalition formation are repeated until all the coalitions have made their coalition formation decisions, resulting in a final CS. The negotiation process described above can be achieved using a common control channel where links can exchange coalition formation messages to perform the proposed distributed coalition formation.

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication.






(a) Transition probability graph for $N = 3$ links, when the number of states in state space follows the Bell number with $1_{[X]}$ representing the indicator function as explained in Eq. 23.




(b) Transition probability graph for $N = 4$ links, when the number of states in the state space follows the Bell number and with $1_{[X]}$ representing the indicator function as explained in Eq. 23.

Fig. 3. Transition probability graph of dynamic coalition formation with GC representing the grand coalition.

### B. Dynamics of Coalition Formation Game Model

In our dynamic game model, a finite set $\mathbf{C}$ of all possible CSs for $N$ links forms the state space of the coalition formation game. Let:

$$\mathbf{C} = \{\mathbf{C}_1, \mathbf{C}_2, ...., \mathbf{C}_{|\mathbf{C}|}\}, \tag{19}$$

where each element of $\mathbf{C}$ is a state representing a CS. The initial state of the game process is the set of singleton coalitions. If the Markov chain is currently in state $\mathbf{C}_k$, it moves to state $\mathbf{C}_l$ at the next step with a transition probability denoted by $P_{\mathbf{C}_k\mathbf{C}_l}$. We say that the coalition game process moves forward when a CS changes due to the merging of two coalitions that results in an internally stable new coalition, and it stays in the same state if no new coalition is formed. For example, transition from the CS state of all singleton coalitions, i.e., $\mathbf{C}_1$ given as $\big\{\{1\},\{2\},....,\{N\}\big\}$ to $\mathbf{C}_2$ or $\mathbf{C}_3$ given as $\big\{\{1,2\},\{3\},....,\{N\}\big\}$ and $\big\{\{1\},\{2,3\},\{4\},....,\{N\}\big\}$ respectively, is possible, because both $\mathbf{C}_2$ and $\mathbf{C}_3$ result from the merger of two singleton coalitions. However, the transition from $\mathbf{C}_2$ to $\mathbf{C}_3$ is not possible because it implies that link 2 will break away unilaterally from the coalition $\{1,2\}$ and merge with $\{3\}$ to form $\{2,3\}$ at the same time [9]. In other words, state transitions will occur only when two coalitions decide to merge.

Each of $n$ coalitions with some probability $p$ proposes a new CS. The probabilities that a single coalition proposes a CS change, no one proposes a CS change, and more than one coalition proposes a CS change are:

$$\Gamma_{NW} = (1-p)^n, \text{ no one proposes a CS change;} \tag{20}$$

$$\Gamma = np(1-p)^{n-1}, \text{ one proposes a CS change; and} \tag{21}$$

$$\Gamma_{SW} = 1 - \Gamma_{NW} - \Gamma, \text{ more than one proposes a CS change .} \tag{22}$$

Any coalition may propose a CS change to another coalition in the current state $\mathbf{C}_k$, and if all the links in the proposed coalition are at least as well off as before the merge, the game moves to $\mathbf{C}_l$. The transition probabilities for the $N$ link coalition game with $B_N$ (see Eq. 1) possible CSs as the state space $\mathbf{C}$ are given as:

$$\begin{aligned} P_{\mathbf{C}_k\mathbf{C}_l} &= \frac{2p(1-p)^{(|\mathbf{C}_k|-1)}}{\mid \mathbf{C}_k \mid -1} 1(V(\mathbf{C}_l), V(\mathbf{C}_k)), \\ P_{\mathbf{C}_k\mathbf{C}_k} &= 1 - \sum_{\mathbf{C}_l \in \Omega_l} P_{\mathbf{C}_k\mathbf{C}_l}, \end{aligned} \tag{23}$$

where

$$1(V(\mathbf{C}_l), V(\mathbf{C}_k)) = \begin{cases} 1 & \text{when } V(\mathbf{C}_l) \geq V(\mathbf{C}_k). \\ 0 & \text{otherwise} \end{cases}$$

$V(\mathbf{C}_k)$ and $V(\mathbf{C}_l)$ are the values of CS states $\mathbf{C}_k$ and $\mathbf{C}_l$, respectively, $\mid \mathbf{C}_k \mid$ represents the number of coalitions in the present CS state $\mathbf{C}_k$, $\mathbf{C}_l$ represents any one of the new possible CS states to which coalitions can transit from $\mathbf{C}_k$, and $\mathbf{\Omega}_l$ represents the set of all new possible CS states to which coalitions can transit from $\mathbf{C}_k$. The set $\mathbf{\Omega}_l$ is given as: $\mathbf{\Omega}_l = \Big\{ \{\mathbf{S}_1 \cup \mathbf{S}_2, \mathbf{S}_3, ..., \mathbf{S}_{|\mathbf{C}_k|}\}, \{\mathbf{S}_1, \mathbf{S}_2 \cup \mathbf{S}_3, ..., \mathbf{S}_{|\mathbf{C}_k|}\}, ...., \{\mathbf{S}_1, \mathbf{S}_2, ...., \mathbf{S}_{(|\mathbf{C}_k|-1)} \cup \mathbf{S}_{|\mathbf{C}_k|}\} \Big\}$. Given $\mid \mathbf{C}_k \mid > 1$ coalitions in the present state $\mathbf{C}_k$, it is possible to transition from $\mathbf{C}_k$ to one of the $\mid \mathbf{\Omega}_l \mid$ possible states, where $\mid \mathbf{\Omega}_l \mid$ can be calculated as:

$$\mid \mathbf{\Omega}_l \mid = \binom{\mid \mathbf{C}_k \mid}{2}. \tag{24}$$

For instance, if the number of coalitions $\mid \mathbf{C}_1 \mid = 4$, then it is possible to move from the state $\mathbf{C}_1$ to one of $\mid \mathbf{\Omega}_l \mid = 6$ different states (besides itself), provided that the coalition formation condition is satisfied. As explained previously, two coalitions can merge only if all links within the proposed coalition $\mathbf{S}$ are at least as well off as before the merge. The indicator function $1(V(\mathbf{C}_l), V(\mathbf{C}_k))$ in Eq. 23 represents the possible agreement or disagreement among the links participating in the coalition game to form the proposed coalition $\mathbf{S}$. As the transition probability at any present state $\mathbf{C}_k$ does not depend upon the prior states of the CSs, the Markov property holds.

From the above specified transition model for the proposed coalition formation game, we can construct the transition matrix $\tilde{P}$ of the Markov chain. The dimension of the transition matrix is $B_N \times B_N$, where $N$ is the number of links playing the coalition formation game and $B_N$ is the Bell number given by Eq. 1. It can be seen from Eq. 1 that $B_N$ grows super exponentially. For large $N$ coalitional interaction becomes an issue, leading to a combinatorial explosion. Moreover, if the number of coalitions playing the coalition formation game is large then there is an increased coalition message collision probability. For large values of $\mid \mathbf{C}_k \mid$, the number of links attempting to play the coalition formation game with probability of success $p$ on behalf of their coalitions grows, and there is an increased probability of more than one coalition attempting to transmit coalition messages at the same time, resulting in coalition packet collisions. Imposing some hierarchy, through clustering, becomes the practical solution to this problem in large scale wireless networks, with links interacting with a small subset of peers . This can be modeled as a network of wireless links only interacting with a limited subset of links known to them to form stable CSs.

One simple and practical way of reducing the complexity of the proposed coalition formation model for the large number of links can be explained as follows. Typically, for many applications each link in the wireless network is required to satisfy a target signal-to-interference-plus-noise ratio (SINR). The presence of such an additional constraint will motivate only those links that have SINR below the target value to participate. This may help in reducing the number of participating links and hence reduce the size of the Markov chain. Moreover, in this context, during coalition formation, once a coalition achieves the target SINR, it will have no further incentives to keep participating in the coalition formation as further increasing the size of coalition incurs further increase in communication cost. The exclusion of the links satisfying the target SINR from the coalition formation may help in reducing the size of the Markov chain and may allow the coalition formation to converge quickly (as the game is only played between the left over links).

The explanation and analysis of the Markov chain model that allows links to leave the coalition formation process is a topic for future study and is beyond the scope of this paper.

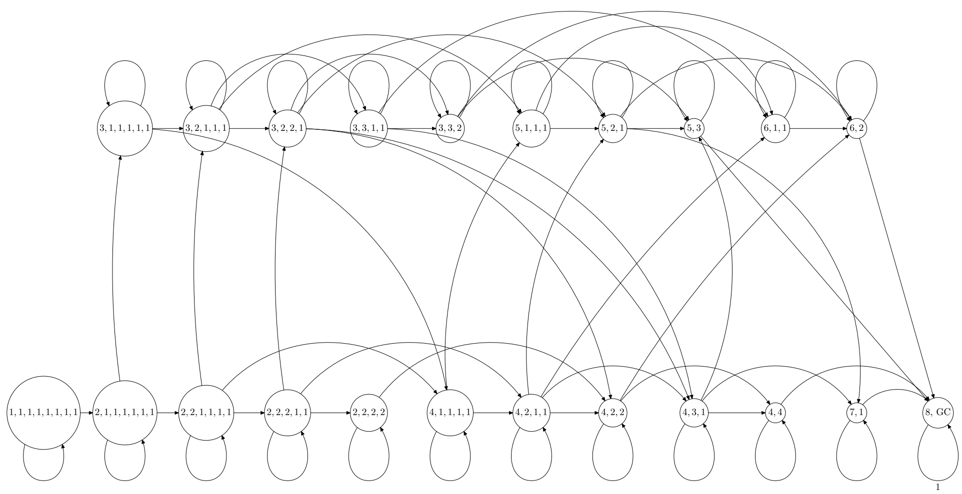



Fig. 4. Transition graph of dynamic coalition formation game for $N = 8$ links, when the number of states in the state space follows the integer partition function.

### *C. Dynamic Game Model with Reduced Complexity*

In Fig. 3, we present examples of the proposed $N$-link dynamic coalition formation game, using $N = 3$ and $N = 4$ links. It can be seen from Figs. 3a and 3b that the size of the state space comprising all possible CSs, i.e., the cardinality of $\mathbf{C}$, grows considerably even for a small increase in the number of links from $N = 3$ to $N = 4$.

To reduce the super exponential growth of the state space of the Markov chain, we introduce a reduced complexity model for the analysis of stable CSs. In this model the state space of the Markov chain represents subspaces of CSs containing coalitions of particular sizes. For example, the state $(2,1,1)$ for $N = 4$ links represents the subspace of all CSs containing three coalitions with one coalition of size two and two singleton coalitions of size one, i.e., $\{\{1,2\},\{3\},\{4\}\}$, $\{\{1,3\},\{2\},\{4\}\}$, $\{\{1,4\},\{2\},\{3\}\}$, $\{\{1\},\{2,3\},\{4\}\}$, $\{\{1\},\{2,4\},\{3\}\}$ and $\{\{1\},\{2\},\{3,4\}\}$. The state space $\bar{\mathbf{C}}$ for this reduced model is equal to all the integer partitions that can be generated for $N$ links. For example, the state space $\bar{\mathbf{C}}$ for $N = 4$ links is the integer partition of $N = 4$, given as: $\bar{\mathbf{C}} = \left\{(1,1,1,1),(2,1,1),(3,1),(2,2),(4)\right\}$. In this model we are not interested in stable CSs of any particular link, but we are only interested in analyzing stable grouping of the CSs according to the sizes of coalitions they contain. The growth rate of the state space of the Markov chain for this model follows an integer partition function, and the growth rate of this function is known to be $\Theta(\frac{e^{\pi\sqrt{2N/3}}}{N})$, where $N$ represent the number of links [19]. This growth is much slower with respect to $N$ than the super exponential growth of the state space $\mathbf{C}$. The proposed reduction in complexity for Markov model is particularly relevant to symmetric network scenarios. In Fig. 4, we present the state transition graph for $N = 8$ links, using integer partition as the state space. The number of integer partition states for this game is only 22, as compared to 4140 states when all possible CSs are used to represent the state space of the Markov process. The transition probability from the present state $\bar{C}_k$ to any next state $\bar{C}_l$ for $N$ links using this model can be calculated as :

$$P_{\bar{C}_k \bar{C}_l} = \sum_{r=1}^{s(\bar{C}_k, \bar{C}_l)} \frac{2p(1-p)^{(|\bar{C}_k|-1)}}{|\bar{C}_k| - 1} 1_r(V(\bar{C}_l), V(\bar{C}_k)), \tag{25}$$

where

$$1_r(V(\bar{C}_l), V(\bar{C}_k)) = \begin{cases} 1 & \text{when } V(\bar{C}_l) \geq V(\bar{C}_k). \\ 0 & \text{otherwise} \end{cases}$$

$V(\bar{C}_l) = \sum_{\mathbf{S}_i \in \bar{C}_l} v(\mathbf{S}_i)$, with $l = 1, 2, ..., |\bar{\mathbf{C}}|$, and $s(\bar{C}_k, \bar{C}_l)$ is a function that represents the number of ways coalitions can combine in $\bar{C}_k$ to reach a particular integer partition state $\bar{C}_l$. For instance, for $N = 8$ links if the coalition game is at present in state $(2, 1, 1, 1, 1, 1, 1)$ then it can be seen from Fig. 4 that the merger of coalitions can result in a transition to either integer partition state $(3, 1, 1, 1, 1, 1)$ or to the integer partition state $(2, 2, 1, 1, 1, 1)$. If the coalition formation process moves from $(2, 1, 1, 1, 1, 1, 1)$ to $(3, 1, 1, 1, 1, 1)$, then to calculate the transition probability, we have $n = |\bar{C}_k| = 7$ coalitions in the present integer partition state $\bar{C}_k$ and $s(\bar{C}_k, \bar{C}_l) = 6$, as a coalition of size two within $\bar{C}_k$ can combine in six different ways with the other singleton coalitions to reach $\bar{C}_l = (3, 1, 1, 1, 1, 1)$. However, if the coalition formation process moves from $(2, 1, 1, 1, 1, 1, 1)$ to $(2, 2, 1, 1, 1, 1)$, we have $s(\bar{C}_k, \bar{C}_l) = 15$ as singleton coalitions within $\bar{C}_k$ can combine in fifteen different ways to reach $\bar{C}_l = (2, 2, 1, 1, 1, 1)$. The indicator function $1_r(V(\bar{C}_l), V(\bar{C}_k))$ in Eq. 25 represents the possibility of agreement or disagreement among the links participating in the coalition game to form a new CS. From Eqs. 23 and 25 and also from Figs. 3a, 3b and Fig. 4 it can be seen that the grand coalition state can be reached in different ways depending on the state transitions. Whenever the grand coalition is formed each link is promised at least the payoff $v_{i,i \in \mathbf{N}}(\mathbf{N}) \geq d_{i,i \in \mathbf{N}}(\mathbf{N})$.

We will quantify by simulation results in Section V that in the proposed coalition formation game, rational payoffs and coalition state transitions depend on the interference perceived by the links.

## V. Analysis of the Dynamic Game

Using the standard framework of coalitional game theory, we analyze the stable CSs for the proposed dynamic model of the coalition game for an interference channel. In analyzing the stable CSs, we need to determine whether the grand coalition is stable, and this can be done using the concept of core.

### A. The Core and the Grand Coalition

In the study of coalition games, a payoff vector $\mathbf{x}$ is said to be in the core if it satisfies the following two properties: 1) $\mathbf{x}$ is an imputation; 2) imputation $\mathbf{x}$ is stable [20].

*Definition 7:* An *imputation* $\mathbf{x}$ is a payoff vector that satisfies the following two conditions: 1) $\sum_{i\in\mathbf{N}} x_i = v(\mathbf{N})$; 2) $x_i \geq v(\{i\})\ \forall\ i$.

*Definition 8:* An imputation $\mathbf{x}$ is *unstable* if there is a coalition $\mathbf{S} \subset \mathbf{N}$ such that $\sum_{i\in\mathbf{S}} x_i < v(\mathbf{S})$. Otherwise, $\mathbf{x}$ is said to be stable.

*Definition 9:* The set, $\gamma$, of stable imputations is called the *core*

$$\gamma = \{\mathbf{x} : \sum_{i\in\mathbf{N}} x_i = v(\mathbf{N}) \text{ and } \sum_{i\in\mathbf{S}} x_i \geq v(\mathbf{S}), \text{ for all } \mathbf{S} \subset \mathbf{N}\}. \tag{26}$$

Due to the myopic nature of links in the coalition formation process, it is possible that for certain rate allocations the core of the game is not empty but links cannot form the grand coalition. In other words, sometimes, even though the grand coalition is advantageous to all links, there is no path to reaching it because the intermediate coalitions are not sustainable. Any utilization of core allocations among wireless links is possible only if the grand coalition is formed. On the other hand, there may be circumstances where there is no path to the grand coalition, even if it ultimately would yield better value, i.e., the core is not empty but the grand coalition cannot be formed.

To illustrate the above situation, we can construct an example, for $N = 3$ links, where the core is non-empty but the grand coalition cannot be formed even if the cost for cooperation is zero.

*Example 5.1:* Let $H = \begin{pmatrix} 1 & .7 & .3 \\ .3 & 1 & .7 \\ .7 & .3 & 1 \end{pmatrix}$ be the link gain matrix for $N = 3$ links, with $W = 3$ and $SNR = 10\text{dB}$ for all links. In this three-link game, due to perceived interference at the myopic links, link 1 wants to cooperate with link 2 but not with 3, link 2 wants to cooperate with 3 but not with 1 and link 3 wants to cooperate with link 1 but not with link 2. Transitions from state $\mathbf{C}_1 = \big\{\{1\},\{2\},\{3\}\big\}$ to any next state $\mathbf{C}_l$ are not possible. Calculating coalition values using Eqs. (3)-(6) and rules explained in Section III and Section IV we obtain: $v(\{1\}) = v(\{2\}) = v(\{3\}) = 1.9573$, $v(\{1,2\}) = v(\{1,3\}) = v(\{2,3\}) = 3.6306$, $v(\{1,2,3\}) = 7.4313$.

To show that the core of this game is not empty, suppose that the links form the grand coalition and divide $v(\{1,2,3\})$. Suppose some payoff vector $\mathbf{x}$ is proposed as a division of $v(\mathbf{N}) = v(\{1,2,3\})$. The solution of the core for the three-link game in example 5.1 consists of all vectors $(x_1, x_2, x_3)$ satisfying

$$\begin{aligned} & x_1 \geq v(\{1\}), \quad x_1 + x_2 \geq v(\{1,2\}), \\ & x_2 \geq v(\{2\}), \quad x_2 + x_3 \geq v(\{2,3\}), \quad x_1 + x_2 + x_3 = v(\{1,2,3\}), \\ & x_3 \geq v(\{3\}), \quad x_1 + x_3 \geq v(\{1,3\}). \end{aligned} \tag{27}$$

Solving (27), we can see that the core for the three-link game is not empty and is given by

$$\gamma = \left\{ \mathbf{x} : x_1 + x_2 + x_3 = 7.4313, x_1 \geq 1.9573, x_2 \geq 1.9573, x_3 \geq 1.9573 \right\}. \tag{28}$$

The three links are treated symmetrically in this game by the core and all Pareto-efficient payoff allocations satisfying $x_i \geq v(\{i\})$ are included in it. However, using coalition values of the three-link game in example 5.1, it can be seen that $v(\{i,j\}) - v(\{i\}) < v(\{j\})$, $\forall i, j$. Therefore, any proposed $v(\{i,j\})$ in the CS state $\mathbf{C}_k = \left\{\{1\},\{2\},\{3\}\right\}$ cannot guarantee that all links within the proposed coalition are at least as well off as without the proposed coalition.

Now that we have shown that a non-empty core does not necessarily lead to the formation of the grand coalition, we will present a condition for the formation of a stable grand coalition for our dynamic coalition game.

*Definition 10:* The *marginal contribution* of any link $i \in \mathbf{S}$ in the presence of other links in any proposed coalition $\mathbf{S}$ is defined as: $m_i(\mathbf{S} \setminus \{i\}) = v(\mathbf{S}) - v(\mathbf{S} \setminus \{i\})$.

*Theorem 5.1:* In the proposed dynamic coalition game a stable grand coalition is formed if in every CS state (or, correspondingly, in every integer partition state) there is at least one proposed coalition $\mathbf{S}$ that satisfies: $m_i(\mathbf{S} \setminus \{i\}) \geq d_i(\mathbf{S}), \forall i \in \mathbf{S}$, for $|\mathbf{S}| > 1$.

*Proof:* First, any link leaving the coalition in any CS state or in any integer partition state could form a singleton coalition and would get the payoff $v(\{i\}) \leq v_{i,i \in \mathbf{S}}(\mathbf{S})$, so no link has an incentive to leave. If in every CS state (or correspondingly in every integer partition state) of the dynamic coalition game there is at least one proposed coalition $\mathbf{S}$ in which the marginal contributions of all links are at least equal to their individual minimum rational payoffs $d_i(\mathbf{S})$, it means that with the merger of coalitions the payoff values of each link participating in the merger of coalitions is non-decreasing. So the rate $R_i$ of each link will increase monotonically with the merging of coalitions. As a result, the wireless links forming coalitions have an incentive to allow merging of coalitions at every state. Hence, a stable grand coalition is formed if in every CS state or in every integer partition state there is at least one proposed coalition $\mathbf{S}$

that satisfies: $m_i(\mathbf{S} \setminus \{i\}) \geq d_i(\mathbf{S}), \forall i \in \mathbf{S}$, for $|\mathbf{S}| > 1$. ■

## *B. Simulation Results*

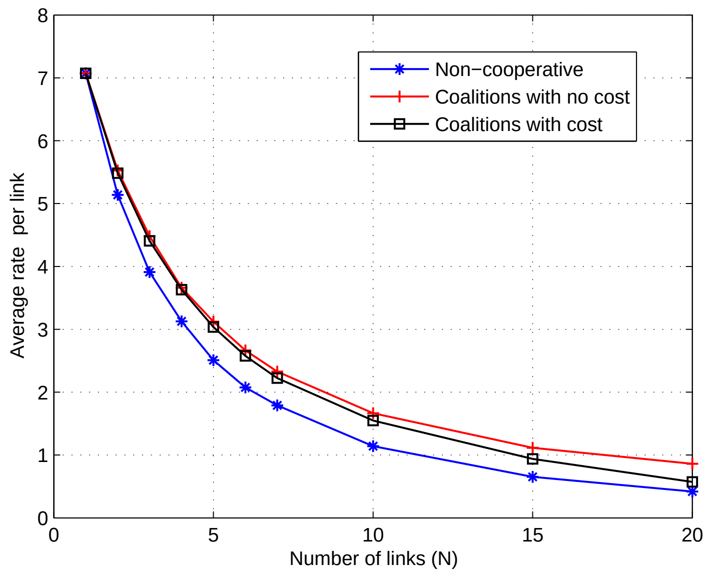
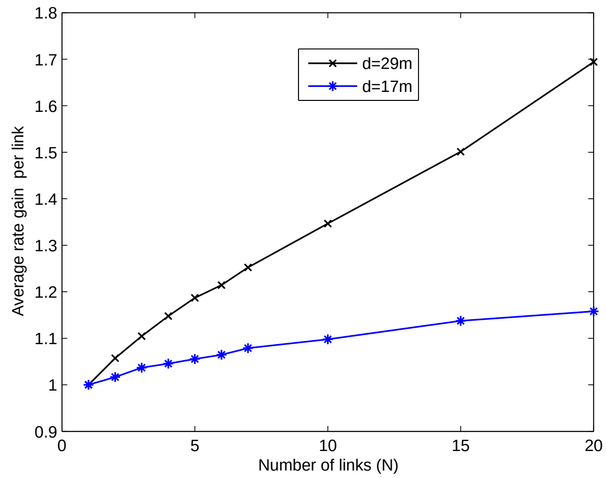



Fig. 5. Average rate per link vs different networks sizes. ($d$, i.e., the distance between a transmitter and its own receiver is 32m)

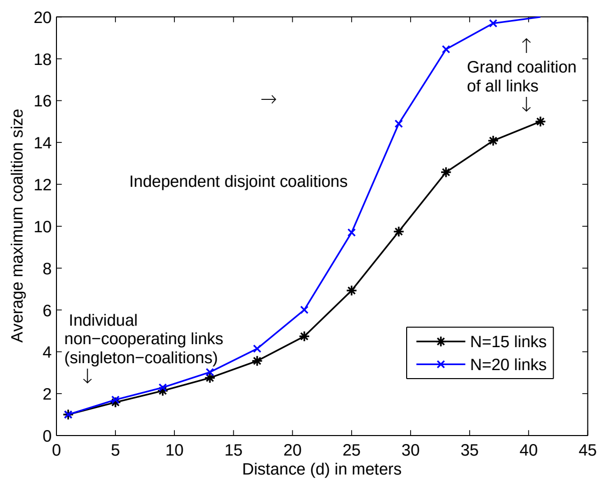
We simulate an ad hoc network comprising $N$ links randomly deployed in a 150m $\times$ 150m area. We set the transmitter power for each link as $P = 100$ mW and the noise variance $N_0 = -40$dBm. For path loss, we set $\alpha = 3$ and $\kappa = 1$. The spectrum band parameter $W$ is set to 5. The coalition formation overhead $\phi$ in Eq. (18) is set to 0.2.

We choose $P = 100$mWatt for transmission power as it is a typical transmission power value for ad hoc networks [21]. Path loss value ($\alpha$) in wireless communications is typically between 2 to 4 [22]. If $\alpha = 2$ is chosen as a parameter (that is the value for free space propagation which can model a channel with few obstructions, such as encountered in rural areas) then there will be little signal propagation loss over distance and as a result links will interfere more with each other and will have greater incentives to form coalitions of large sizes. If $\alpha = 4$ is chosen as a parameter (that is the value for urban channel modeling) then there will be high signal propagation loss over distance and as a result links will interfere less with each other and will have incentives to form coalitions of small sizes. We choose $\alpha = 3$ for our simulation purposes which would represent the channel model for suburban area.

For a fixed area, increasing the number of links $N$ will lead to more interference and decreasing the number of links will lead to less interference. To analyze the coalition formation process under varying




(a) Average rate gain per link due to coalition formation with no cost as compared with the non-cooperative strategy for different network sizes. $d$ represents the distance between a transmitter and its own receiver.




(b) Average maximum coalition size for different distances between a transmitter and its own receiver with network sizes of $N = 15$ and 20 links.

Fig. 6. Average rate gain per link due to coalition formation with no cost and average maximum coalition sizes for different number of network links.

interference conditions, we vary the number of links $N$ (between 1 to 20) for a fixed network area of 150mx150m and evaluate the average rate per link and average maximum coalition sizes (see Fig. 5 and Fig. 6b).

In Fig. 5, we show the average rate per link for different network sizes, when all links act independently, when links can form coalitions but there is no cost to coalition formation, and when links can form coalitions but incur some cost in the coalition formation process. It can be seen in Fig. 5 that the distributed process of coalition formation yields an improvement in the average link rates as compared to the non-cooperative strategy. In Fig. 6(a), we compare the gain in average rate per link due to coalition formation with the non-cooperative strategy for different network sizes. In this figure, we show that although coalition formation yields significant average rate gains when compared with the non-cooperative strategy, these gains are reduced as the distance between the transmitter and its own receiver is decreased. Since the performance of the coalition formation solution is dependent on the distance between the transmitters and their own receivers, we show average maximum coalition sizes for different distances in Fig. 6b. It can be seen from Fig. 6b that the coalition formation solution results in the formation of the grand coalition (for network sizes of 15 and 20 links) whenever the distances between the transmitters and their own receivers is greater than 40m for the given simulation scenario. Therefore, the terminal state grand coalition of the finite Markov chain of the proposed dynamic coalition game is an absorbing state and, in game theoretic terms, a Nash equilibrium for the distances between transmitters and their own receivers greater than 40m. However, if there is no path to the grand coalition state, i.e., the stable grand coalition cannot be formed, then any state of the proposed dynamic coalition game that satisfies the coalition structure internal and external stability condition is an absorbing state, as links have no incentive to leave that internal and external stable state. We verify this in Fig. 6b: as the distance between the transmitters and their own receivers decreases, the average maximum coalition size, i.e., the number of links in a coalition also decreases. In other words, as the distance between the transmitters and their own receivers decreases, the network for such scenarios is composed of independent disjoint coalitions (internally and externally stable coalitions) of smaller sizes. It can also be seen from Fig. 6b that as the distance between the transmitters and their own receivers is reduced to 12m then the coalition formation solution results in a network structure mostly composed of individual non-cooperative links, i.e., all singleton coalitions. In other words for the distances of less than 12m between a transmitter and its own receiver the coalition formation solution mostly coincides with the non-cooperative solution.

*Definition 11:* A coalition game is *superadditive* if for every pair of coalitions $\mathbf{S_i}$ and $\mathbf{S_j}$,

$$v(\mathbf{S_i}) + v(\mathbf{S_j}) \leq v(\mathbf{S_i} \cup \mathbf{S_j}), \text{ if } \mathbf{S_i} \cap \mathbf{S_j} = \emptyset. \tag{29}$$

Superadditivity in coalition games implies that forming the grand coalition is efficient. To show that superadditivity is a more restrictive requirement to form the grand coalition, as compared to the formulated condition in Theorem 5.1, we consider the following simulation set up: $N = 3$ links are deployed with direct channel gain $h_{ii}$ between the transmitter and its own receiver normalized to one and $SNR = \frac{P}{N_0 W} = 10$dB.

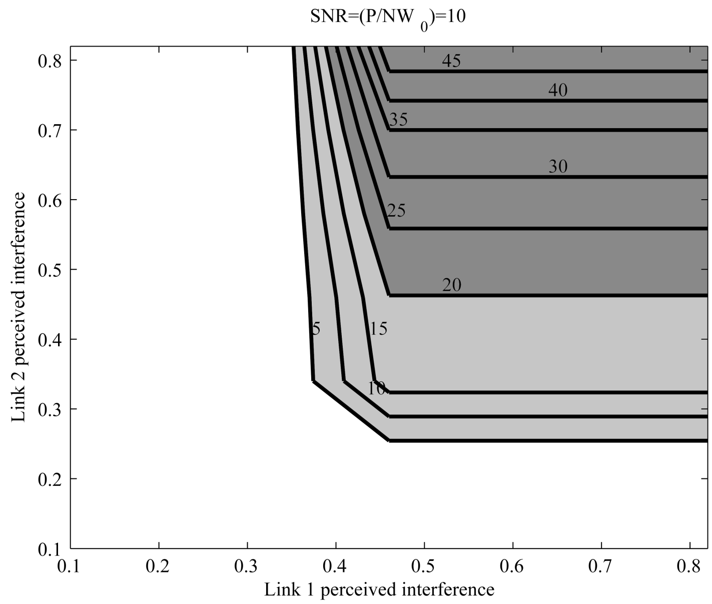


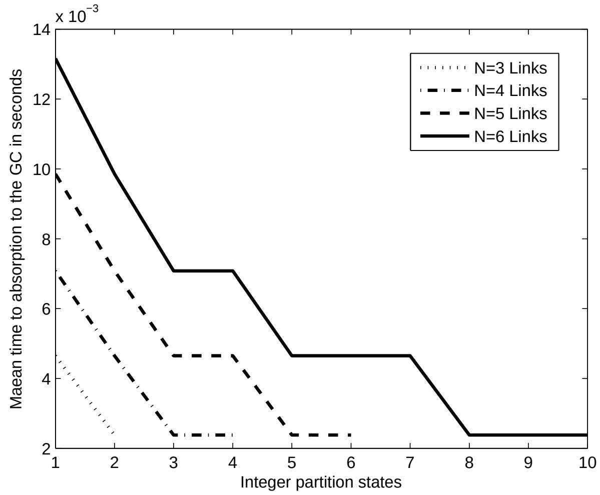

Fig. 7. GC (light grey and dark grey) and superadditive (dark grey) regions with percent coalition gains are shown for various interference channel gains of links 1 and 2 interference varied while for the third link $\sum_{j,j\neq i} h_{ji} = 0.85$. For instance percent gain for GC values are calculated as: $\frac{v(\mathbf{N})-(\sum_{i=1}^{|\mathbf{N}|} v(\{i\}))}{\sum_{i=1}^{|\mathbf{N}|} v(\{i\})} \times 100$.

The coalition formation game always starts from the initial state of all singleton coalitions. To analyze the coalition formation game at a given SNR for the various interference environments, we control the asymmetry between the three links by varying the cross gains $h_{ji}$ each time the coalition game is played, where $0 < h_{ji} \leq 1, j \neq i$ and $i, j \in \mathbf{N}$. It can be seen from Fig. 7 that the proposed coalition game in an interference channel is superadditive for the perceived high symmetric interference gains at the receivers. In Fig. 7, the interference gains of links 1 and 2 are varied while for the third link interference gain is kept fixed at $\sum_{j,j\neq i} h_{ji} = 0.85$. It can be seen from Fig. 7 that inequality (29), i.e., superadditivity, is a more restrictive condition for forming the stable grand coalition than our formulated condition.

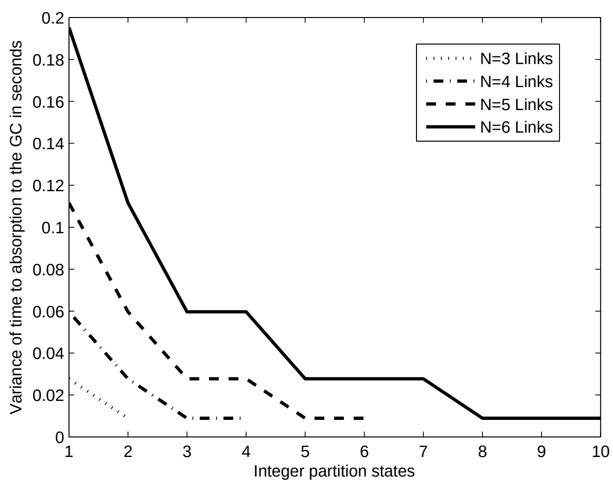
## *C. Dynamic Game Transients*

Using standard theory of absorbing Markov chains one can calculate the mean $\mu$ and variance $\sigma^2$ of the time for the dynamic coalition game starting from the initial state of all singleton coalitions to reach stable CSs. Let $\bar{P}$ represent the state transition probability matrix of an absorbing Markov chain in canonical form [24], [25], [26]:

$$\bar{P} = \begin{pmatrix} I & O \\ R & Q \end{pmatrix},$$




(a)




(b)

Fig. 8. Mean time and its variance for the coalition game to form the GC from different transient states. The dwell time in each state is set to 1msec. All the links are assumed to be enduring moderate to high interference [23].

where $I$ is an identity matrix, $O$ is a matrix with all zero entries, $R$ is the matrix of transition probabilities from transient to absorbing states and $Q$ is the matrix of transition probabilities between the transient states. The matrix $F = (I - Q)^{-1}$ is called the *fundamental matrix* for $\overline{P}$. Using $F$, one can calculate the mean $\mu$ and variance $\sigma^2$ of the time for the game before the coalition game process converges to the absorbing state [24]:

$$\mu = F\tau, \quad \sigma^2 = (2F - I)\mu - \mu_{sq}, \tag{30}$$

where $\mu$ is a column vector whose $i$th entry $\mu_i$ is the expected number of discrete steps or time intervals before the process reaches a stable CS, i.e., an absorbing state of the Markov chain, given that the process starts in state $\mathbf{C}_i$ (or correspondingly $\bar{C_i}$), $\mu_{sq}$ is a column vector whose entries are the square of the entries of $\mu$ and $\tau$ is a column vector whose components are the respective state "dwell" times, i.e., the time it takes to make a coalition formation decision. [1] All the links are assumed to be enduring moderate to high interference. Using the Markov chain model in which the state space follows an integer partition function, in Fig. 8a and 8b we illustrate the mean and variance of the time for the coalition game to reach the absorbing state of grand coalition from initial state $\bar{C_i}$ for $N = 3,4,5$ and 6 links. In Fig. 8a and 8b, simulation results are generated by assuming that all the links are enduring moderate to high interference [23], probability of coalition formation proposal is $p = 0.3$, the direct channel gain $h_{ii}$ between the transmitter and its own receiver is normalized to one, and $SNR = \frac{P}{N_0 W} = 10$dB. It can be seen from Fig. 8a and 8b that the mean and variance of the time to converge to the absorbing state of the grand coalition decreases with the number of coalitions participating in the coalition game. For instance in state $\bar{C_1}$, i.e., the state of all the singleton coalitions, the number of coalitions playing the game is highest so it takes longer to converge to the grand coalition from $\bar{C_1}$ as compared to the other states. The mean and variance of the time to converge to the grand coalition from states 3 and 4 are same in Fig. 8a and 8b, as the number of coalitions participating in the two states are the same. The two figures in Fig. 9 illustrate the mean and variance of the time for the coalition game to converge to grand coalition with different probabilities of coalition formation proposal. Fig. 9a shows that for small $p$, $\mu$ is high because the mean time between coalition formation messages is too long. If $p$ is high then the mean time between coalition formation messages is shorter but the number of coalition message collisions is higher, resulting in a longer time between CS changes. This suggests that depending on the number of players in the game there is an optimum value for $p$. We illustrate this optimum value for $N = 6$ links in Fig. 9.

## VI. Conclusions

We apply a coalitional game theoretic framework to the study of stable coalition structures (CSs) in an interference channel. We show that a coalition game in an interference channel is a game that generates positive externalities and have PFF, and present its conversion to CFF. Our work employs an absorbing Markov chain model to model the equilibrium state of the grand coalition or the equilibrium state of internally and externally stable CSs for the coalition game in an interference channel. Using a Markovian model of the coalition game, we analyze the dynamics of the coalition formation game and the stability

[1] In wireless networks, coalition formation decision typically involves transmission and reception of coalition formation messages (packets). The time required to transmit/receive and decode a message may range from few micro seconds to few milliseconds. We take 1 millisecond dwell time as an example.

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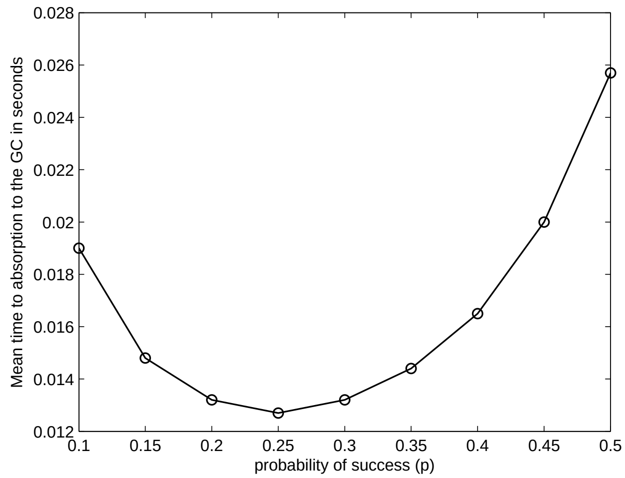
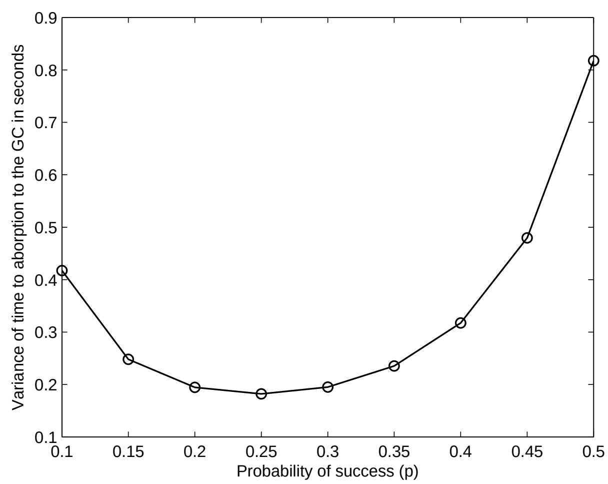



(a)




(b)

Fig. 9. Mean time and its variance for the coalition game to form the GC from the transient state of all singleton coalitions for different probabilities of success $p$. The dwell time in each state is set to 1msec. All $N = 6$ links are assumed to be enduring moderate to high interference [23].

of different CSs in an interference channel. When a coalition game for various interference environments is analyzed in terms of CSs, the number of the states in the state space of the Markov chain follows the Bell number. When the state space of the Markov chain is generated by grouping the CSs according to the sizes of coalitions they contain, the number of states follows the integer partition function. We have shown that, given sufficient time, the coalition process converges either to the grand coalition or to internally and externally stable CS, depending on the interference perceived by the links. We analyze the mean $\mu$ and variance $\sigma^2$ of the time for the game to reach the stable CSs. We demonstrate that for certain interference channel gains, although the grand coalition can yield optimal payoffs, due to the myopic nature of wireless links the grand coalition cannot be formed and links cannot obtain the optimal core payoffs. We then formulate a condition for the formation of the stable grand coalition. We also show that the restrictive condition of superadditivity for the stable grand coalition formation reduces the region of stable coalition structures, as compared to our formulated condition.

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