Low-Cost Routing in Selfish and Rational Wireless Ad Hoc Networks

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Abstract—Numerous routing protocols have been proposed for wireless networks. A common assumption made by the majority of these protocols is that each wireless node will follow the prescribed protocol without any deviation. This may not be true in practice since wireless nodes could be owned by users who perform in their own interests. We then have to design routing protocols that still work properly even for networks composed of selfish nodes. In this paper, we propose a unicast routing protocol to address this issue under the assumption that all networking nodes are rational. Here, a node is rational if it always chooses a strategy that maximizes its benefit. We assume that each node has a privately known cost of relaying a unit of data for other nodes. In our protocol, each wireless node has to declare a cost for forwarding a unit of data. When a node wants to send data to the access point, it first computes the least cost path to the access point and then computes a payment to each node on this path. We present a pricing mechanism such that the profit of each relay node is maximized when it declares its true cost. We also give a time optimal method to compute the payment in a centralized manner. We then discuss in detail how to implement the routing protocol in the distributed manner. We conduct extensive simulations to study the ratio of the total payment over the total cost incurred by all relay nodes. We find that this ratio is small in practice. Our protocol works when the wireless nodes will *not* collude and we show that no truthful mechanism can avoid the collusion of any pair of two nodes. We also give a truthful mechanism when a node only colludes with its neighbors.

Index Terms—Noncooperative computing, unicast, game theory, wireless ad hoc networks.

1 Introduction

Wireless networks have become increasingly important with the requirement for enhanced data and multimedia communications in ad hoc environments. While infrastructured networks are common, there are a growing number of applications that require multihop infrastructureless mobile wireless networks. In a multihop wireless network, each wireless node can only send signal to nodes within some transmission range.

A source node communicates with far off destinations by using intermediate nodes as relays. A common assumption made by the majority of the wireless ad hoc routing protocols is that each wireless node will follow the prescribed protocols without any deviation, e.g., a node is always assumed to relay data packets for other nodes if it is asked to do so by the routing protocols. However, this may not be true in reality: The wireless nodes could be owned by individual users and, thus, they will perform in their own interests. The wireless nodes are often powered by batteries only, thus it is not in the best interest of a wireless node to always forward the data packets for other nodes. If a node refuses to relay the data while the routing protocol assumed that it will, the throughput of the network may decrease and even the network connectivity may be broken *de facto*. In other words, selfish wireless nodes may hinder the functioning of the network completely. Thus, a stimulation mechanism is required to encourage the users to provide service to other nodes.

Dealing with selfish users has been well-studied in game theory and economics. Recently, there have been a sequence of results [1], [2], [3], [4], [5], [6] published in the theoretical computer science area that tried to solve various problems when the agents are selfish and rational. Here, an agent is *rational* if it always chooses a strategy that maximizes its own gain. A common setting in all these results is that each agent incurs a cost if it is selected to provide the service. For example, in wireless networks, each node will incur an energy cost (and possibly memory cost) when it is asked to relay the data for other nodes. Several protocols [7], [8], [9], [10], [11], [12], [13] have also been proposed recently to address the noncooperative issue in wireless ad hoc networks. Some protocols [7], [10], [11] pay the relay node a virtual *nuglet* when it relays a data packet for other nodes and charge the source node h nuglets when it initiates a unicast with h-hops. However, the concept of nuglet does not reflect the actual cost of a node relaying the data packet and, thus, nodes may still refuse to relay data packets. In this paper, we are interested in designing a routing protocol that will compensate a relaying node with a monetary value that is at least its actual cost.

We consider a set of wireless nodes $V = \{v_0, v_1, \cdots, v_{n-1}\}$. Each node v_i is associated with an average cost c_i to forward a data packet and this cost is only known to node v_i itself. Here, the cost c_i may also include the minimum amount of profit z_i that node v_i wants to make if it is chosen to relay packets for other nodes. For simplicity, in this paper, we assume that the cost c_i is *fixed* and node v_i knows this monetary value. This is also a common assumption made in all mechanism design results [4], [6], [14] that can achieve strategyproofness. If its cost changes, node v_i has to declare this new cost, as we will see later, so its profit is maximized. Notice that our results can be simply extended to the scenario when c_i reflects the cost of

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Manuscript received 16 Apr. 2004; revised 29 Dec. 2004; accepted 16 Jan. 2005; published online 15 Mar. 2006.
For information on obtaining reprints of this article, please send e-mail to: tmc@computer.org, and reference IEEECS Log Number TMC-0138-0404.

node v_i relaying data for one communication session instead of a data packet.

We assume that, in order to stimulate cooperation among all wireless nodes, every wireless node is willing to pay other nodes for relaying its data to and from the access point (or other wireless nodes in general). For simplicity, we use the access point as the sole destination. We will use node v_0 to represent the access point of the wireless network to the wired network and, in addition, the routing from each node to the access point is connection-oriented. Our routing protocol works as follows: First, each node v_j in the network declares a cost d_j for relaying a data packet of unit size, which could be different from its true cost c_j. Second, the source node v_i computes the least cost path (LCP), denoted by $\mathbf{P}(v_i, v_0, d)$, from v_i to the access point v_0 according to the declared cost vector $d = (d_0, d_1, \cdots, d_{n-1})$. After that, a payment $p_i^j(d)$ to node v_j is computed (either in a centralized or in a distributed way) for every node v_j. The data is then routed along the computed LCP. Each node on the LCP is asked to relay the data packets and is compensated by the computed payment. The *utility* or called *profit* by some researchers of node v_j is $u^j(d) = p_i^j(d) - c_j$ if node v_j relays data for v_i; otherwise, it is $p_i^j(d)$. Naturally, it is preferred that each node v_j declares a cost $d_j = c_j$ (called *truthful or strategyproof* in this paper). Since we assumed that wireless nodes are selfish and rational, there is no guarantee that any wireless node will reveal its cost truthfully unless it is convinced that it cannot do better by declaring a different cost. The first objective of this paper is then to design a payment scheme such that each node v_j will always maximize its profit when it declares its true cost, i.e., $d_j = c_j$, *no matter what* other wireless nodes do. The second objective of this paper is to implement the protocol efficiently and truthfully.

By assuming that the nodes will not collude with each other, we present a strategyproof pricing mechanism such that the profit of each wireless node is maximized when it declares its true cost. In our protocol, the payment to a node v_k on the LCP is d_k plus the difference between the cost of the least cost path without using v_k and the cost of the least cost path. The payment to any node not on the LCP is always 0. Our protocol does not need the routing to be performed repeatedly. To simplify our analysis, we assume that the network topology remains the same for the period of routing of current traffic request from a source node to the access point. Our main contributions of this paper are follows: First, we present a centralized method to compute the payment in time $O(n \log n + m)$, where n is the number of wireless nodes, and m is the number of wireless links. Clearly, this is asymptotically optimum since we have to spend $O(n \log n + m)$ time just to compute a least cost path for routing. Second, we discuss in detail how to implement our routing protocol in a distributed manner. Notice that it is not straightforward that we can implement a protocol in a distributed manner, even without considering the communication complexity. The difficulty here is how to rely on these selfish wireless nodes to *truthfully* implement our protocols that will prevent them from lying about its cost. Notice that since we pay more than its actual cost of a node to relay the data packets, the total payment to all node's relaying packets is clearly at least the same as the actual cost of the least cost path. It is easy to construct a worst-case example in which we could pay much more than the actual cost. We then conduct extensive simulations to study the ratio of the payment to all relay nodes by the source node over the total cost incurred by all relay nodes. We studied both the ratio averaged for all source nodes and the maximum ratio among all source nodes. We find that these ratios are all small when the cost of wireless nodes are randomly distributed in an interval.

The rest of the paper is organized as follows: In Section 2, we briefly discuss what is algorithmic mechanism design and what is the network model used in this paper and formally define the problem we want to solve. We present our pricing mechanism in Section 3. We discuss in detail how to compute the payment fast and how to compute it in a distributed manner. In Section 4, we show that there is no pricing mechanism that can prevent an arbitrary pair of nodes from colluding. We then give a mechanism that can prevent the neighbors from colluding with each other. We also study the case when the cost is incurred on communication links instead of wireless nodes. We conduct extensive simulations to show that the overpayment to the relay nodes according to the pricing scheme is small compared with the actual cost of the least cost path. We review the priori arts on dealing with noncooperative computing and wireless networks in Section 5. We conclude our paper in Section 6 with a discussion of possible future works.

2 Technical Preliminaries

2.1 Algorithmic Mechanism Design

Conventionally, in economics and mechanism design theory, the scenarios in which the agents act according to their own self-interests are modeled as follows: There are n agents and each agent i, for $i \in \{1, \cdots, n\}$, has some private information t^i, called its *type*. The type t^i is a node's cost to forward a packet in unicast scenario. All agents' types $t = (t^1, t^2, \cdots, t^n)$ define a type vector, which is called the *profile*. Given a reported profile, there is an output specification $\mathcal{O}$ that maps each type vector t to an allowed output. For each possible output o, agent i's preferences are given by a valuation function w^i that assigns a real number $w^i(t^i, o)$, which does not depend on other agents' types.

Given a reported profile $a = (a^1, \cdots, a^n)$, a mechanism defines an *output* $\mathcal{O}(a)$ and a *payment* vector

$$p(a) = (p^1(a), \cdots, p^n(a)),$$

where $p^i = p^i(a)$ is the money given to each participating agent i. Agent i's *utility* is $u^i(a) = w^i(t^i, \mathcal{O}(a)) + p^i(a)$. By assumption of rationality, agent i always tries to maximize its utility $u^i(a)$ by choosing its action a^i. A mechanism satisfies the *incentive compatibility* (IC) if each agent maximizes its utility by reporting its type t^i *regardless* of what other agents do. A mechanism satisfies the *individual rationality* (IR) (or called *voluntary participation*) if each agent's utility of participating in the action is nonnegative. A mechanism is *strategy-proof* (or called *truthful*) if it satisfies both IC and IR properties.

One of the most celebrated positive results in mechanism design is what is usually called the family of Vickrey-Clarke-Groves (VCG) mechanisms by Vickrey [16], Clarke [17], and Groves [18]. VCG mechanisms apply to mechanism design maximization problems where the objective function $g(o,t)$ is the sum of all agents' valuations, i.e., $g(o,t) = \sum_i w^i(t^i, o)$, and the set of possible outputs is assumed to be finite. This maximization mechanism design problem is often called *utilitarian*. A mechanism $M = (\mathcal{O}(t), p(t))$ belongs to the family of VCG mechanisms if 1) the output $o = \mathcal{O}(t)$ computed based on the type vector t maximizes the objective function $g(o,t) = \sum_i w^i(t^i, o)$, and 2) the payment to an agent i is of format $p^i(t) = \sum_{j \neq i} w^j(t^j, \mathcal{O}(t)) + h^i(t^{-i})$. Here, $h^i()$ is an arbitrary function of t^{-i}. Groves [18] proved that a VCG mechanism is truthful. Green and Laffont [19] proved that, under mild assumptions, VCG mechanisms are the only truthful implementations for utilitarian problems.

Let a^{-i} denote the vector of strategies of all agents except i, i.e., $a^{-i} = (a^1, a^2, \cdots, a^{i-1}, a^{i+1}, \cdots, a^n)$. Let $a|^i b = (a^1, a^2, \cdots, a^{i-1}, b, a^{i+1}, \cdots, a^n)$, i.e., each agent $j \neq i$ uses strategy a^j, except that the agent i uses strategy b.

2.2 Network Model

We consider a set $V = \{v_0, v_1, \cdots, v_{n-1}\}$ of n wireless nodes. Here, v_0 is used to represent the access point (AP) of the wireless network. Let $G = (V, E)$ be the communication graph defined by V, where E is the set of links (v_i, v_j) such that the node v_i can communicate directly with the node v_j. We assume that G is node biconnected. In other words, the remaining graph, by removing any node v_i and its incident links from G, is still connected. The biconnectivity of the communication graph G will prevent the monopoly of nodes as will see later in addition to provide fault tolerance.

In wireless networks, if two nodes cannot communicate directly, they communicate through multihop wireless links by using some intermediate nodes to relay the message. Consequently, each node in the wireless network also acts as a router to forward data packets for other nodes. We assume that each wireless node v_i has a fixed cost c_i of relaying/sending a data packet to any (or all) of its outgoing neighbors. This cost c_i is a private information, only known to node v_i. All n nodes together define a cost vector $c = (c_0, c_1, \cdots, c_{n-1})$, which is the profile of the network G. When a node v_i wants to make a guaranteed profit z_i, its declared "cost" should be $c_i + z_i$. It will be seen later that node v_i makes a profit at least z_i when it is chosen to relay the data. In the remaining of our study, we assume that the source node will always pay the relay nodes for relaying its data, i.e., the source node has an infinitely large valuation of the data being sent to the destination. Our results can be easily extended to deal with the case when the source node v_i has a fixed valuation b_i of the data being sent to the destination. The source node chooses to send the data via relay nodes if and only if the total payments to all these relay nodes is no more than b_i. It is easy to show that the source node will also be truthful about b_i.

In this paper, we restrict our attentions to a unicast between any node v_i and the access point v_0 only. Our results can be easily extended to the routing between an arbitrary pair of source node v_i and destination node v_j.

2.3 Statement of Problem

If a node v_i wants to send data to the access point v_0, typically, the least cost path (with minimum total relaying cost) from node v_i to node v_0, denoted by $\mathtt{P}(v_i, v_0, c)$, is used to route the packets. Consider a path $\Pi(i,0) = v_{r_s}, v_{r_{s-1}}, \cdots v_{r_1}, v_{r_0}$ connecting node v_i and node v_0, i.e., $v_{r_s} = v_i$ and $v_{r_0} = v_0$, and node v_{r_j} can send signal directly to node $v_{r_{j-1}}$. The cost of the path $\Pi(i,0)$ is defined as $\sum_{j=1}^{s-1} v_{r_j}$, which excludes the costs of the source node and the target node.

To stimulate cooperation among all wireless nodes, node v_i pays some nodes of the network to forward the data to the access point. Thus, each node v_j on the network declares a cost d_j, which is its claimed cost to relay the packets. Note that, here, d_j could be different from its true cost c_j. Then, node v_i computes the least cost path $\mathtt{P}(v_i, v_0, d)$ to connect to the access point v_0 according to the declared cost vector $d = (d_0, d_1, \cdots, d_{n-1})$. For each node v_j, a payment $p_i^j(d)$ is computed according to the declared cost vector d. The *utility* of node v_j is $u^j(d) = p_i^j(d) - x_j(i) \cdot c_j$, where $x_j(i) \in \{0,1\}$ indicates whether v_j relays the packet for v_i. We always assume that the wireless nodes are rational: It always tries to maximize its utility $u^j(d)$.

We assume that the cost c_i is based on per packet or per session, whichever is appropriate. If the cost is per packet and a node v_i wants to send s packets to the access point v_0 in one session, then the actual payment of v_i to a node v_k is $s \cdot p_i^k$ for that session.

If the payment scheme is not well-designed, a node v_j may improve its utility by lying its cost, i.e., declares a cost $d_j \neq c_j$. The objective of this paper is then to design a payment scheme such that each node v_j maximizes its utility as long as it declares its true cost, i.e., $d_j = c_j$. Using the standard assumption from economic model, we assume that the wireless nodes do *not* collude to improve their utilities. We will relax this assumption later.

3 The Pricing Mechanism

3.1 Payment Scheme

Assume that the node v_i has to send packets to v_0 through the relay of some other nodes. It pays these relay nodes to compensate their costs for carrying the transit traffic incurred by v_i. The output $\mathcal{O}(d)$ of the mechanism is the path connecting v_i and v_0 with the minimum cost, which is known as $\mathtt{P}(v_i, v_0, d)$. The payment to a node v_k is 0 if $v_k \notin \mathtt{P}(v_i, v_0, d)$. Otherwise, its payment is

$$p_i^k(d) = \|\mathtt{P}_{-v_k}(v_i, v_0, d)\| - \|\mathtt{P}(v_i, v_0, d)\| + d_k.$$

Here, $\mathtt{P}_{-v_k}(v_i, v_0, d)$ denotes the least cost path between node v_i and v_0 without using node v_k, and $\|\Pi\|$ denotes the total cost of a path Π.

This payment falls into the VCG mechanism, so it is strategy-proof. In other words, if $d_k = c_k$, node v_k maximizes its utility $p_i^k(d) - x_k(i) \cdot c_k$. Every node participating in the relay will have a *nonnegative* profit; every node that does not relay the traffic will have profit 0. Notice that when a node originally is not chosen with its true cost, its profit will become *negative* if it tries to lie its cost such that it is chosen. Clearly, to make sure that the payment is well-defined, we need that the network is biconnected, i.e., the path

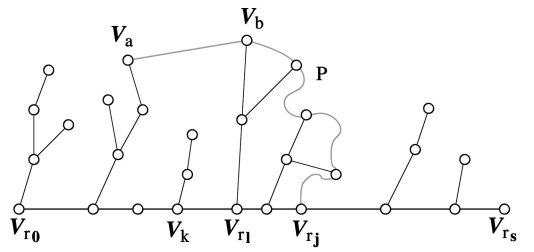

Fig. 1. Computing v_k-avoiding shortest path.

$\mathrm{P}_{-v_k}(v_i, v_0, d)$ does exist. Otherwise, node v_k can charge a monopoly price since it is a critical node to connect v_i and v_0.

Designing a truthful payment scheme for unicast is straightforward. One of our main contributions in this paper is a time optimal centralized method to compute the payment, which will be described in detail in the following section.

3.2 Fast Payment Computing

Assume that the access point has collected all nodes' costs, and the network structure G. Then, this access point can compute the payment to all relay nodes in a centralized manner. The very naive way to calculate the payment for all nodes on $\mathrm{P}(v_i, v_0, d)$ is to calculate every node's payment using Dijkstra's algorithm. In the worst case, there will be $O(n)$ nodes on $\mathrm{P}(v_i, v_0, d)$, so this naive algorithm will result in a time complexity $O(n^2 \log n + nm)$. In [20], Hershberger and Suri provided a fast payment calculation algorithm for *edge weighted* graph (by assuming the edges are rational agents). Nardelli et al. [21] studied a similar question of finding the most vital node of a shortest path in an edge weighted graph and gave a method to do so in time $O(m + n \log n)$. Borrowing some ideas from [20], we present an $O(n \log n + m)$ time complexity algorithm for fast payment calculation in a *node weighted* graph.

Consider a node $v_k \in \mathrm{P}(v_i, v_0, d)$ and we want to compute $\|\mathrm{P}_{-v_k}(v_i, v_0, d)\|$. The basic idea of our algorithm is for a pair of nodes v_a, v_b such that $v_a v_b \in G$, we calculate the path $\mathrm{P}_{-v_k}(v_i, v_a, d)$ and $\mathrm{P}_{-v_k}(v_b, v_0, d)$ separately. Then, by concatenating $\mathrm{P}_{-v_k}(v_i, v_a, d)$, link $v_a v_b$, and $\mathrm{P}_{-v_k}(v_i, v_a, d)$, we obtain the path with the minimum cost from v_i to v_0 without node v_k and having $v_a v_b$ on it. Choosing the minimal cost path for all edges $v_a v_b \in G$, we find $\|\mathrm{P}_{-v_k}(v_i, v_0, d)\|$. See Fig. 1 for an illustration.

Algorithm 1 computes the payment for all nodes on the least cost path. The correctness of this algorithm comes from the following observations of the shortest v_k-avoiding path.

Algorithm 1 Fast VCG Payment Computing 1

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1:  Find the Shortest Path Tree (SPT) $SPT(v_i)$ and $SPT(v_0)$
      rooted at $v_i$ and $v_0$, respectively. Assume that
      $\mathrm{P}(v_i, v_0, d) = v_{r_0} v_{r_1} \cdots v_{r_{s-1}} v_{r_s}$, where $v_{r_0} = v_i$ and
      $v_{r_s} = v_0$. For a node $v_k \in \mathrm{P}(v_i, v_0, d)$, let $L(v_k)$ be the cost
      of LCP from $v_i$ to $v_k$ and $R(v_k)$ be the cost of LCP from
      $v_k$ to $v_0$.
2:  for every node $v_k$ in $G$ do
3:    Find a node $v_{r_l} \in \mathrm{P}(v_i, v_0, d)$ such that removing
        node $v_{r_l}$ causes that node $v_k$ neither connects to $v_i$ nor
        connects to $v_j$ in the tree $SPT(v_i)$.
4:    Set $v_k.level = l$.
5:  Mark all nodes WHITE.
6:  Mark all nodes whose level is $s$ BLACK.
7:  for $l = s - 1$ down to 1 do
8:    repeat
9:      For each WHITE node $v_k$ whose level is $l$, find its
          BLACK neighbor $v_a$ such that $d_a + \|\mathrm{P}_{-v_{r_l}}(v_a, v_0, d)\|$
          is minimized.
10:     Set $R^{-l}(v_k) = c_a + \|\mathrm{P}_{-vr_l}(v_a, v_0, d)\|$;
11:     For all nodes $v_k$ with level $l$, find the node with
          minimum $R^{-l}(v_k)$. Let this node be $v_j$. Mark $v_j$
          BLACK.
12:   until all nodes with level $l$ except $v_{r_l}$ are BLACK.
13:   Mark node $v_{r_l}$ as BLACK.
14: For node $v_k$ with level $l$, find a neighbor $v_s$ whose
      $c^{-l}(v_k) = L(v_s) + R^{-l}(v_k) + d_s + d_k$ is minimized among
      all neighbors with level smaller than $l$. Denote such
      minimum value as $c^{-l}(v_k)$.
15: Among all nodes with level $l$, choose the node $v_k$ whose
      $c^{-l}(v_k)$ is minimal and set $c^{-l}$ to this value, i.e.,
      $c^{-l} = \min_{v_k.level=l} c^{-l}(v_k)$.
16: For each node $v_{r_l} \in \mathrm{P}(v_i, v_0, d)$, where $l$ starts from $s - 1$
      to 1, we use a heap $H$ to find the path containing an
      edge $v_a v_b$ with minimum cost such that
      $v_a.level < l < v_b.level$. The heap $H$ has nodes $\overline{v_a v_b}$
      corresponding to all such edges $v_a v_b \in G$. The value of a
      node $\overline{v_a v_b}$ is $L(v_a) + R(v_b) + d_a + d_b$. Find the node with
      the minimal value in $H$ and compare this value with
      $c^{-l}$, and the minimal of these two values is set as
      $\|\mathrm{P}_{-v_{r_l}}(v_i, v_0, d)\|$.
17: Calculate the payment to node $v_{r_l}$ as follows
          $p_i^{r_l} = \|\mathrm{P}_{-v_{r_l}}(v_i, v_0, d)\| - |\mathrm{P}(v_i, v_0, d)\| + d_{r_l}$.
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Lemma 1. *Assume that, for a node $v_{r_l} \in \mathrm{P}(v_i, v_0, d)$,*

$$\mathrm{P}_{-v_{r_l}}(v, v_0, d) = v_{l_0} v_{l_1} \cdots v_{l_{t-1}} v_{l_t},$$

where $v_{l_0} = v_i$ and $v_{l_t} = v_0$. If $v_{l_a}.level \geq l$, then $v_{l_b}.level \geq l$ for all $b > a$.

Proof. We prove it by contradiction. Assume that there exists a pair of nodes a and b such that $v_{l_b}.level < l$, $v_{l_a}.level \geq l$, and $b > a$. Notice that $\mathrm{P}(v_i, v_{l_b}, d)$ does not contain node v_{r_l} since $v_{l_b}.level < l$. Thus, replacing path $v_{l_0} v_{l_1} \cdots v_{l_{t-1}} v_{l_t}$ by path $\mathrm{P}(v_i, v_{l_b}, d)$ concatenated with $v_{l_{b+1}} \cdots v_{l_{t-1}} v_{l_t}$ will result in a v_{r_l}-avoiding path with a smaller weight since the path $\mathrm{P}(v_i, v_{l_b}, d) \subset SPT(v_i)$ will only use nodes with level at most $v_{l_b}.level < l$, while the subpath $v_{l_0} v_{l_1} \cdots v_{l_{b-1}} v_{l_b}$ uses node v_{l_a} with level at least l. Notice that $\mathrm{P}(v_i, v_{l_b}, d)$ is the least cost path connecting v_i and v_{l_b}. This finishes our proof. □

Lemma 2. *For a node v_k such that $v_k.level = l$, then $\mathrm{P}(v_k, v_0, d)$ cannot contain any relay node v_{r_a} with $a < l$.*

Proof. Again, we prove it by contradiction. Assume there exists a node v_k such that $v_k.level = l$, and $\mathrm{P}(v_k, v_0, d)$ contains a node v_{r_a} with $a < l$. Obviously, $\mathrm{P}(v_i, v_k, d)$ contains path $v_{r_0} v_{r_1} \cdots v_{r_{l-1}} v_{r_l}$. For simplicity, we assume that path $\mathrm{P}(v_i, v_k, d)$ is composed of two subpaths $v_{r_0} v_{r_1} \cdots v_{r_{l-1}} v_{r_l}$ and P_1 as shown in Fig. 2.

Similarly, we assume that $\mathrm{P}(v_k, v_0, d)$ is composed of two subpaths P_2 and $v_{r_a} v_{r_{a+1}} \cdots v_{r_{s-1}} v_{r_s}$. Let P_3 be the

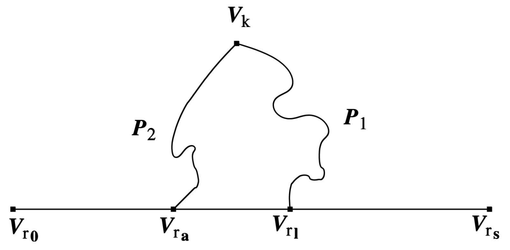

Fig. 2. Observation of v_{r_l}-avoiding shortest path.

subpath $v_{r_a}v_{r_{a+1}}\cdots v_{r_{l-1}}v_{r_l}$. It is easy to show that $\|P_2\| + \|P_3\| \leq \|P_1\|$ from the property of least cost path $\mathtt{P}(v_i, v_k, d)$ and $\|P_1\| + \|P_3\| \leq \|P_2\|$ from the property of least cost path $\mathtt{P}(v_k, v_0, d)$. Consequently, we have $\|P_3\| \leq 0$, which is a contradiction. This finishes our proof. □

Similarly, we have:

Lemma 3. *Consider the path* $\mathtt{P}_{-v_{r_l}}(v_k, v_0, d)$. *If there exists a node* $v_{k'}$ *on this path with* $v_{k'}.level < v_k.level$, *then node* v_k *cannot appear on* $\mathtt{P}_{-v_{r_l}}(v_i, v_0, d)$.

The proof of this lemma is omitted due to space limit. Lemma 3 allows us to only focus on the path connecting v_k and v_0 that avoids nodes $v_{k'}$ with $v_{k'}.level < v_k.level$.

Now, we analyze the time complexity of this algorithm. First, the shortest path tree can be calculated in $O(n \log n + m)$ and, with the SPT, it only takes us linear time to find $L(v_k)$ and $R(v_k)$. Clearly, we can compute the level for all nodes in time $O(n)$. We can calculate the value $R^{-l}(v_k)$ for all nodes v_k in time $O(n \log n + m)$. Assume the number of nodes with label l is n_l and node v_k's degree is $deg(v_k)$, then it will take at most $\sum_{v_l.level=l} deg(v_k) + n_l \log n_l$ time to find c^{-l}. Summing l from 1 to $s-1$, the time complexity as $\sum deg(v_k) + \sum_{l=1}^{s-1} n_l \log n_l < 2m + n \log n$. Thus, computing $R^{-l}(v_k)$ and c^{-l} will take time complexity of $O(n \log n + m)$. The heap operations have at most m insertions, m deletions, and n extract-min operations, which takes time $O(m + n \log n)$. Overall, the time complexity is still $O(m + n \log n)$.

3.3 Distributed Algorithm for Payment Calculation

Unlike wired or cellular networks, wireless ad hoc networks are lack of a centralized authority. Thus, it is more desirable to compute the payment in a distributed manner. Next, we discuss how to compute the payment of a node to all the relay nodes truthfully in a distributed manner. Assume that there is a fixed destination node v_0. Our distributed algorithm will compute the payment of each node v_i to all its relay nodes. The distributed algorithm has two stages. First, all nodes together find the Shortest Path Tree (SPT) rooted at node v_0. We assume that the SPT tree does not have a loop. This step can be easily implemented using Dijkstra's algorithm, so we omit this one. Assume that we already formed a shortest path tree T rooted at node v_0, and every node knows its parent and children in tree T. Second, every node v_i computes its payment p_i^k in a distributed manner using Algorithm 2, which is based on the algorithm presented in [22].

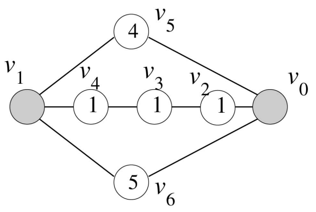

Fig. 3. The node has the incentive to lie about his shortest path.

Algorithm 2 Distributed payment computing by a node v_i.

```
1: Set $p_i^k \leftarrow \infty$, if $v_k \in \mathtt{P}(v_i, v_0, d)$; otherwise, $p_i^k \leftarrow 0$.
2: Broadcasts its entries $p_i^k$ to its neighbors.
3: **while** $v_i$ receives an updated price from a neighbor $v_j$ **do**
4:   **if** $v_j$ is the parent of $v_i$ **then**
5:     $p_i^k \leftarrow \min(p_i^k, p_j^k)$ if $v_k \in \mathtt{P}(v_i, v_0, d)$.
6:   **else if** $v_i$ is the parent of $v_j$ **then**
7:     $p_i^k \leftarrow \min(p_i^k, p_j^k + d_i + d_j)$ if $v_k \in \mathtt{P}(v_i, v_0, d)$.
8:   **else**
9:     for every $v_k \in \mathtt{P}(v_i, v_0, d)$, $v_i$ updates $p_i^k$ as follows.
10:    **if** $v_k \in \mathtt{P}(v_j, v_0, d)$ **then**
11:      $p_i^k \leftarrow \min(p_i^k, p_j^k + d_j + \|\mathtt{P}(v_j, v_0, d)\| - \|\mathtt{P}(v_i, v_0, d)\|)$;
12:    **else**
13:      $p_i^k \leftarrow \min(p_i^k, d_k + d_j + \|\mathtt{P}(v_j, v_0, d)\| - \|\mathtt{P}(v_i, v_0, d)\|)$
14: Broadcasts its entries $p_i^k$ to its neighbors.
```

Whenever some entry p_i^k stored at node v_i changes, the entry p_i^k is sent to all neighbors of v_i by node v_i. When the network is static, the price entries decrease monotonically and converge to stable values after a finite number of rounds (at most n rounds).

3.4 Compute the Payment Truthfully

While it is quite obvious to conceive that the node v_i has the incentive not to correctly calculate his payment p_i^k in the second stage, it is not so straightforward to notice that the node v_i also has the incentive to lie about his shortest path even in the first stage. We give an example to show that even we can guarantee that node v_i calculates his payment truthfully in the second stage, it is not unnecessary for us to worry about nodes' lying in the first stage. In Fig. 3, the shortest path between v_0 and v_1 should be $v_1v_4v_3v_2v_0$. It is easy to calculate that v_1's payments to nodes v_2, v_3, and v_4 are all exactly 2. Then, the overall payment by node v_1 is 6. If node v_1 lies that it is not a neighbor of v_4, then its shortest path becomes $v_1v_5v_0$. Now, it only needs to pay v_5 5 to send a packet. Thus, node v_1 benefits by lying about its neighborhood connection information, which consequently changes the SPT. This problem rises from the fact that the least cost path is not necessarily the path that you pay the least. This observation also raises the concern whether a distributed method presented in [22] does compute the payment correctly since a node can lie not only about its cost, but also about its neighborhood information.

Notice that the distributed method to compute the payment relies on the selfish node v_i to calculate the payment p_i^k to node v_k, which cannot prevent node v_i from manipulating the calculation in its favor. In [22], the authors suggested to use the following approach: All agents are required to sign all messages that they sent and to verify all messages that they received from their neighbors. They claimed that their protocol can be modified so that all forms of cheating by agents are detectable. Observe that, they did not consider the possible scenario when an agent could lie about the topology (the scenario we just discussed in previous paragraph). Notice that even using their approach, all nodes must keep a record of messages sent to and received from its neighbors so that an audit can be performed later if a disagreement happens.

Next, we present a new distributed method that prevents nodes from lying about the topology, and miscalculating the payment, and it does not need to store all messages. The nodes use Algorithm 3 to compute the shortest distance to source and then use Algorithm 4 to compute the payment to the relay nodes. It is easy to verify that they are truthful and no node will lie about its neighbor information and will follow the payment calculation procedure. For the example illustrated in Fig. 3, node v_1 has to use the shortest path $v_1v_4v_3v_2v_0$ to compute the payment according to our new protocol since node v_4 knows the existence of this shortest path and it will detect the lie by node v_1 if node v_1 chose to use path $v_1v_5v_0$ instead. The problem remaining is how to make it more efficient.

Algorithm 3 Truthful Distributed SPT Construction by v_i

```
1: Every node $v_i$ has two variables: $D(v_i)$ stores the shortest
   distance to $v_0$ and $FH(v_i)$ stores its parent on SPT.
   Initially, if $v_0$ is $v_i$'s neighbor then set $D(v_i)$ to 0 and
   $FH(v_i)$ to $v_0$; else set $D(v_i)$ to $\infty$ and $FH(v_i)$ to NULL.
   Node $v_i$ broadcasts its information to its neighbors.
2: while $v_i$ received information from its neighbor $v_j$ do
3:   if $D(v_i) > D(v_j) + c_j$ then
4:     $D(v_i) \leftarrow D(v_j) + c_j$, and $FH(v_i) \leftarrow v_j$;
5:   if $v_i \neq FH(v_j)$ and $D(v_i) + c_i < D(v_j)$; or $v_i = FH(v_j)$
     and $D(v_i) + c_i \neq D(v_j)$ then
6:     node $v_i$ contacts $v_j$ directly using reliable and secure
       connection, asking $v_j$ to update his $D(v_j)$ to
       $D(v_i) + c_i$ and $FH(v_j)$ to $v_i$. After the necessary
       updating, $v_j$ must broadcast his information.
7: Node $v_i$ broadcasts its information to its neighbors.
```

Algorithm 4 Truthful Distributed Payment Computation by v_i

```
1: Set $p_i^k \leftarrow \infty$, if $v_k \in \mathrm{P}(v_i, v_0, d)$; otherwise, $p_i^k \leftarrow 0$.
2: Broadcasts its entries $p_i^k$ to its neighbors.
3: while $v_i$ received information $p_j^k$ from its neighbor $v_j$ do
4:   $v_i$ updates $p_i^k$ using Algorithm 2 (Steps 4-15).
5:   when $p_i^k$ changes, $v_i$ broadcasts the value of $p_i^k$, and the
     ID of the node $p_j$ that triggered this change.
6:   if $v_i$ triggered the change for this $p_j^k$ from $v_j$, $v_i$
     recalculates $p_j^k$ for $v_j$ using Algorithm 2 (Steps 4-15) to
     verify it. If his answer and the payment sent from $v_j$ do
     not match, node $v_i$ then notifies $v_j$ and other nodes.
     Node $v_j$ will then be punished accordingly, e.g., $v_j$ is
     dropped from the network by all nodes.
```

4 Other Issues of Pricing Mechanism

4.1 Collusion of Nodes

So far, we have assumed that the wireless nodes do *not* collude to improve their utilities. In practice, the nodes may collude with each other in the hope to gain as a group. There are several possible ways such that some nodes can collude. For example, if two nodes v_{k_1} and v_{k_2} know that the removal of them will disconnect some nodes from the access point, then these two nodes can collude to declare arbitrarily large costs and charge a monopoly price together. Notice that, by declaring much higher costs together, one node's utility may decrease, but the sum of their utilities is guaranteed to increase (thus, they share the increased utilities).

We point out here that the collusion of nodes discussed here is different from the traditional *group strategyproof* concept studied in [23], [24]. A pricing mechanism is said to be group strategyproof in [23], [24] if any subset of agents colludes, then each agent of this subset cannot improve its utility without decreasing the utility of some other agent. Clearly, this formulation of group strategyproofness cannot capture the scenario when the profit can be transferred among all colluding nodes, which happens very often in real world. We then formally define what is *k-agents strategyproof* mechanism as follows:

Definition 1. *A mechanism is said to be k-*agents strategyproof *if, when any subset of agents of size k colludes, the* overall *utility of this subset is made worse off by misreporting their types. A mechanism is* true group strategyproof *if it is k-*agents strategyproof *for any k.*

Clearly, we cannot design a *true group strategyproof* mechanism for the unicast routing problem studied here: If all nodes but node v_i collude and declare arbitrarily high costs, then node v_i has to pay a payment arbitrarily higher than the actual payment it needs to pay if these nodes do not collude.

Directly from the incentive compatibility property, for any truthful mechanism, we have:

Lemma 4. *Assume node v_i's valuation is of the form $w^i(o, c_i)$. For any strategyproof mechanism, if the output o keeps unchanged, then the payment and utility of node v_i do not depend on c_i.*

Furthermore, for any 2-agents strategyproof mechanism, we have a stronger conclusion:

Lemma 5. *Suppose every node v_i's valuation is of the form $w^i(o, c_i)$. For any 2-agents strategyproof mechanism, as long as the output o does not change, node v_i's payment and utility do not depend on the profile c.*

Proof. From Lemma 4, we know $w^i(o, c_i)$ does not depend on c_i since we assume the output o is not changed when node v_i declares d_i instead of c_i. Thus, we just need to prove that its utility does not depend on any other nodes' declared cost. We prove it by contradiction. Assume its utility $u^i(d)$ depends on a node v_k's declared cost d_k, then there exist two numbers $d_{k_1} \neq d_{k_2}$ such that $u^i(d|^k d_{k_1}) \neq u^i(d|^k d_{k_2})$. Without loss of generality, we assume that $u^i(d|^k d_{k_1}) > u^i(d|^k d_{k_2})$. From Lemma 4, we have $u^k(d|^k d_{k_1}) = u^k(d|^k d_{k_2})$ since output o is not changed. Consider the case with original profile $c = d|^k d_{k_2}$. Node v_i can ask v_k to lie its cost to d_{k_1}, thus increase v_i's utility while keeping v_k's utility unchanged, which violates the incentive compatibility of 2-agents strategy-proof mechanism. □

In the following discussions, we restrict our attention to the unicast scenario. Remember that x_k denotes whether a node v_k is on the least cost path or not. Let D_1^k be the set of profiles such that $x_k = 1$, i.e., node v_k is on the LCP; D_0^k be the set of profiles such that $x_k = 0$, i.e., node v_k is not on the LCP. Clearly, $D_1^k \bigcup D_0^k$ comprises all possible profiles. From Lemma 5, we have the following:

Lemma 6. *Assume that $\mathcal{A}$ is a 2-agents strategyproof mechanism for unicast and its output $\mathcal{O}$ is the LCP connecting the source and target. For any node v_k, if x_k does not change, then p^k calculated by $\mathcal{A}$ is independent of d.*

Proof. We prove it by contradiction. Suppose node v_k's payment depends on d, then there exist two profiles $(d|^i d_{i_1})$ and $(d|^i d_{i_2})$ such that $d_{i_1} > d_{i_2}$, $p^k(d|^i d_{i_1}) \neq p^k(d|^i d_{i_2})$, and $x_k(d|^i d_{i_1}) = x_k(d|^i d_{i_2})$. There are two cases here.

Case 1: $x_i(d|^i d_{i_1}) = x_i(d|^i d_{i_2})$. Clearly, the LCP path remains the same when node v_i declares cost d_{i_1} or d_{i_2}. From Lemma 5, we have $p^k(d|^i d_{i_1}) = p^k(d|^i d_{i_2})$, which is a contradiction. Thus, this case is impossible.

Case 2: $x_i(d|^i d_{i_1}) \neq x_i(d|^i d_{i_2})$. Since $d_{i_1} > d_{i_2}$, this case means that when $d_i = d_{i_1}$, v_i is not on the LCP, and when $d_i = d_{i_2}$, v_i is on the LCP. Now, fixing d^{-i} and increasing node v_i's declared cost from d_{i_2} to d_{i_1}, there must exist $a_i \in [d_{i_2}, d_{i_1}]$ such that v_i is on LCP if $d_i < a_i$, v_i is not on LCP if $d_i > a_i$, and it is unknown when $d_i = a_i$. For node v_i, its utility and payment do not depend on its own declared cost d_i. From Lemma 5, its payment is a constant $\underline{P_i}$ when $d_i < a_i$ (since the output remains the same for every $d_i < a_i$) and another constant $\overline{P_i}$ when $d_i > a_i$. From the incentive compatibility of node v_i, we have $\underline{P_i} - d_i \geq \overline{P_i}$, for any $d_i \leq a_i$, since we have to prevent node v_i from lying its cost from d_i to a number larger than a_i. Similarly, to prevent v_i from lying down its cost from a number larger than a_i to a $d_i \leq a_i$, we need $\underline{P_i} - d_i \leq \overline{P_i}$, for any $d_i \leq a_i$. Thus, we have $\underline{P_i} - \overline{P_i} = a_i$.

Suppose $p^k(d|^i d_{i_1}) = p^k(d|^i d_{i_2}) + \delta$. We first consider the case $\delta < 0$. Considering the graph with profile $c = (d|^i d_{i_1})$, clearly node v_i is not on the LCP and its utility is $\overline{P_i}$. Thus, the sum of node v_k and v_i's utility, when node v_i declares cost d_{i_1}, is $u^k(d|^i d_{i_1}) + u^i(d|^i d_{i_1}) = p^k(d|^i d_{i_1}) + \overline{P_i} - x_k \cdot c_k$, where $x_k = x_k(d|^i d_{i_1}) = x_k(d|^i d_{i_2})$. Now, consider the scenario when v_i declares its cost as $d_{i_2} \leq a_i$. The sum of node v_k and v_i's utility becomes (since x_k remains the same)

$$\begin{aligned} &u^k(d|^i d_{i_2}) + u^i(d|^i d_{i_2}) \\ &= p^k(d|^i d_{i_2}) + \underline{P_i} - x_k c_k - d_{i_2} \geq p^k(d|^i d_{i_2}) + \overline{P_i} - x_k c_k \\ &= p^k(d|^i d_{i_1}) + \overline{P_i} - x_k c_k - \delta > u^k(d|^k c_i) + u^i(d|^k c_i). \end{aligned}$$

This implies that v_i and v_k can benefit together by asking v_i to lie its cost from d_{i_1} to d_{i_2}.

We then consider the case $\delta > 0$. Consider the graph with profile $c = (d|^i c_i)$, where $c_i = a_i - \epsilon$, and

$$0 \leq \epsilon \leq \min\left\{\frac{\delta}{2}, a_i - d_{i_2}\right\}.$$

Clearly, node v_i is on the LCP and its utility is $\underline{P_i} - c_i$. Thus, the sum of node v_k and v_i's utility, when $d_i = c_i$, is

$$\begin{aligned} &u^k(d|^i c_i) + u^i(d|^i c_i) = p^k(d|^i c_i) + \underline{P_i} - x'_k c_k - c_i \\ &= p^k(d|^i d_{i_2}) + \overline{P_i} - x'_k c_k + \epsilon, \end{aligned}$$

where $x'_k = x_k(d|^i c_i)$. Now, consider the scenario when v_i declares its cost as d_{i_1}. Notice that $d_{i_1} \geq a_i$. The sum of node v_k and v_i's utility becomes

$$\begin{aligned} &u^k(d|^i d_{i_1}) + u^i(d|^i d_{i_1}) \\ &= p^k(d|^i d_{i_1}) + \overline{P_i} - x_k c_k = p^k(d|^i d_{i_2}) + \overline{P_i} - x_k c_k + \delta \\ &> p^k(d|^i d_{i_2}) + \overline{P_i} - x'_k c_k + \epsilon = u^k(d|^k c_i) + u^i(d|^k c_i). \end{aligned}$$

The last inequality comes from $x_k(d|^i d_{i_1}) = x_k \leq x'_k = x_k(d|^i c_i)$ (proof follows) and $\delta > \epsilon$. This implies that v_i and v_k can benefit together by asking v_i to lie its cost from $a_i - \frac{\delta}{2}$ to d_{i_1}.

At last, we prove that when node v_i declares a cost $b_i \in [d_{i_2}, a_i]$ while d^{-i} is fixed, $x_k(d|^i b_i) \geq x_k(d|^i d_{i_1})$. We only have to prove for the case $x_k(d|^i d_{i_1}) = 1$ and we prove it by contradiction. Assume that there is a $b_i \in [d_{i_2}, a_i]$ such that $x_k(d|^i b_i) = 0$. Let $\Pi(d)$ be the total cost of a path Π under cost profile d. Let Π_1 be the least cost path connecting the source and target using profile $d|^i b_i$. Observe that $v_i \in \Pi_1$. By assumption, $v_k \notin \Pi_1$. Let Π_2 be the least cost path connecting the source and target using profile $d|^i d_{i_2}$. Remember that $v_i \in \Pi_2$ and $v_k \in \Pi_2$. Thus, Π_1 and Π_2 are different paths. From the optimality of Π_1 under cost profile $d|^i b_i$, we have $\Pi_1(d|^i b_i) < \Pi_2(d|^i b_i)$. From the optimality of Π_2 under cost profile $d|^i d_{i_2}$, we have $\Pi_1(d|^i d_{i_2}) > \Pi_2(d|^i d_{i_2})$. On the other hand,

$$\begin{aligned} \Pi_1(d|^i b_i) &= \Pi_1(d|^i d_{i_2}) + b_i - d_{i_2} > \Pi_2(d|^i d_{i_2}) + b_i - d_{i_2} \\ &= \Pi_2(d|^i b_i) > \Pi_1(d|^i b_i), \end{aligned}$$

which is a contradiction. This finishes our proof. □

The above lemma implies that, for any 2-agents strategy-proof mechanism for unicast, the payment to any node v_k, regardless of the cost profile, is a constant as long as v_k is on the LCP; the payment to any node v_k, regardless of the cost profile, is another constant as long as v_k is *not* on the LCP.

Theorem 1. *There is no 2-agents strategyproof mechanism for unicast problem if the output is the LCP.*

Proof. We prove it by contradiction. Assume $\mathcal{A}$ is a 2-agents strategyproof mechanism. From Lemma 6, we know that if node v_k is on LCP then its payment is $\underline{P}$, else its payment is $\overline{P}$. Now, consider a profile d of their declared costs, and v_k in on LCP. Given a fixed d^{-k}, there exists $a_k > 0$ such that v_k is on LCP if and only if $d_k \leq a_k$. It is easy to see that $a_k = \mathrm{P}_{-v_k}(v_i, v_0, d) - \mathrm{P}(v_i, v_0, d|^k 0)$ and $\overline{P} - \underline{P} = a_k$ (otherwise, node v_k can lie about its cost to improve its utility). In other words, $\overline{P} - \underline{P} = \|\mathrm{P}_{-v_k}(v_i, v_0, d) - \mathrm{P}(v_i, v_0, d|^k 0)\|$ depends on d, which is a contradiction to the requirement that both $\overline{P}$ and $\underline{P}$ are fixed constants. This finishes our proof. □

Theorem 1 relieves us from designing any k-agents strategyproof when the objective is to use the least cost path for routing. In the following discussions, we study how to design a truthful mechanism such that it can prevent nodes from colluding with its one-hop neighbors. Notice that the VCG payment scheme discussed in Section 3.1 does not prevent a node from colluding with its neighbors at all. It is not difficult to construct an example such that, for a node $v_k \in \mathrm{P}(v_i, v_0, d)$, the path $\mathrm{P}_{-v_k}(v_i, v_0, d)$ uses a node v_t that is a neighbor of v_k and $v_t \notin \mathrm{P}(v_i, v_0, d)$. Then, v_t can lie its cost up to increase the utility of node v_k.

Assume that node v_i pays other nodes to relay the data to another node v_j. Let $N(v_k)$ be the set of neighbors of node v_k, including node v_k itself. Thus, to have a payment scheme that prevents collusion between any two neighboring nodes, it is necessary that the graph resulted by removing $N(v_k)$ still has a path connecting v_i and v_j. Therefore, we assume that graph $G \backslash N(v_k)$ is connected for any node v_k. Similar to the payment scheme presented in Section 3.1 when nodes do not collude, we design the following payment scheme $\tilde{p}$ that avoids the collusion between any two neighboring nodes. The payment $\tilde{p}_i^k(d)$ to a node v_k is

$$\tilde{p}_i^k(d) = \|\mathrm{P}_{-N(v_k)}(v_i, v_0, d)\| - \|\mathrm{P}(v_i, v_0, d)\| + x_k(i) \cdot d_k,$$

where $\mathrm{P}_{-N(v_k)}(v_i, v_0, d)$ is the least cost path connecting v_i and v_j in graph $G \backslash N(v_k)$ without using any node in $N(v_k)$. Notice that the payment to a node $v_k \notin \mathrm{P}(v_i, v_0, d)$ could be positive when node v_k has a neighbor on $\mathrm{P}(v_i, v_0, d)$. This is a sharp difference to our first payment scheme based on VCG.

We then prove that the payment scheme $\tilde{p}$ is indeed truthful.

Theorem 2. *The payment scheme $\tilde{p}$ is a strategyproof mechanism that prevents any two neighboring nodes from colluding.*

Proof. Clearly, each individual node will be truthful since our mechanism belongs to the family of VCG mechanisms. Notice for any two neighboring nodes v_k and v_l, their utilities can be written as

$$\begin{cases} u^k(c) &= \sum_{t=0}^{n-1} v^t(o(c), c_t) + h^{-k}(c^{-N(v_k)}) \\ u^l(c) &= \sum_{t=0}^{n-1} v^t(o(c), c_t) + h^{-l}(c^{-N(v_l)}). \end{cases}$$

Summarizing them, we get

$$u^l(c) + u^k(c) = 2\sum_{t=0}^{n-1} v^t(o(c), c_t) + h^{-k}(c^{-N(v_k)}) + h^{-l}(c^{-N(v_l)}).$$

Notice that $h^{-k}(c^{-N(v_k)}) + h^{-l}(c^{-N(v_l)})$ does not depend on d_l and d_k since v_k and v_l are neighbors of each other. In addition, $\sum v^i(o(c), c_i)$ is maximized when they reveal their true costs. Thus, v_k and v_l will maximize their total utilities by revealing their true costs. □

It is easy to show that the above payment scheme is optimum in terms of the payment to each individual node when the output is the least cost path. The proof is omitted due to space limit.

Furthermore, we can extend the above scheme to a more general case when we want to prevent some groups of nodes from colluding. Let $\{Q(v_1), Q(v_2), \cdots, Q(v_n)\}$ be a set of subsets of nodes, i.e., $Q(v_k) \subset V$. We then show how to design a truthful mechanism such that any node v_k *cannot* collude with other nodes in $Q(v_k)$ to increase their total utilities. For simplicity, assume that $v_k \in Q(v_k)$, for $1 \le k \le n$. It is easy to show that the following mechanism is truthful: 1) the output is the least cost path connecting the source v_i and destination v_0; 2) the payment $\tilde{p}_i^k(d)$ to a node v_k is

$$\tilde{p}_i^k(d) = \|\mathrm{P}_{-Q(v_k)}(v_i, v_0, d)\| - \|\mathrm{P}(v_i, v_0, d)\| + x_k(i) \cdot d_k.$$

Obviously, we need graph $G \backslash Q(v_k)$ to be connected for any node v_k.

4.2 Link Cost Instead of Node Cost

So far, we assumed that the cost of a node forwarding data to *any* neighbor is same, i.e., the cost is incurred on each node. However, each node could have different costs of forwarding data to different neighbors by using power adjustment technique. Thus, we assume that each wireless node v_i has a private type $c_i = (c_{i,0}, c_{i,1}, \cdots, c_{i,n-1})$. Here, $c_{i,j}$ is its power cost to support the link to a node v_j. If node v_i cannot reach node v_j, then the power cost is assumed to be ∞. Obviously, $c_{i,i} = 0$.

Notice that this model of network is different from the network models used by previous strategyproof pricing mechanisms [4], [22] for unicast. In their models, either a link is an agent, which has a computational power and private cost type, or a node is an agent, which has computational power and the private scalar cost to carry the transit traffic. In our new model here, we treat each node as an agent and it has some private type which is a vector. The valuation of a node is *solely* determined by which incident link is used in the optimal solution. Specifically, given an output (a path $v_{i_s}, v_{i_{s-1}}, \cdots v_{i_1}, v_{i_0}$ connecting node v_i to v_0, where $v_i = v_{i_s}$ and $v_0 = v_{i_0}$), the valuation $w^{i_k}(c^{i_k}, o)$ of the node v_{i_k} is $-c_{i_k, i_{k-1}}$. Given the declared types by all wireless nodes, the mechanism will compute a path that maximizes the valuation of all nodes, and a pricing scheme p that is strategyproof.

The strategyproof pricing mechanism works as follows. First, each node v_i declares its cost vector d_i, which is a n-ary vector itself. We then define a *directed* and *weighted* graph $G = (Q, E, W)$, where the weight of a directed link v_iv_j is $d_{i,j}$. A least cost directed path $\mathrm{P}(v_i, v_0, d)$ is computed to connect v_i to v_0, which is the output. Let $x_{k,j}(d, i, 0)$ be the indicator of whether a directed link v_kv_j is on the directed path from v_i to v_0. The payment $p_i^k(d)$ to node v_k is $\sum_j x_{k,j}(d, i, 0) d_{k,j} + \Delta_{i,k}$. Here, $\Delta_{i,k}$ is the improvement of the least cost path from v_i to the access point due to the existence of node v_k. In other words,

$$\Delta_{i,k} = \sum_{r,j} x_{r,j}(d|^k\infty, i, 0) d_{r,j} - \sum_{r,j} x_{r,j}(d, i, 0) d_{r,j}.$$

Notice, to calculate the least cost $v_k\text{-avoiding-path}$, we set $d_{k,j} = \infty$ for each node v_j.

It is not difficult to show that, under the above payment model, every node (except the source node) could not lie about its cost vector to improve its profit. We assume that the source node will not lie about its cost vector. Then, the above payment scheme is truthful. We can show that the fast payment scheme based on Algorithm 1 can be modified to compute the payment in time $O(n \log n + m)$ when each node is an agent in a link-weighted directed network. Notice that, in this network model, the source node could lie about its cost vector such that its total payment to all relay nodes will decrease.

4.3 Ratio of Total Payment over Total Cost of the Path

Clearly, node v_i pays each node on $\mathrm{P}(v_i, v_0, c)$ more than its actual cost to make sure that it will not lie about its cost. The overpaid value is the improvement of the least cost path due to the existence of node v_k. It is not difficult to construct a network example such that the overpayment of a node v_i could be arbitrarily large. But, on the other hand, when we conducted extensive simulations to study the amount of overpayment when the cost of each node is chosen independently and uniformly from a range and the network topology is a random graph, the result shows that the large overpayment usually will not happen in the real world.

Let $p_i = \sum_{v_k \in \mathrm{P}(v_i, v_0, c)} p_i^k(c)$, i.e., the total payment of node v_i to the relay nodes. The metrics of the overpayment used in our simulations are *Total Overpayment Ratio* (TOR), *Individual Overpayment Ratio* (IOR), and *Worst Overpayment Ratio*. The TOR of a graph is defined as $\sum_i p_i / \sum_i \|\mathrm{P}(v_i, v_0, c)\|$, i.e., the total payment of all nodes over the total cost of all LCPs. The IOR of a graph is defined as $\frac{1}{n}\sum_i \frac{p_i}{\|\mathrm{P}(v_i, v_0, c)\|}$, i.e., the average overpayment ratio over all n nodes. The worst overpayment ratio is defined as $\max_i \frac{p_i}{\|\mathrm{P}(v_i, v_0, c)\|}$, i.e., the maximum overpayment ratio over all n nodes. We found that the IOR and TOR are almost the same in all our simulations and they take values around 1.5. In all our simulations, the average and the maximum are taken over 100 random instances.

In the first simulation, we randomly generate n nodes uniformly in a $2{,}000m \times 2{,}000m$ region. The transmission range of each node is set as $300m$. The cost of each node v_i to forward a packet to any neighbor is $c_1 + c_2 \cdot r^{\kappa}$, where r is fixed at 300. Here, random number c_1 takes value from 300 to 500 and c_2 takes a random value from 10 to 50. The ranges of c_1 and c_2 we used here reflect the actual power cost in one second of a node to send data at $2Mbps$ rate. The number of nodes in our simulations varies among $100, 150, 200, \cdots, 500$. We choose two different κ values 2 and 2.5. Fig. 4a illustrates the difference between IOR and TOR when graph model is UDG and $\kappa = 2$. We found that these two metrics are almost the same and both of them are stable when the number of nodes increases. Fig. 4d illustrates the overpayment respecting to the hop distance to the source node. The average overpayment ratio of node stays almost stable regardless of the hop distance to the source. The maximum overpayment ratio decreases when the hop distance increases, which is because large hop distance to the source node will smooth off the oscillation of the relay costs' difference: For a node closer to the access point, the second shortest path could be much larger than the shortest path, which in turn incurs large overpayment; for a node far away from the access point, the second shortest path has total cost almost the same as the shortest path, which in turn incurs small overpayment. Keep in mind that the payment indeed increases when the hop distance to the source increases. Fig. 4b and Fig. 4c illustrate the overpayment for UDG graph when $\kappa = 2$ and $\kappa = 2.5$, respectively.

In our second simulation, we study the performance of our payment methods when each individual nodes have different transmission ranges instead of the same transmission ranges studied before. The cost c_i of a node v_i to send a packet to its neighbors is $c_1 + c_2 \|r_i\|^{\kappa}$. Here, the transmission range r_i of node i takes a random value from $[100, 500]$. Fig. 4e and Fig. 4f illustrate the overpayment for random networks when $\kappa = 2$ and $\kappa = 2.5$, respectively. Similar observations were obtained for these simulations.

4.4 Other Issues about the Pricing Mechanism

Other possible attacks. There are some other attacks possible to the scheme. A source node may refuse to pay by claiming that he is not the source of the communication and thus should not pay for it. To counter this attack, we require that each node sign the message when it initiates the message, the relay nodes will verify the signature.

Another possible attack is *free riding*: A relay node v_k on the route $\mathrm{P}(v_i, v_0, c)$ may attempt to piggyback data on the packets sent between the initiator v_i with the goal of not having to pay for the communications to node v_0. To counter this attack, the initiator v_i pays the relay node v_k only when it receives a signed acknowledgment from the access point v_0.

Where to pay. We briefly discuss how the payment is charged. All payment transactions are conducted at the access point v_0. Each node v_i has a secure account at node v_0. There are two scenarios here. The first scenario is that the access point v_0 receives a data from v_i. Node v_0 verifies the truthfulness of the source and then pays each node v_k on the LCP p_i^k and charges that from node v_i. The second scenario is that a node v_i retrieves data from node v_0. A relay node v_k on the LCP will send a signed acknowledgment after relaying the data to the next hop v_j. The acknowledgment to v_0 by v_k includes the authentication from v_j that v_k does send data to v_j. Node v_0 then pays node v_k and charges node v_i accordingly after receiving this signed acknowledgment.

A better way to prevent these attacks described before may be to combine the efficient pricing mechanism presented here with the payment management method presented by Jacobsson [8], which we leave as a future work.

Mobility and dynamic cost. Our protocols assumed that the network topology does not change for the period time of computing the least cost path, computing the payment, and routing packets from the source to the access point. When wireless nodes move, the above protocol still works as long as the network topology does not change. When the mobility triggers the change of the network topology, our protocol has to update the path, and the payment calculated accordingly. A node will be compensated for only the portion of the traffic it relayed.

For simplicity, we assumed that the cost of each node is fixed for the moment. When the cost of a node changes, the node has to declare its new cost. This will trigger the change of the least cost paths and consequently the payment to

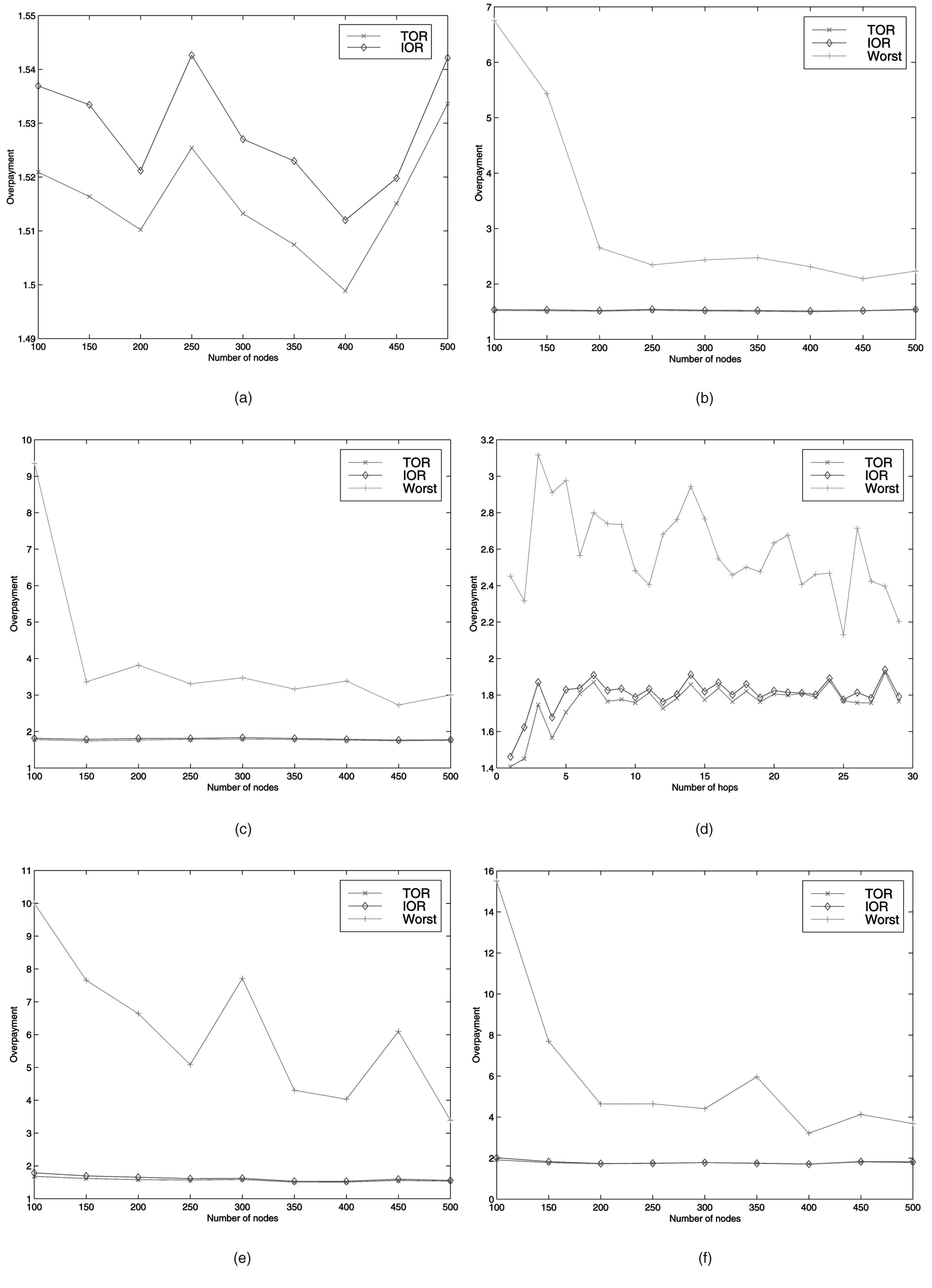

Fig. 4. Overpayment ratios IOR, TOR, and the worst ratio for UDG and random graphs. (a) IOR and TOR difference. (b) UDG with $\kappa = 2$. (c) UDG with $\kappa = 2.5$. (d) The affect of hops. (e) Random graph with $\kappa = 2$. (f) Random graph with $\kappa = 2.5$.

nodes in the network. The cost could change due to its battery power changes, the competition among other nodes changes, and so on.

So far, we only concentrate on one session of routing. For the successive sessions, the source node and the access point already know the "actual" cost of all relay nodes. It is

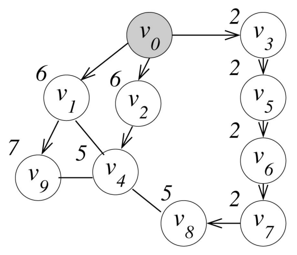

Fig. 5. An example of wireless network and the node cost. The directed links form the shortest path tree from v_0 to every v_i.

natural to ask how much we should pay the relay nodes for later sessions now? One may think that we could pay these nodes their declared costs now. It is not difficult to construct an example such that this simple mechanism is not truthful: A relay node could misreport its cost initially; although its payment for the first session could decrease, but the gain in later sessions is large enough to cover this initial loss. We actually can prove that for successive sessions the source node still has to pay the relay nodes based on the payment scheme described before. The result is omitted here due to space limit. Notice that another reason we could not pay a relay node v_k its declared cost d_k is that node v_k could simply refuse to relay since it gains nothing by relaying.

Resale the path. Another possible collusion happens after the payment is calculated and during the process of actually routing the packets. Let $p_i = \sum_{k=0}^{n-1} p_i^k$, i.e., the total payment of node v_i to all relay nodes on the least cost path $\mathtt{P}(v_i, v_0, c)$. Assume that $p_i > p_j + \max(p_i^j, c_j)$ for some neighbor v_j of v_i. Notice that $\max(p_i^j, c_j) = x_j(i)p_i^j + (1 - x_j(i))c_j$ since if v_j is on LCP $\mathtt{P}(v_i, v_0, c)$, then $p_i^j \geq c_j$ and $p_i^j = 0 < c_j$ otherwise. Here, $x_j(i)$ is the indicator function whether node v_j is on $\mathtt{P}(v_i, v_0, c)$. Then, v_i and v_j can collude in favor of them as follows: 1) v_j sends the data packets for v_i and v_j pays all relay nodes on path $\mathtt{P}(v_i, v_0, c)$. 2) v_i pays v_j the cost $p_j + \max(p_i^j, c_j)$, which covers the payment by v_j. 3) v_i and v_j split the difference $p_i - (p_j + \max(p_i^j, c_j))$, which is the saving of node v_i from colluding with node v_j. Notice that it is possible that $p_i > p_j + \max(p_i^j, c_j)$ for some neighbor v_j of v_i. Fig. 5 illustrates such an example of such collusion. It is easy to compute that $p_8 = 20$, $p_4 = 6$, and $p_8^4 = 0$. Notice $c_4 = 5$. Thus, v_8 can ask v_4 to forward the data packets using its LCP to v_0. Node v_8 pays node v_4 a price $6 + 5 = 11$ to cover its payment p_4 and its cost c_4, and half of the savings, which is 4.5. Thus, the total payment of node v_8 is only 15.5 now, which is less than p_8 and node v_4 also increases its utility from 0 to 4.5. Currently, we are not aware of any method that can prevent this from happening. We may argue, on the other side, that it may be not necessary to prevent this from happening since this actually encourages some kind of cooperations among nodes.

5 Related Work

Routing has been part of the algorithmic mechanism-design from the beginning. Nisan and Ronen [4] provided a polynomial-time strategyproof mechanism for optimal route selection in a centralized computational model. In their formulation, each edge e of the graph is an agent and has a private cost c_e of sending a unit data along this edge. Their payment scheme falls into the family of VCG mechanisms. Feigenbaum et al. [22] then addressed the truthful low cost routing in a different network model. They assumed that each node k incurs a transit cost c_k for each transit packet it carries. Their payment scheme again is also a VCG mechanism. They also gave a distributed method such that each node i can compute the payment to node k for carrying the transit traffic from node i to node j if node k is on the LCP from i to j. Since the mechanism is truthful, any node cannot lie its cost to improve its profit in their distributed algorithm. However, as they pointed out [22], it is unclear how to prevent these selfish nodes from running a different algorithms in computing a payment that is more favorable to themselves since we have to rely on these nodes to run the distributed algorithm, although we know that the nodes will input their true values. Anderegg and Eidenbenz [25] recently proposed a routing protocol for wireless ad hoc networks based on the VCG mechanism. They assumed that each node is a selfish agent, and incurs a privately known cost when communicating with a neighbor. In their model, the cost from a node v to a node u may be different from the cost from node u to a node v, i.e., being asymmetric.

Some researchers use totally different methods to deal with selfish wireless networks. We briefly review some of them as follows. Marti et al. [9] proposed a scheme based on credibility of nodes. They call a node *misbehaving* if it originally agrees to relay traffic but does not actually. The routing protocol will avoid using these misbehaving nodes. Buttyan et al. [7], [8], [11], [10] presented a sequence of methods to stimulate cooperation among nodes and to prevent nodes from overloading the network. A key idea behind their protocols is that we should compensate nodes providing a service, and charge nodes receiving a service. In their protocols, each node maintains a counter, called *nuglet counter*, in a tamper resistant hardware module. The value of nuglet always remains positive. When a node sends packets to other node, it pays each relay node 1 nuglet, and its nuglet counter is decreased by the hops of the path used. These approaches can be viewed as a fixed price payment. Srinivasan et al. [12] proposed two acceptance algorithms, which are used by each wireless node to decide whether to relay the traffic for other node. In [13], Srinivasan et al. proposed an acceptance algorithm called GTFT and they proved that GTFT results in Nash equilibrium. Salem et al. [26] presented a charging and rewarding scheme for packet forwarding in multihop cellular networks. In their network model, there is a base-station to forward the packets. They use symmetric cryptography to cope with the lying.

6 Conclusion

In this paper, we gave a strategyproof pricing mechanism that stimulates cooperation for unicast among wireless ad hoc networks. In our strategyproof scheme, each node v_k first declares its cost of relaying data for other nodes, node v_i then computes the least cost path to the access point v_0, and a payment is also computed for each relay node on the least cost path. We presented the first centralized algorithm with optimal time-complexity to compute such payment. We also discussed in detail how to implement this scheme on each selfish node in a distributed manner. We showed that although each node is selfish, the proposed scheme guarantees that each node will declare its true cost and also follow the designed protocol. As all truthful mechanisms, the proposed scheme pays each relay node more than its declared cost to prevent it from lying. We conducted extensive simulations and found that practically the overpayment is small when the cost of each node is a random value between some range.

Our protocol assumes that nodes will not collude. We showed that *no* truthful mechanism can prevent all pairs of nodes from colluding to improve their utilities. We designed a truthful payment scheme that can prevent nodes from colluding with its neighbors. Our payment scheme is optimum in terms of the individual payment.

In this paper, we assumed that the cost of a node is fixed (at least for one communicating session). In practice, the cost of a node may be dynamic, e.g., it may depend on other ongoing traffics, depend on its remaining battery power, or even depend on the competition by other nodes. A successful study of the static cost model will pave a way for studying the dynamic cost model. We leave designing payment schemes for dynamic cost model as a future work.

ACKNOWLEDGMENTS

The work of the second author is partially supported by the US National Science Foundation grant number CCR-0311174.

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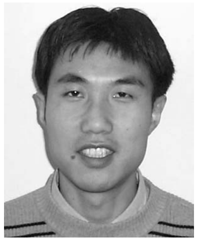

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