Interference Avoidance in Networks with Distributed Receivers

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Abstract—Direct extensions of distributed greedy interference avoidance (IA) techniques developed for centralized networks to networks with multiple distributed receivers (as in ad hoc networks) are not guaranteed to converge. Motivated by this fact, we develop a waveform adaptation (WA) algorithm framework for IA based on potential game theory. The potential game model ensures the convergence of the designed algorithms in distributed networks and leads to desirable network solutions. Properties of the game model are then exploited to design distributed implementations of the algorithm that involve limited feedback in the network. Finally, variations of IA algorithms including IA with respect to legacy systems and IA with combined transmit-power and WA adaptations are investigated.

Index Terms—Interference avoidance, waveform adaptation, distributed networks, game theory.

I. INTRODUCTION

NETWORKS are becoming less structured and increasingly involve distributed decision making and dynamic spectrum sharing. Nodes in a distributed network are required to independently adapt in a way that reduces interference and consequently facilitates multi-user communications. This paper investigates strategies for WA that allow nodes in the network to avoid or minimize the interference at their receivers and that are amenable to a distributed implementation in networks with non-colocated receivers.

Distributed WA techniques for IA have been extensively investigated for networks with a centralized receiver or equivalently networks with co-located receivers ([1], [2], [3] and references within). Due to the inherent structure of these networks, the interference profiles for different users are symmetric. This property leads to the convergence of simple iterative WA schemes where each user greedily adapts to maximize or improve the signal to interference and noise ratio (SINR) at its receiver [4]. In addition, this property ensures that each greedy adaptation decreases the total-sum-correlation (TSC) of the network. Minimization of TSC in CDMA networks with a central receiver is equivalent to the maximization of sum-capacity. Hence greedy adaptations could lead to globally optimal waveforms in centralized networks. WA in multi-cell networks, where each user communicates with multiple

collaborative base-stations/receivers, is investigated in [5] and [6]. It is assumed in these papers that each user's adaptations are based on the composite of the received signals at all base-stations. This assumption makes the interference profiles of users symmetric and allows the greedy WA algorithms to be easily extended to multi-cell networks.

Greedy WA algorithms are extended to networks with non-colocated receivers (i.e. in networks such as ad hoc networks where users might not have common receivers) in [7]. However, in these networks the direct application of greedy WA algorithms does not always lead to convergence. This is caused by the asymmetry of the mutual interference between users at different receivers, leading the users to adapt their sequences in conflicting ways and resulting in resource allocation cycles. (Convergence of the algorithms for distributed networks developed in [7] can only be established for networks with 2 users.) Since each adaptation, in general, requires considerable feedback from the receiver to the transmitter, these allocation cycles are expensive with respect to the network overhead and are undesirable from a network performance perspective. Distributed network scenarios in which such situations arise have been identified in [6] and [8].

Game theory is a branch of applied mathematics that models interactions between rational decision makers with formalized incentive (preference) structures and provides tools to predict and analyze the outcome of these interactions. Game theoretic models can hence be used for the design and analysis of distributed algorithms in which individual nodes adapt their actions and contend for a common resource. The optimality, convergence, steady-states, and stability of these algorithms can then be investigated by using properties of game models. A survey on the use of game theory to analyze wireless ad-hoc networks is presented in [9].

In this paper, we draw on concepts from game theory to develop iterative WA techniques for IA that converge to a desirable state in networks (including networks with distributed receivers). Specifically, we design an adaptation framework based on potential game theory. Game theory has been previously used to design and analyze WA algorithms in [3], [4], [10], [11] and [12]. In [3] and [4], greedy WA for centralized networks is cast as a potential game with a weighted form of the TSC function as the potential function. The potential game formulation is then used to develop new convergence properties for the algorithms in centralized networks. However, it can be shown that the TSC is no longer a potential function when greedy WA is directly extended to distributed networks. A potential game model is also used in [11] to design dynamic frequency selection algorithms. However, the

developed algorithms are again based on greedy adaptations and only converge in networks with symmetric channels. In [12] convex non-cooperative games are used to investigate the convergence and Nash Equilibria of combined power and waveform adaptation games that allow users/players to obtain desired target SINRs in the network. However, the formulation also uses greedy adaptations where convergence can only be established in centralized networks. In [10] separable game theory is used to develop joint power and waveform adaptation games. Conditions that allow the existence of Nash Equilibria for these joint games are identified, and it is shown that these conditions are satisfied for greedy adaptation in centralized networks. However, greedy adaptations in distributed networks might not have Nash Equilibria. In our proposed formulation, as opposed to greedy adaptations, each user adapts to improve some measure of the influence caused by its actions on the other users in the network in addition to improving its own performance. This ensures the convergence of the adaptation techniques in networks with any structure (including distributed networks). Properties of potential games are then used to construct distributed implementations of the adaptation scheme. We also present some variations of the IA algorithm including IA with respect to legacy systems and a multi-parameter IA scheme where users adapt both their transmit power and their waveforms. Such combined power and WA adaptation algorithms have been previously analyzed only for centralized networks ([13], [12], [10] and references within). Based on the framework presented in our paper, combined power and WA games to achieve achieve target SINRs have been further developed in [14].

The rest of the paper is organized as follows: The system model for the network under consideration is described in Section II. Game theory concepts used in this paper are briefly discussed in Section III. In Section IV, the WA for IA problem is cast as a potential game and the Nash equilibria of the game are identified. Based on the potential game formulation, a best response algorithm for WA is designed in V. However, it is seen that the distributed implementation of the algorithm requires considerable feedback in the network. Properties of potential games are used to design distributed implementations of the algorithm that require limited feedback in the network in Section VI. Some variations of the IA algorithm including IA with respect to legacy systems and IA with combined power and WA adaptations are presented in Section VII. Finally, Section VIII concludes the paper.

II. SYSTEM MODEL

In this paper, we develop a WA framework based on potential game theory for IA in distributed networks. A potential game model (briefly reviewed in the next section) is chosen as this model provides a framework where each user's adaptation can be designed to iteratively increase a global network performance measure, leading to algorithms that are simple, easy to analyze, and useful from a network perspective. The distributed network we consider is made up of a cluster of transmit and receive nodes as in an ad hoc network (Figure 1). Interference is caused to a transmit-receive node pair by transmissions between other node-pairs in the network. The network can thus be viewed as a collection of interfering
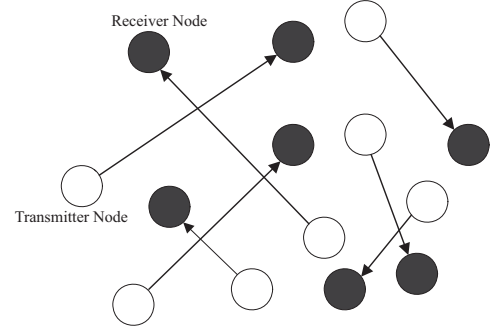

Fig. 1.   Example network with multiple un-coordinated receivers

links similar to the model considered in [15]. As is also mentioned in [15], this network model is a generalization of a cellular-type network with co-located or centralized receivers and hence the results presented here are applicable to the centralized network scenario as well. Interference between nodes is influenced by the correlation between the waveforms of user nodes, transmit power levels, and the channel characteristics.

The transmit waveforms of user-nodes are represented using a signal-space characterization [2] [16]. Let the transmit signal of a user, corresponding to a single information symbol, span an interval of T seconds and be almost limited to bandwidth $\pm W$. Then the transmit waveform of a user can be represented in about $2WT$ orthonormal basis functions [2]. (Note that these functions can overlap in frequency or time.) Let the signature sequence associated with a user be the projection of the transmit waveform of the user in these orthonormal basis functions (or dimensions). Let N denote the number of orthonormal dimensions used to represent a transmit waveform of a user (i.e. transmission dimensions available to the network) and K denote the number of transmitting nodes in the network. Column vector $s_k \in \mathbb{R}^N$ is used to denote the signature sequence associated with transmitting node k. The signature sequences are allowed to have real values (as opposed to bi-polar sequences). Without loss of generality, the signature sequences are assumed to have unit norm and hence are constrained to the N dimensional sphere $S = \left\{ s_k \in \mathbb{R}^N : \|s_k\|^2 = 1 \right\}$. The transmit power level of the kth node is denoted by p_k and the fading coefficient of the channel between the kth transmit node and the jth receive node is denoted by g_{kj}. The channel is assumed to be constant over all signal dimensions and over the time required for the adaptation process. The data symbol (assumed to be of zero-mean and unit-variance) transmitted from the kth transmit node is denoted by b_k. The symbols sent by each transmitter are assumed to be independent and to have zero mean and unit variance. The received signal at the jth receive node is then

$$r_j = \sum_{k=1}^{K} \sqrt{p_k} \, g_{kj} s_k b_k + z, \tag{1}$$

where $r_j \in \mathbb{R}^N$ and the vector $z \in \mathbb{R}^N$ models zero mean additive Gaussian noise with variance σ^2. For notational simplicity, similar to a majority of the literature on IA, we assume that waveforms from multiple users are synchronized at the receivers. However, the schemes presented in this paper

can be easily extended to asynchronous systems (this is briefly discussed in Section VII-C).

III. BRIEF SUMMARY OF POTENTIAL GAME THEORY

Consider a normal form game [17] represented as the tuple $\Gamma = \langle \mathcal{K}, \{A_k\}_{k \in \mathcal{K}}, \{u_k\}_{k \in \mathcal{K}} \rangle$. Here, $\mathcal{K} = \{1, 2, \dots, |\mathcal{K}|\}$ is the set of players of the game. The set of actions available for player k is denoted by A_k and the utility function associated with each player k by u_k. If the set of all available actions for all players is represented by $A = \underset{k \in \mathcal{K}}{\times} A_k$, then $u_k : A \to \mathbb{R}$. Player k prefers an action profile $a \in A$ over an action profile a' if $u_k(a) \geq u_k(a')$. A Nash Equilibrium (NE) for a game is an action profile from which no player can increase its utility by unilateral deviations. An action profile, $a \in A$, is a NE if and only if $u_k(a) \geq u_k(b_k, a_{-k}) \ \forall k \in \mathcal{K}, b_k \in A_k$. Here, $(b_k, a_{-k}) = (a_1, \dots, a_{k-1}, b_k, a_{k+1}, \dots, a_{|\mathcal{K}|})$ refers to the action profile in which the action of user k is changed from a_k to b_k, while the actions of the other players remain the same. Nash equilibria form the steady states of the game.

Suppose that a normal form game is played repeatedly and in a myopic fashion. At each stage of the game, a set of players are chosen to make decisions (i.e. adapt their actions) according to a decision timing rule. The two main classes of decision timing rules are asynchronous, where no two users make a decision at the same time instant, and synchronous, where all users make a decision at the same time instant. Some examples of asynchronous decision rules are *round robin* where at each step all users sequentially update, *random update* where at each step a randomly chosen user updates and *random set update* where at each step a randomly chosen subset of users simultaneously update (detailed definitions can be found in [11]). Note that it is assumed that the impact of the action updates at any instant can be observed at the subsequent time instant. The decision making players choose actions that improve their utility functions. The criteria for a particular choice of action or the *decision making rule* give rise to the best and better response dynamics defined below:

1) *Best response dynamic:* At each stage, a player k deviates from $a_k \in A_k$ to an action $b_k \in A_k$ if $u_k(b_k, a_{-k}) \geq u_k(c_k, a_{-k}), \ \forall c_k \in A_k$. Note that a NE is an action profile, $a \in A$, such that a_k is a best response for every player $k \in \mathcal{K}$.

2) *Better response dynamic:* At each stage, a player k deviates from $a_k \in A_k$ if there exists an action $b_k \in A_k$ such that $u_k(b_k, a_{-k}) > u_k(a_k)$.

A normal form game together with a decision timing rule and a decision making rule can be used to construct an adaptation algorithm.

A potential game ([11] and [18]) is a normal form game such that any changes in the utility function of a player due to a unilateral deviation by the player are reflected in a global function referred to as the potential function. The existence of a potential function thus captures the effect of the actions of all individual users and gives us a network game whose convergence and fixed points are easy to analyze. In addition, if the potential function is also a global network performance measure, these games give a framework where users can serve the greater good by following their own best interest.

Based on the relationship between the potential function and the utility functions of the players in the game, potential games can be grouped into several classes. We focus on *exact potential games* (EPGs) [11], defined below, for the WA framework developed in this paper.

Definition 1: A normal form game is an *EPG* if there exists a function $V : A \to \mathbb{R}$, known as the *exact potential function*, that satisfies $u_k(a) - u_k(\hat{a}_k, a_{-k}) = V(a) - V(\hat{a}_k, a_{-k})$, $\forall k \in \mathcal{K}, a \in A$ and $\hat{a}_k \in A_k$.

The NE of a potential game include maximizers of the potential function. A best response dynamic with an asynchronous decision rule will converge to a NE of the game in EPGs with continuous utility functions and compact action spaces. A better response dynamic with an asynchronous decision rule will also converge in these games. However, in the latter scenario, they might not necessarily converge to a NE of the game. Additional properties such as a better response with a finite minimum step size or a random better response can however be used to establish the convergence of these games to the NE as shown in [4].

IV. POTENTIAL GAME FORMULATION FOR WAVEFORM ADAPTATION

In this section, we cast the WA problem in ad hoc networks as an EPG. The node-pairs in the network are the players of the game ($\mathcal{K} = \{1, \dots, K\}$). The transmit waveforms available to the transmit nodes ($A_k = S, \ \forall k \in \mathcal{K}$) are the action sets. We now seek to to design a utility function for the nodes that leads to a potential function which is also desirable from a network perspective.

A. WA as a Potential Game

The SINR at a receive-node is a good indicator of the throughput and performance of the particular user node-pair. Hence, the sum of Inverse SINRs (SISINR) of users (or in other words, weighted sum-interference-and-noise, wherein the interference at each user's receive-node is divided by the power received from its transmitter) in a distributed network is a possible measure of network performance.

The interference and noise seen at the kth receiver in a distributed network with K user node-pairs is given by

$$i_k = \sum_{j=1, j \neq k}^{K} \sqrt{p_j} g_{jk} s_j b_j + z. \tag{2}$$

The inverse SINR at the kth receiver, assuming a matched filter, is given by

$$I_k(s_k, s_{-k}) = \frac{s_k^H \left(\sum_{j=1, j \neq k}^{K} s_j s_j^H p_j g_{jk}^2 + R_{zz} \right) s_k}{p_k g_{kk}^2} \tag{3}$$
$$= \frac{s_k^H R_{ii,k} s_k}{p_k g_{kk}^2}.$$

Here, $R_{ii,k}$ is the interference-plus-noise-crosscorrelation matrix given by $R_{ii,k} = E\left[i_k i_k^T\right]$ and $R_{zz} = E\left[z z^T\right]$ is the noise covariance matrix. If the noise process is white, R_{zz} is

a scalar multiple of the identity matrix. The SISINR of the network is given by

$$I_{sum}(s) = \sum_{k=1}^{K} \frac{s_k^H R_{ii,k} s_k}{p_k g_{kk}^2}. \qquad (4)$$

To allow a WA update by each user in the network to reduce the above function (the weighted sum-interference-and-noise in the network), the negative of the SISINR function is taken to be the potential function of the game. The terms of the potential function, involving the kth user can be separated to yield,

$$V(s) = -I_{sum}(s) = -\frac{s_k^H \left(\sum\limits_{j=1,j\neq k}^{K} s_j s_j^H p_j g_{jk}^2 + R_{zz} \right) s_k}{p_k g_{kk}^2}$$

$$-\sum_{j=1,j\neq k}^{K} \frac{s_j^H s_k s_k^H s_j p_k g_{kj}^2}{p_j g_{jj}^2}$$

$$-\underbrace{\sum_{j=1,j\neq k}^{K} \frac{s_j^H \left(\sum\limits_{l=1,l\neq k,j}^{K} s_l s_l^H p_l g_{lj}^2 + R_{zz} \right) s_k}{p_j g_{jj}^2}}_{\text{Non-contributing terms}}.$$

$$(5)$$

The effect of the actions of the kth user is only perceived in the first two terms. Hence a simple formulation of a utility function for the kth user, such that the negative of the SISINR function is an exact potential function of the game, is given by

$$u_k(s_k, s_{-k}) = -s_k^T X_k s_k, \quad \text{where}$$

$$X_k = \frac{R_{ii,k}}{p_k g_{kk}^2} + \sum_{j\neq k,j=1}^{K} \frac{s_j s_j^T p_k g_{kj}^2}{p_j g_{jj}^2}. \qquad (6)$$

Note that for a unilateral deviation by the kth user, from signature sequence s_k to sequence $\hat{s}_k$, $u_k(s) - u_k(\hat{s}_k, s_{-k}) = V(s) - V(\hat{s}_k, s_{-k})$. It can also be seen that the utility for a user is made of two terms: the inverse-SINR of the user at its receive node and the interference caused by the user to all the other users in the network. A user thus benefits by reducing the interference caused to the other users in the network in addition to reducing the interference at its own receiver. In this way, as opposed to greedy IA games, each user's utility function incorporates a measure of the influence of its actions on the other users in the system.

B. Nash Equilibria of the Game

The utility function for the kth user (Equation (6)) can be re-written as follows since the sequences are normalized:

$$u_k(s_k, s_{-k}) = -\frac{s_k^T X_k s_k}{s_k^T s_k}. \qquad (7)$$

Matrix X_k can be observed to be a symmetric matrix since it consists of terms that are the weighted cross-correlations of the transmit sequences of users and which are hence symmetric. It is also positive definite since the diagonal terms are positive

and also greater than zero due to the inclusion of the non-zero noise power terms. Hence the utility function can be identified to be a negative weighted Rayleigh quotient of X_k. This is maximized by the eigenvector corresponding to the minimum eigenvalue of X_k [19]. The best response of the user to the current state of the network is, therefore, given by the minimum eigenvector of X_k. At the NE, by definition, each user's current action is equal to the best response of the user to its utility function (in other words the NE is equivalent to the fixed points of a best-response algorithm). The NE of the game can thus be characterized by:

$$X_k s_k = a_{min,k} s_k, \quad k \in \mathcal{K}, \qquad (8)$$

where $a_{min,k}$ is the minimum eigenvalue of matrix X_k.

As mentioned before, the NE of a potential game include the maximizers of the potential function. Since the potential function given by Equation (5) is continuous and bounded, the potential function is guaranteed to have at least one maximum (Weierstrass theorem [20]) and hence at least one NE. The following theorem characterizes the global maximizers of the potential function for a subset of possible network scenarios.

Theorem 1: In an under-loaded (number of sequences is less than the number of dimensions, i.e., $K < N$) and equally-loaded (number of sequences is equal to the number of dimensions, i.e., $K = N$) network scenario with a white noise process ($R_{zz} = \sigma^2 I_{N \times N}$), the potential function $V(s)$ (5) is maximized by a set of orthogonal sequences.

Proof: Let s^O be an orthogonal sequence set with $K \leq N$ sequences (i.e. $s_i^T s_j = 0, \forall i, j \in \mathcal{K}$ and $i \neq j$). Then, the value of the potential function in a network with a white noise process is given by

$$V(s^O) = -\sum_{k=1}^{k} \frac{\sigma^2}{p_k g_{kk}^2}. \qquad (9)$$

Now consider a sequence set s that is not orthogonal. There exists at least two sequences s_i and s_j in the sequence set such that $s_i^T s_j = a \neq 0$. Therefore the value of the potential function is given by

$$V(s) \leq -\sum_{k=1,k\neq i,k\neq j}^{k} \frac{\sigma^2}{p_k g_{kk}^2} - \frac{a^2 p_j g_{ji}^2 + \sigma^2}{p_i g_{ii}^2} - \frac{a^2 p_i g_{ij}^2 + \sigma^2}{p_j g_{jj}^2}$$

$$< -\sum_{k=1}^{k} \frac{\sigma^2}{p_k g_{kk}^2} = V(s^O). \qquad (10)$$

This shows that the set of orthogonal sequences maximize the potential function for the given network scenario. As an example, if the signal dimensions denote different frequency bands, the best allocation when $K < N$, is to choose orthogonal frequencies. Note that since the potential function is given by the negative of the weighted sum interference in the network, the global maximizer of the potential maximizer corresponds to a desirable solution for the network. ∎

V. BEST RESPONSE ALGORITHM

As mentioned before, an algorithm can be formulated for WA by using a decision timing rule to allow users to update their waveforms according to a decision making rule

with respect to the utility function designed in the previous section. In this section, we design an algorithm where users update their transmit waveforms according to the best response dynamic using an asynchronous timing rule.

A. Algorithm Description

The best response of a user to the current network state with respect to the the utility function (6) is the minimum eigenvector of X_k. The WA algorithm for IA can therefore be formally written as follows:

Best-response-based SISINR WA Algorithm

1) Fix the transmit-power levels and initialize codeword s_k for each user.
2) For each $k \in \mathcal{K}$,
    a) Let a_k be the minimum eigenvector of X_k. If $a_k \neq s_k$, replace s_k by a_k.
3) Repeat step 2 until a fixed point or some termination criteria is reached

B. Convergence and Fixed Points

It can be seen from the analysis in Section IV-A that each user update increases the value of the potential function and hence iteratively decreases the weighted sum interference in the network. As mentioned before, EPGs exhibit best response convergence to the NE of the game and the proposed EPG has at least one NE. Consequently, at least one fixed point exists for the proposed algorithm and the fixed points of the SISINR WA algorithm are characterized by Equation (8). Also, as mentioned before, the NE of a potential game include the maximizers of the potential function. Therefore, the proposed algorithm could lead to solutions that minimize the weighted interference in the network and hence are desirable from the network perspective.

The following theorem shows that for a subset of network scenarios (under-loaded and over-loaded), the proposed algorithm leads to the optimal network solution (or the global maximizer of the potential function). We do not have specific theoretical results for other network scenarios (including over-loaded networks). The performance of the algorithm in these scenarios will be analyzed via simulations.

Theorem 2: In an under-loaded ($K < N$) and equally-loaded ($K = N$) network scenario with a white noise process ($R_{zz} = \sigma^2 I_{N \times N}$), the fixed point of the best-response-based SISINR WA algorithm correspond to sets of orthogonal sequences for the users in the network, which are global maximizers of the potential function and optimal solutions for the network.

Proof: Let s^* be a fixed point of the algorithm (equivalent to the NE of the game under a best response as is the case here). Then by (8), $X_k s_k^* = a_{min,k} s_k^*$, $k \in \mathcal{K}$. Here, $a_{min,k}$ is the minimum eigenvalue of matrix X_k. Let $\hat{X}_k = X_k - \frac{R_{zz}}{p_k g_{kk}^2}$. Then, when the noise is white, the fixed point can be characterized by $\hat{X}_k s_k^* = \hat{a}_{min,k} s_k^*$, $k \in \mathcal{K}$. Here, $\hat{a}_{min,k}$ is the minimum eigenvalue of matrix $\hat{X}_k$. If $K \leq N$ and if the sequence set is orthogonal, $a_{min,k} = 0, \forall k$ since the cross-correlation between any two sequences is zero. We shall prove

by contradiction that if the sequence set is not orthogonal or in other words, if $a_{min,k} \neq 0$, $\forall k$, the sequence set is not a fixed point.

Since the solution sequence set is not orthogonal, there exists a user $k \in \mathcal{K}$ for whom $X_k s_k^* = a s_k^*$, where a is a positive number that is not zero. Consider this user k and the matrix $X = \hat{X}_k + s_k^* s_k^{*T}$. Since the K sequences are not orthogonal and $K \leq N$, the matrix has less than N linearly independent rows or columns. Therefore, the matrix has at least one eigenvalue that is zero [19]. Also since the matrix is symmetric with positive diagonal values, all the eigenvalues of the matrix are non-negative. Therefore zero is the smallest eigenvalue of the matrix X. It follows that zero is also the smallest eigenvalue of matrix $\hat{X}_k$. This is due to the fact that $\hat{X}_k$ is a symmetric matrix with positive diagonal elements and hence has non-negative eigenvalues. In addition, since $\hat{X}_k = X_k - s_k^* s_k^{*T}$, the eigenvalues of $\hat{X}_k$ are lesser than or equal to the eigenvalues of X_k.

Since $a > 0$, user k can switch to the eigenvector of $\hat{X}_k$ that corresponds to the eigenvalue of zero. However, s^* is then not a fixed point of the algorithm which contradicts our initial assumption. ∎

Note that, in the proposed algorithm, we use a round-robin decision update scheme. However, the algorithm can converge with any asynchronous decision update rule. This allows easier implementations of the proposed algorithm in practical networks.

C. Simulation-based Performance Evaluation

A distributed network is simulated by placing K transmit and receive nodes uniformly in a circular region with radius R ($R = 5$m in the simulations). The power at a receive node from a transmit-node at a distance of rm from the transmit node is assumed to be given by $\frac{p_k}{r^\alpha}$, where p_k is assumed to be the power received from a transmit node at a distance of 1m and α is the path-loss exponent ($\alpha = 3$ in the simulations). All user-nodes are assumed to transmit at the same power-level of 100mW. The path loss at a distance of 1m is assumed to be 40dB (therefore $p_k = -50$dBw). The received signals are assumed to be corrupted by additive white Gaussian noise with -70dBw power per transmission bandwidth. Note that one iteration of the algorithm in the simulation results corresponds to one waveform adaptation by a single user-node unless indicated otherwise.

Figure 2 and Figure 3 illustrate the convergence of the SISINR algorithm in different over-loaded network scenarios. (Note that the figure also includes curves for a greedy game. These curves will be explained in Section V-D.) It can also be seen from the latter figure that in an over-loaded scenario, multiple fixed points can exist for the WA algorithm. This is as opposed to equally or under-loaded network scenarios where, as shown in Theorem 2, the algorithm always converges to an orthogonal sequence configuration (simulation results are seen to corroborate Theorem 2 but are not included here for the sake of brevity). Hence, to evaluate the quality of the multiple fixed points in the over-loaded scenario, we compare them with solutions numerically obtained by a Lagrangian global search algorithm described below.
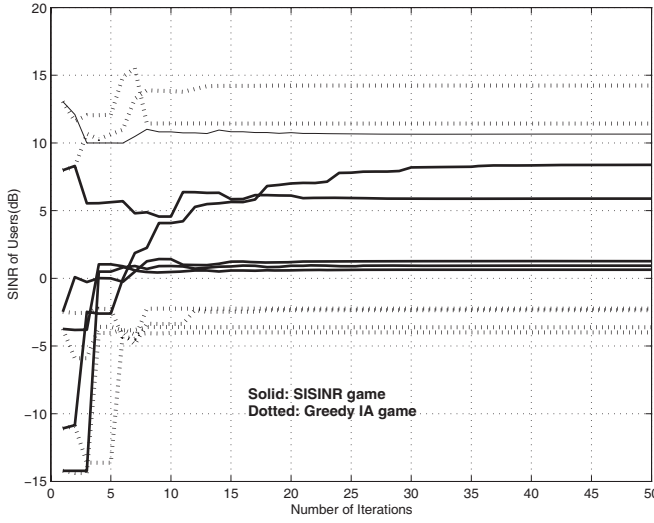
Fig. 2. Comparison of the SINRs of users for the SISINR WA algorithm and the greedy IA algorithm. The network has 6 users sharing 3 signal dimensions.
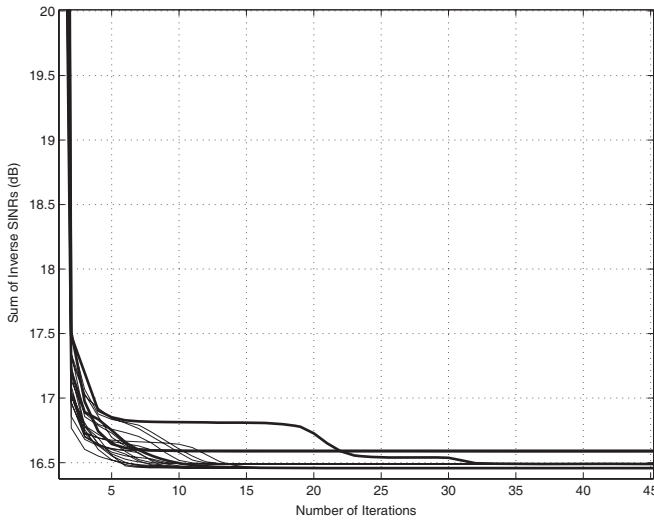

Fig. 3. Weighted sum-interference-plus-noise function for a SISINR WA algorithm with 12 user node-pairs sharing 6 dimensions. Plots show convergence from different random initial choice of waveforms to different fixed points.

The optimization problem to find a sequence set that minimizes the sum interference function (4) with constraints on the power of the sequences can be written as follows:

$$\text{P1:} \quad \min_s \sum_{k=1}^{K} \frac{s_k^H R_{ii,k} s_k}{p_k g_{kk}^2} \quad \text{subject to:} \quad s_k^T s_k = 1, \quad \forall k \quad (11)$$

This can reformulated as the following Lagrangian function

$$f_L = \sum_{k=1}^{K} \frac{s_k^H R_{ii,k} s_k}{p_k g_{kk}^2} + \sum_{k=1}^{K} \lambda_k \left(s_K^T s_k - 1\right)^2. \quad (12)$$

Here, $\lambda_k, \quad k \in \mathcal{K}$ are the Lagrangian multipliers. A gradient search algorithm similar to that in [21] can be used to find the stationary point of the Lagrangian function. The derivative of f_L with respect to λ_k and of f_L with respect to s_k for $k \in \mathcal{K}$ are
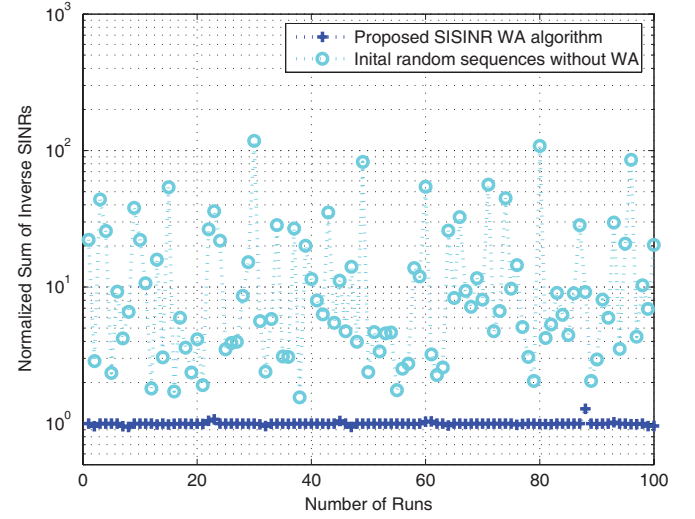

Fig. 4. Plot of the values of the Weighted sum-interference-plus-noise function at the end of the proposed SISINR WA algorithm divided by the values of the function that result from the Lagrangian search algorithm over 100 realizations of an overloaded-network with 5 users sharing 4 signal dimensions. Also plotted are the normalized values of the function for the random sequences used to initialize the algorithms (normalized by dividing the values by the values that result at the end of the Lagrangian search algorithm).

$$\frac{df_l}{d\lambda_k} = \left(s_k^T s_k - 1\right)^2 \text{ and}$$

$$\frac{df_L}{ds_k} = \frac{2R_{ii,k} s_k}{p_k g_{kk}^2} + \sum_{j=1, j \neq k}^{K} \frac{2g_{kj}^2 p_k s_j s_j^t s_k}{p_j g_{jj}^2} + 4\lambda_k \left(s_k^T s_k - 1\right) s_k \quad (13)$$

respectively. At each iteration of the algorithm, s_k and λ_k for $k \in \mathcal{K}$ are updated as follows:

$$s_k \leftarrow s_k - \mu \frac{df_L}{ds_k} \quad \text{and} \quad \lambda_k \leftarrow \lambda_k + \mu \frac{df_L}{d\lambda_k} \quad (14)$$

Here, μ is a step size for the updates. Figure 4 plots the normalized value of the sum interference function (4) for independent instantiations of the network and initial sequences. (Note that for the Lagrangian search algorithm only solutions that satisfy the constraints are retained.) It is seen that the proposed algorithm and the Lagrangian search algorithm significantly reduce the interference in the network. In addition, the performance of the proposed algorithm is very similar to that obtained by the Lagrangian algorithm which searches across the global sequence space (the solutions are equivalent in 95% of the runs) indicating that the solutions obtained from the proposed algorithm are nearly optimal.

The WA algorithm involves considerable network overhead. Hence, it is important to evaluate the gains provided by the algorithm over a simple random-access scheme which does not require any network overhead. This is done by setting up the following network simulation: We consider a network with K users and N dimensions. Transmission time is divided into time-slots of duration T. The probability that a user has a packet to transmit in a transmission slot (of duration T) is denoted by λ. In the random-access-scheme (similar to a multi-band ALOHA scheme discussed in the literature [22]), a user that has a packet to transmit, randomly chooses
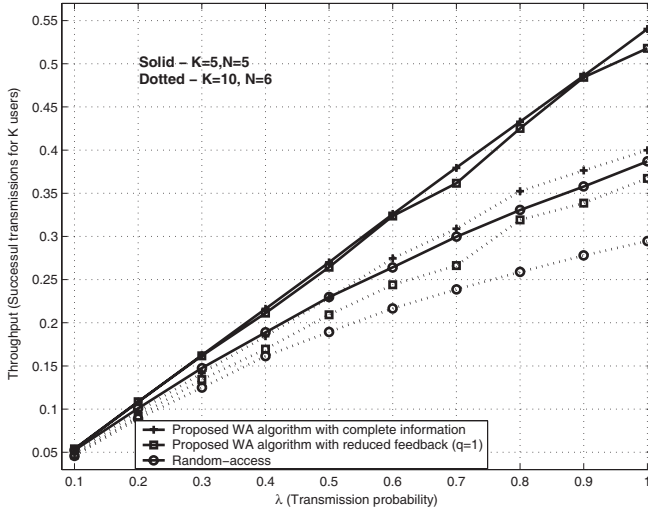
Fig. 5.   Throughput comparison with random-access in a distributed network. The plot is generated by averaging over multiple instantiations of a network.
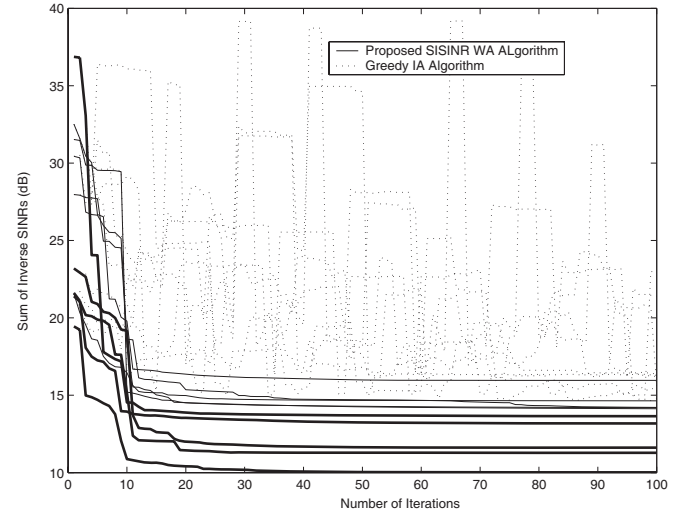

Fig. 6.   Comparison of the weighted sum-interference-plus-noise function for the SISINR WA algorithm and the greedy IA algorithm in multiple arbitrary networks. The network has 12 users sharing 6 signal dimensions.

a dimension from the N available transmission dimensions. (For e.g. if the transmission dimensions are frequency bands, a band is randomly chosen from the N available frequency bands.) Nodes that transmit on the same dimension interfere with each other. In the WA scheme, a user that has a packet to transmit, transmits using its current signature sequence. The signature sequences of all K users are adapted continually for the first $K * 10$ slots (i.e. the WA algorithm is run for 10 round-robin iterations). A transmission by user is assumed to be unsuccessful if the SINR at its receiver is less than 0dB. (Note that this is slightly different from ALOHA models usually used in the literature where a collision or unsuccessful transmission is assumed if multiple users transmit in the same dimension.) Retransmissions of unsuccessful packet transmissions are ignored. Throughput results are obtained over 1000 time-slots. Results are then averaged over multiple network instantiations. Figure 5 plots the results for an equally-loaded network (with $K = 5$ and $N = 5$) and an over-loaded network (with $K = 10$ and $N = 6$). It is seen that for small-transmission probabilities (small values of λ), the performance of both schemes are similar. (Note that the plots also include a reduced-feedback WA scheme. This scheme will be discussed in the next section.) This is due to the fact that for lower transmission probabilities, the probability of finding an unused dimension is larger while using the random-access scheme. However, when the transmission probability increases, the WA scheme, especially in the equally-loaded scenario, offers considerable gains over the random-access scheme. Hence WA schemes are useful in high-traffic networks.

D. Comparison with Greedy WA Game

In this sub-section, we compare the performance of the proposed SISINR WA algorithm with the performance of a greedy WA algorithm (similar to algorithms discussed in [7]). In the greedy IA algorithm, the utility function of each user is given by the negative of the inverse SINR at its receiver, i.e., $u_k^g(s_k, s_{-k}) = -s_k^H R_{ii,k} s_k$. The utility function decreases with interference power and does not incorporate the effect of

the user's action on the other users in the network. The best response of the kth user is the minimum eigenvector of $R_{ii,k}$. Note again that the greedy algorithm does not converge in all network scenarios (example scenarios are identified in [8]).

Figure 6 shows the weighted sum-interference-plus-noise function for the proposed algorithm and the greedy algorithm over independent instantiations of the network. It is observed that, in general, the proposed SISINR WA algorithm results in lower interference in the network as compared to the greedy IA algorithm. It can also be seen that the greedy IA algorithm leads to a cyclic allocation of resources in some network scenarios. Also, since the utility function in the proposed algorithm takes into account the effect of the particular user's actions on the other users in the network, the proposed algorithm is seen to lead to fairer allocation of resources than the greedy IA algorithm. This is illustrated in Figure 2 which plots the SINRs of the users in the network for the two adaptation algorithms. It is seen that the SINRs of the users are more closely distributed in the case of the proposed algorithm than the greedy IA algorithm. (Note that a network scenario where the greedy algorithm converges is chosen for this particular simulated example.) Also, when averaged over allocations in different arbitrary network scenarios, the proposed SISINR algorithm results in a Theil's entropy measure (an inequality index where measure 0 indicates equal distribution and higher values indicate indicate more unequal distribution of resources [23]) of 1.3998 while the greedy IA algorithm results in a measure of 2.9691.

E. Comparison with Waterfilling Algorithms

In this subsection, we compare solutions obtained from the proposed SISINR best-response algorithm with those obtained from greedy waterfilling algorithms discussed in [7] and [24]. As in [25] and [24], we consider a representative system with two transmitters and two receivers ($K = 2$). Also similar to the system considered in [7], to enable each transmit-receive pair to waterfill the available signal dimensions (N), N sequences are associated with each transmit-receive pair. (Note that the

system is now always an overloaded system since the total number of sequences in the system, $2N$, is greater than N.) Let the sequences associated with transmitter-k ($k \in \{1, 2\}$) be denoted by $s_m^k, m = 1, \ldots, N$. Let the interference-plus-noise cross-correlation matrix for sequence-m of user-k be denoted by $R_{ii,m}^k$. The matrix can be specified as

$$R_{ii,m}^k = \sum_{j=1, j \neq m}^{N} s_j^k s_j^{kT} p_k g_{kk}^2 + \sum_{j=1}^{N} s_j^l s_j^{lT} p_l g_{lk}^2. \quad (15)$$

Here, $l \in \{1, 2\}$ such that $l \neq k$. The greedy waterfilling algorithm [7] is an extension of the greedy WA algorithm discussed in Section V-D and is as follows:

Best-response-based SISINR WA Algorithm

1) Initialize codewords for each user.
2) For each codeword of user-1, $s_m^1, m = 1, \ldots, N$, replace s_m^1 with minimum eigenvector of $R_{ii,m}^1$
3) Repeat step 2 until a fixed point or some termination criteria is reached
4) For each codeword of user-2, $s_m^2, m = 1, \ldots, N$, replace s_m^2 with minimum eigenvector of $R_{ii,m}^2$
5) Repeat step 4 until a fixed point or some termination criteria is reached
6) Repeat steps 2-5 until a fixed point or some termination criteria is reached

In the algorithm, greedy IA (steps 2-3 for user-1 and steps 4-5 for user-2) is sequentially applied at each user considering the other user's interference as Gaussian noise. Greedy IA at a user's receiver while the interference from the other user is fixed, is guaranteed to converge to a solution that waterfills over the spectrum of its interference-plus-noise covariance matrix [7] [26].

Note that in the greedy waterfilling algorithm discussed above, each sequence only adapts to the interference at its receiver. Our proposed algorithm, on the other hand, also adapts to the interference caused by it to other waveforms (sequences) in the system. Hence, in our algorithm in step 2 and 4, s_m^1 and s_m^2 are replaced by the minimum eigenvector of X_m^1 and X_m^2 respectively, where,

$$X_m^k = \frac{R_{ii,m}^k}{p_k g_{kk}^2} + \frac{\sum_{j=1, j \neq m}^{N} s_j^k s_j^{kT} p_k g_{kk}^2}{p_k g_{kk}^2} + \frac{\sum_{j=1}^{N} s_j^l s_j^{lT} p_k g_{kl}^2}{p_l g_{ll}^2}$$

$$= 2 \frac{\sum_{j=1, j \neq m}^{N} s_j^k s_j^{kT} p_k g_{kk}^2}{p_k g_{kk}^2} + \sum_{j=1}^{N} s_j^l s_j^{lT} \left(\frac{p_k g_{kl}^2}{p_l g_{ll}^2} + \frac{p_l g_{lk}^2}{p_k g_{kk}^2} \right). \quad (16)$$

Here, $k \in \{1, 2\}$ and $l \in \{1, 2\}$ such that $l \neq k$. For ease of analysis, we consider the case where, $p_1 = p_2$ and $g_{11} = g_{22}$ (transmitters have equal power and are equi-distant from their respective receivers). It can be seen that our proposed algorithm would be similar to performing greedy waterfilling in a system where the gain from transmitter-1 to receiver-2 and the gain from transmitter-2 to receiver-1 are both given by

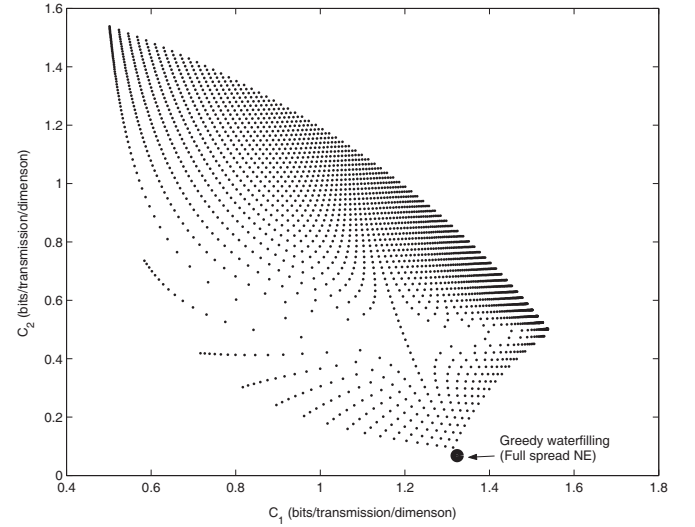$$g_{12}^* = g_{21}^* = \sqrt{\frac{g_{12}^2 + g_{21}^2}{2}} \quad (17)$$

Fig. 7. Comparison of feasible rate regions of the proposed SISNR algorithm and the greedy waterfilling algorithm. The feasible rate region of the greedy waterfilling algorithm is restricted to the FS NE point on the figure. The remaining points (including the FS NE) correspond to the feasible rate region of our proposed algorithm. $g_{11} = g_{22} = 1$, $g_{12} = \sqrt{10}$, $g_{21} = \sqrt{0.09}$, $P_1 = P_2 = 10$ and $N_0 = 0.01$.

Simultaneous waterfilling regions as a function of channel gains (g_{12} and g_{21}) are identified in [25]. By substituting g_{12}^* and g_{21}^* for g_{12} and g_{21} respectively, the framework in [25] can be used to find the rate regions for our proposed algorithm. These resultant rate regions are discussed below.

Scenario-1: $g_{12} = g_{21}$

In this scenario, $g_{12}^* = g_{21}^* = g_{12} = g_{21}$ and our proposed algorithm performs similar to the greedy waterfilling algorithm.

Scenario-2: $g_{21} \neq g_{21}$

We first consider the scenario where $g_{12}^2 g_{21}^2 < 1$. In this scenario, it is shown in [25] that only a unique NE, where both users spread over all available dimensions, exists for the simultaneous waterfilling game. This referred to as the full-spread (FS) NE [24]. It is shown that the FS NE is not desirable in general since separating users in different solutions leads to more efficient solutions (with respect to the sum of the rates achievable by users in the network). With our proposed algorithm, the equivalent channel gains are given by Equation (17). Hence in a large number of network scenarios, where the greedy algorithm is restricted to the inefficient FS NE, our algorithm results in a larger and more desirable rate region. This is illustrated in Figure 7 which shows the rate region where $g_{12}^2 = 10$ and $g_{21}^2 = 0.09$ (resulting in $g_{12}^2 g_{21}^2 = 0.9 < 1$).

In other network scenarios, it is that in general the rate region achievable with the proposed algorithm is larger and more balanced (potentially leading to a fairer allocation of resources) than that achievable by the greedy waterfilling algorithm (similar results are also indicated in Section V-D). This is illustrated in Figure 8 which shows the rate region where $g_{12}^2 = 100$ and $g_{21}^2 = 1$.

In theorem-6 of [24], it is shown that the rate region achievable by non-cooperating (self-interested) users is given by rate vectors that are component-wise larger or equal to
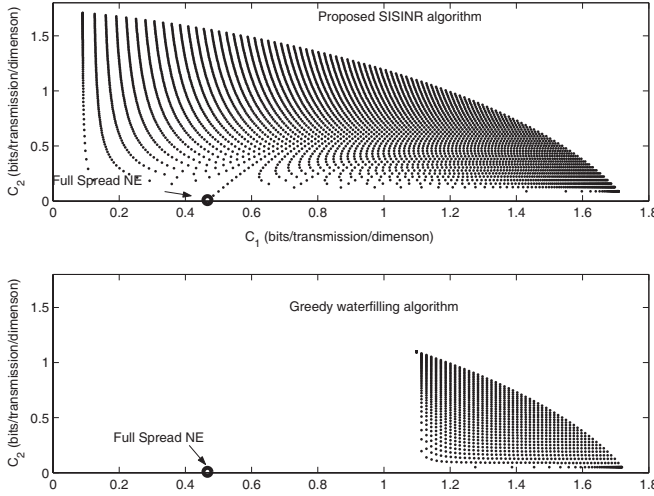
Fig. 8. Comparison of feasible rate regions of the proposed SISNR algorithm and the greedy waterfilling algorithm. $g_{11} = g_{22} = 1$, $g_{12} = 10$, $g_{21} = 1$, $P_1 = P_2 = 10$ and $N_0 = 0.01$.

the rate vector at the FS NE (and might not include the proportionally-fair rate vector). In Figures 7 and 8 this rate region lies to the right of the indicated FS NE. It can be seen from the figures that the proposed algorithm results in a comparatively larger feasible rate-region (and in most scenarios is seen to include the proportionally-fair rate vector).

It can be concluded from the above discussion that since, in our proposed algorithm, users incorporate a measure of their effect on the performance of other users in the network, a larger and more fairer rate region than that of greedy IA algorithms is feasible.

F. Implementation of Algorithm

The proposed SISINR best-response algorithm can be implemented at a centralized controller for the network which has access to information about all the transmit-receive node-pairs in the network. The proposed algorithm is preferable compared to a global search algorithm since it is much simpler to implement (for example the Lagrangian-search algorithm discussed in the previous section requires many more iterations than the proposed algorithm and might not always converge to solutions that satisfy the constraints) and, as shown in the previous section, leads to similar network solutions. Further, the proposed iterative algorithm can easily incorporate variations in channel conditions or network characteristics. A global search will require the entire procedure to be repeated for each variation. Note that the implementation of both algorithms at the centralized controller requires the channel information of all links in the network.

Alternatively, in the absence of a centralized controller (as is common in distributed ad hoc type networks), the algorithm can be implemented at each node-pair in the network. However, this would require any node making adaptation decisions to have access to the signature sequences of all the transmit-nodes in the network and the received power levels at all the receive-nodes from each transmit-node. This can be accomplished by requiring each transmit node to broadcast its sequence and transmit power level and each receive-node to

broadcast the channel coefficients for all links at the beginning of the adaptation process and requiring the adapting node to broadcast its new signature sequence after each adaptation. However, this process considerably increase the overhead of the network especially since the signature sequences can have any real-value. This motivates the investigation of reduced feedback schemes for the SISINR game in the next section.

VI. ALGORITHMS WITH REDUCED FEEDBACK

In this section, properties of potential games, especially the better response convergence properties, are exploited to design alternate implementations of the WA algorithm that result in reduced feedback in the network.

A. Random better response scheme

In this scheme, a transmit-node adapts to a signature sequence chosen at random and sticks to the adaptation if it decreases the weighted sum-interference in the network. This algorithm is formally stated as follows:

Random Better Response SISINR Waveform Adaptation Algorithm

1) Fix the transmit-power levels and initialize codeword s_k for each user.
2) For each $k \in \mathcal{K}$
   a) Set count $i = 0$.
   b) Choose a random sequence $\hat{s}_k \in S$. Adapt transmit-node k to sequence $\hat{s}_k$.
   c) All receivers in the network that are in transmit-node-k's transmission range (i.e. can hear transmit-node-k), send change in Inverse-SINR (ISINR) due to sequence adaptation by transmit-node k.
   d) If sum change in ISINR is positive, set $s_k = \hat{s}_k$. Else set $i = i+1$. If $i \leq T$ (some positive number), repeat steps 2.b to 2.d.
3) Repeat step 2 until a fixed point is reached or some termination criteria is met.

Convergence and Fixed Points: Since NE are the only fixed points of a random better response [4], the random better response based SISINR WA algorithm theoretically converges to a NE of the game. However, since the signature sequences are real-valued, the action space for the random better response scheme is very large and convergence could be very slow. The slow convergence speed could make this scheme impractical though it involves minimal feedback in the network in each iteration. Convergence speed can be increased by having a directed better response scheme as described in the next subsection.

B. Gradient-based better response scheme

In this scheme each transmit-node adapts according to the interference environment at its corresponding receiver and does not require information of the interference profiles at other receivers. The interference power at the kth receive-node is given by $I_k(s_k, s_{-k})$. The gradient of $I_k(s_k, s_{-k})$

with respect to sequence s_k is given by

$$d_k\left(s_k, s_{-k}\right) = \frac{du_k\left(s_k, s_{-k}\right)}{ds_k}$$
$$= \frac{2\left(s_k^T s_k\right) R_{ii,k} s_k - 2\left(s_k^T R_{ii,k} s_k\right) s_k}{\left(s_k^T s_k\right)^2}. \quad (18)$$

Note that the signature sequence s_k is usually known at the receiver corresponding to user k. Also, the interference-plus-noise cross-correlation matrix, $R_{ii,k}$, can be expressed as $R_{ii,k} = E\left[r_k r_k^H\right] - p_k g_{kk} s_k s_k^H$, assuming bit transmissions from multiple users are uncorrelated. Hence $R_{ii,k}$, and consequently, $d_k\left(s_k, s_{-k}\right)$, can be estimated by computing and averaging the received correlation matrix (assuming no other user adapts during measurement) with no additional overhead in the network.

In this reduced feedback scheme, the kth receive-node finds $q\ (q \in \{1, \ldots, N\})$ dimensions in which vector $d_k\left(s_k, s_{-k}\right)$ has the largest magnitude. The variable q can be used to control the amount of feedback from the receive-node to the transmit-node. The receive-node then finds the step size λ that maximizes $I_k\left(s_k, s_{-k}\right)$ in the direction specified by the q chosen dimensions with the largest magnitude (referred to here as the *ascent direction* and denoted by a_q). For example, if the $d_k\left(s_k, s_{-k}\right)$ is a 4-dimensional vector given by $d_k = \begin{bmatrix} 1 & -3 & 4 & 2 \end{bmatrix}^T$ and $q = 2$, the receive-node finds the optimum step size along the ascent direction $a_q = \begin{bmatrix} 0 & -3 & 4 & 0 \end{bmatrix}^T$. The optimum step-size ϵ_k along a_q is the solution to the following optimization problem and can be solved using a simple line search procedure:

$$\max_{\lambda \neq 0} -\frac{(s_k - \lambda a_q)^T R_{ii,k}(s_k - \lambda a_q)}{(s_k - \lambda a_q)^T (s_k - \lambda a_q)}. \quad (19)$$

The transmit-sequence of the user is then adapted along this direction if interference in the network is decreased. This ensures that each adaptation reduces the potential function (Equation (5)). The proposed algorithm can be formally stated as follows:

Gradient-based Better Response SISINR Waveform Adaptation Algorithm

1) Fix the transmit-power levels and initialize codeword s_k for each user. Also choose a value for variable q .
2) For each $k \in \mathcal{K}$
   a) Set count $i = 0$.
   b) Calculate gradient $d_k\left(s_k, s_{-k}\right)$, corresponding ascent direction a_q and the optimum step-size ϵ_k along a_q at receive-node k.
   c) Feedback a_q and ϵ_k to the transmit-node k.
   d) Adapt transmit-node k to sequence $\hat{s}_k = \frac{s_k + \epsilon_k a_q}{\sqrt{(s_k + \epsilon_k a_q)^H (s_k + \epsilon_k a_q)}}$
   e) All receivers in the network that are in transmit-node-k's transmission range, send change in ISINR at the receive-node due to sequence adaptation by transmit-node k.
   f) If the sum change in ISINR is positive, set $s_k = \hat{s}_k$. Else set $i = i+1$. If $i \leq T$ (some positive number), set $\epsilon_k = 0.5\epsilon_k$ and repeat steps 2.b to 2.f.
3) Repeat step 2 until a fixed point is reached.

Descent-based schemes, similar to the algorithm proposed here, where sequences are adapted along the direction of the negative gradient (gradient-descent) or minimum eigenvector (lagged-IA) of the interference-plus-noise cross correlation matrix have been investigated for centralized networks in [16] and [12]. However, these schemes do not incorporate the effect of a user's adaptation on other users in the network and hence are not guaranteed to converge when directly extended to distributed networks.

The implementation of the proposed algorithm only requires feedback of a $q + 1$-dimensional $(q < N)$ vector from the receive-node corresponding to an adapting transmit-node and negligible feedback from the other nodes in the network. The proposed scheme thus substantially decreases the overhead in the network compared to the best-response iteration based SISINR WA algorithm.

Convergence and Fixed Points: As mentioned before, a potential game with a better response dynamic converges and hence the gradient-based better response SISINR WA algorithm also converges. However, the set of fixed points of the algorithm might be larger than the set of NE. This due to the fact that the gradient (18) is zero for all eigenvectors (not just for the minimum eigenvector) and the fact the gradient of only a part the utility function of a user is taken into consideration for the WA process. Hence the adaptations can get stuck at sub-optimal points. Convergence to the NE can be forced by using a random better response spacer step (wherein each user randomly chooses a sequence that improves its utility function). In the under-loaded or equally-loaded network scenario with white-noise, the optimal configuration is to assign orthogonal signature sequences to each user. Hence, the random better response spacer step can be used whenever a receive-node notices that after the convergence of the WA algorithm, the signature sequence of its corresponding transmit-node is not orthogonal to the interference sub-space at the receive-node. In the over-loaded scenario, the random-better response step can be added at regular pre-determined intervals. Since NE are the only fixed points of a random better response, the gradient-based algorithm with the spacer steps also theoretically converges to a NE.

C. Simulation-based Performance Evaluation

Figure 9 shows the performance of the random better response and the gradient-based better response algorithms (without the random spacer step) in an over-loaded network (with $K = 10$ and $N = 6$). Multiple runs of the better response algorithms from different initial sequences are illustrated in the plot. It is seen that both algorithms substantially reduces the interference in the network. However, as mentioned before, the random better response algorithm converges very slowly. It can be observed that the random better response scheme has not yet converged in 30 round-robin iterations. The gradient-based better response schemes, on the other hand, converge in around 20 round-robin iterations. Note that the gradient-based schemes converge to suboptimal fixed points. However, as mentioned before, a random spacer step can be used to force convergence to the NE.

The figure also illustrates the convergence of the gradient-based algorithm for two different values of q. It is seen
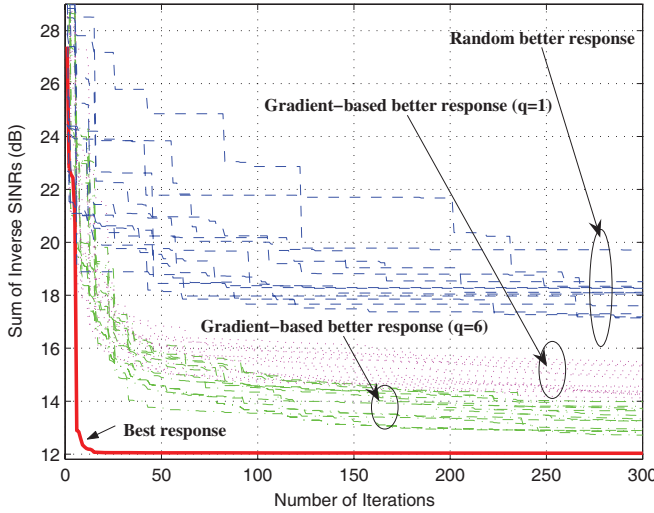
Fig. 9. Convergence comparison of the best-response-based, the gradient-better-response-based and the random-better-response-based SISINR WA algorithms. The network has 10 users sharing 6 signal dimensions. Plot illustrates convergence from different random initial sequences.

that, in general, increasing q increases the rate at which the interference in the network is reduced (number of iterations required for a specific decrease in the interference level). It is also seen that increasing q decreases the amount of interference in the network after convergence. However increasing q also increases the feedback in each iteration. Hence, the optimal q for a given network size can be found based on the requirements of the amount of feedback that can be accommodated in the network, the convergence time for the network and the amount of interference that can be tolerated in the network.

D. Feedback only from a limited number of nodes

An alternate way to reduce the feedback in the network is to reduce the number of user-nodes whose effect is included in the utility function of a user-node. This can be done by taking into consideration the fact that receive-nodes are not much affected by transmissions from far away transmit nodes (or transmissions from nodes whose channel coefficients are very small). The utility function of a user-node can thus be modified as follows:

$$\hat{u}_k(s_k, s_{-k}) = -\frac{s_k^H R_{ii,k} s_k}{p_k g_{kk}^2} - s_k^H \left(\sum_{j \in \mathcal{J}_k} \frac{s_j s_j^H p_k g_{kj}^2}{p_j g_{jj}^2} \right) s_k.$$

$$(20)$$

Here, $R_{ii,k}$ is the interference-plus-noise cross-correlation term that can be measured at the kth user's receive-node and $\mathcal{J}_k$ is the set of receive-nodes which are close to the transmit-node corresponding to the kth user and hence are most affected by transmissions from the kth user. Hence, the utility function of the kth user can be calculated with the help of feedback from only the nodes in set $\mathcal{J}_k$. Note that the weighted sum-interference-plus-noise function does not monotonically decrease, since Equation (5) is no longer an exact potential function for the modified utility function in Equation (5). However, by judicious choice of the set $\mathcal{J}_k$, the fluctuations in the interference function can be made

negligible. Note that this scheme can be used in conjunction with the gradient-based scheme, to further reduce the feedback in the network.

VII. VARIATIONS OF IA ALGORITHM

A. Variation of Weighted Interference Algorithm

The framework can be used to construct other WA algorithms for IA. For instance, a WA algorithm, which reduces the sum interference in the network weighted by the received power of individual users, defined below, can easily be constructed using an EPG framework.

$$V(s) = -\sum_{k=1}^{K} s_k^T \left(\sum_{j=1, j \neq k}^{K} s_j s_j^T p_j g_{jk}^2 p_k g_{kk}^2 \right) s_k. \quad (21)$$

Note that this function is different in spirit to the function given in Equation (5), since here, there is more incentive to provide lower interference to users with larger received powers while in the previous game, stronger users are assumed to be able to tolerate more interference. Therefore, this game might not lead to fair resource allocations but could result in larger sum capacities for the network.

A simple formulation of the utility function for user k, such that function (21) is an EPG is:

$$u_k(s_k, s_{-k}) = -s_k^H \left(\sum_{j \neq k, j=1}^{K} s_j s_j^H p_j g_{jk}^2 p_k g_{kk}^2 \right) s_k$$

$$- s_k^H \left(\sum_{j \neq k, j=1}^{K} s_j s_j^H p_k g_{kj}^2 p_j g_{jj}^2 \right) s_k. \quad (22)$$

In [6], a WA game is described for a multi-cell network where each transmit node communicates with multiple collaborating base-stations/receivers. Each user iteratively finds sequences that minimizes the sum of interferences at all its receivers weighted by the received power of the user at these receivers. This game is similar to the game discussed in this section. Hence the framework can be directly used to construct WA games for multi-cell networks.

B. IA with respect to Legacy System

Consider a legacy static radio system with which the adaptive distributed network co-exists. Then the SISINR WA game can be directly extended to avoid the interference from or to avoid interfering with the legacy radio system. Let $R_{L,k}$ be the weighted covariance matrix of the sum of the signals received from the legacy transmitters at either the legacy receivers (if it is possible to access this information) or the kth receive-node. The utility function of kth user can be modified as follows to incorporate the effect of the legacy transmission:

$$u_k^L(s_k, s_{-k}) = -s_k^H R_{L,k} s_k - \frac{s_k^H R_{ii,k} s_k}{p_k g_{kk}^2}$$

$$- s_k^H \left(\sum_{j \neq k, j=1}^{K} \frac{s_j s_j^H p_k g_{kj}^2}{p_j g_{jj}^2} \right) s_k. \quad (23)$$

The exact potential function corresponding to this modified utility function is given by

$$V_L(s) = -\sum_{k=1}^{K} s_k^H \left(\frac{R_{ii,k}}{p_k g_{kk}^2} + R_{L,k} \right) s_k. \qquad (24)$$

Each user now adapts to a signature sequence that maximizes the utility function given by Equation (23). The existence of a potential function guarantees that each user adaptation monotonically increases the potential function and consequently decreases the interference in the network (which now constitutes the legacy system as well). Since the sum interference in a network is bounded from below, at least one fixed point (formed by a maximizer of the potential function which is also a NE) exists for the game and the algorithm is guaranteed to converges to a NE for the game.

Note that avoiding the interference to a legacy receiver requires information of the interference profile at the legacy receiver unless the channel between the legacy system and the adapting node-pair is symmetric. However, interference from a legacy system to an adaptive distributed network can easily be avoided by using measurements at the receive-nodes corresponding to the adapting transmit-nodes.

C. Extension to Asynchronous Systems

As discussed in Section IV-A, the utility function of a user is made of two terms, the weighted interference at its receive node and the weighted interference caused by the user to the other users in the network. When using the signal space representation for waveforms, interference caused to a user is simply the projection of the received interfering signal on the orthonormal dimensions used by the user-node. Hence the potential game framework and convergence results that we developed hold even if the signals at receivers are asynchronous or if the channels are frequency selective.(Note that the convergence properties of greedy WA algorithms for centralized synchronous DS-CDMA networks likewise hold when directly extended to asynchronous centralized networks as demonstrated in [6] and [27].) We illustrate this by considering the following generalization of our system. Let G_{kj} represent a $N \times N$ matrix that maps the power of the waveform received from the kth user at the jth receiver in a time interval of T seconds averaged over multiple symbols (under the assumption that the information symbols transmitted by users are independent, have zero mean and unit variance). Then, the ISINR of user k is given by

$$I_k(s_k, s_{-k}) = \frac{s_k^H \left(\sum_{j \neq k} G_{jk} s_j s_j^H G_{kj}^H + R_{zz} \right) s_k}{p_k g_{kk}^2} \qquad (25)$$

Note that the above generalized channel matrix G_{kj} allows us to accommodate multipath channel or symbol/chip asynchronicity. For example, consider the symbol-asynchronous (but chip-synchronous) system considered in [27]. Let the d_{kj} represent the relative time delay of user j with respect to the time-delay of user k. Then, matrix G_{kj} for this symbol

asynchronous system is given by

$$G_{kj} = \begin{bmatrix} 0^{1 \times (N-d_{kj})} & 1 & 0^{1 \times (d_{kj}-1)} \\ 0^{1 \times (N-d_{kj}+1)} & 1 & 0^{1 \times (d_{kj}-2)} \\ \vdots & \ddots & \vdots \\ 0^{1 \times (N-1)} & & 1 \\ 1 & & 0^{1 \times (N-1)} \\ 0 & 1 & 0^{1 \times (N-2)} \\ \vdots & \vdots & \ddots & \vdots \\ 0^{1 \times (d_{kj}-1)} & 1 & 0^{1 \times (N-d_{kj})} \end{bmatrix}^H \qquad (26)$$

The utility of the kth user is given by

$$
\begin{aligned}
u_k(s_k, s_{-k}) &= \frac{s_k^H \left(\sum_{j \neq k} G_{jk} s_j s_j^H G_{kj} + R_{zz} \right) s_k}{p_k g_{kk}^2} \\
&+ \frac{s_j^H \left(\sum_{j \neq k} G_{kj} s_k s_k^H G_{kj}^H \right) s_j}{p_j g_{jj}^2} \\
&= s_k^H \left(\frac{\sum_{j \neq k} G_{jk} s_j s_j^H G_{kj} + R_{zz}}{p_k g_{kk}^2} + \sum_{j \neq k} \frac{G_{kj}^H s_j s_j^H G_{kj}}{p_j g_{jj}^2} \right) s_k.
\end{aligned} \qquad (27)
$$

It can be seen that the sum SINR of the network given by the following expression is still a potential function for the game.

$$I_{sum}(s) = \sum_{k=1}^{K} I_k(s_k, s_{-k}) \qquad (28)$$

Therefore, the WA strategies developed in the previous sections can be directly extended to asynchronous systems. The schemes iteratively reduce the sum-interference levels and are guaranteed to converge in asynchronous systems as well. However, note that the characterization of exact solutions of the algorithms in asynchronous systems is not straightforward. For example, in the under-loaded scenario, the solution of the proposed algorithm consists of orthogonal sequences. However, in asynchronous systems, the optimum sequences (and steady state of the network) depend upon the delays in the system.

D. Algorithm for Combined Power and WA

By studying the utility function for the kth user in Equation (6) and the potential function given by Equation (5), it can be observed that the SISINR game is an EPG with respect to the signature sequences s_k, $k \in \mathcal{K}$ as well as the transmit power-levels p_k, $k \in \mathcal{K}$ of the user node-pairs. In this section, motivated by the existence of an EPG, we investigate a joint power and WA algorithm for IA.

In this algorithm, each user adapts both its power and waveform to derive the best utility for the current state of the network. The game can be played as follows: Each user, in an asynchronous fashion, chooses a transmit waveform and power level that maximizes its utility for the current state of the network. The game can be played by assuming a finite number of transmit power levels for the users or by assuming the power level to range between a minimum (PL) and maximum (PU) power level. In the former case, a user finds a waveform

corresponding to each of its power levels which maximizes its utility function for the current state of the network. The user then selects the waveform and power level that corresponds to the maximum utility. In the latter case, the user adapts to a power-level $\hat{p}_k$ which is the power level that maximizes the utility of user-node k the current waveform and the current network state and which is defined as follows:

Let $\overline{p} = \sqrt{\frac{X_1}{X_2}}$, where

$$X_1 = s_1^H \left(\sum_{j \neq i, j=1}^{K} \frac{s_j s_j^H p_j g_{ji}^2}{g_{ii}^2} \right) s_i \tag{29}$$

and

$$X_2 = s_i^H \left(\sum_{j \neq i, j=1}^{K} \frac{s_j s_j^H g_{ij}^2}{p_j g_{jj}^2} \right) s_i \tag{30}$$

Then $\hat{p}_k$ is given by

$$\hat{p}_k = \begin{cases} P_L, & \text{if } \overline{p} < P_L \\ P_U, & \text{if } \overline{p} > P_U \\ \overline{p} & \text{otherwise} \end{cases} \tag{31}$$

The power and WA algorithm can be formally stated as follows:

Best-response-based SISINR Power and Waveform Adaptation Algorithm

1) Initialize the transmit-power level, p_k, and codeword, s_k, for each user.
2) For each $k \in \mathcal{K}$
   a) Replace s_k by the eigenvector corresponding to the minimum eigenvalue of X_k
   b) Replace p_k by $\hat{p}_k$
   c) Repeat steps 2.a and 2.b until a fixed point or a termination criterion is reached.
3) Repeat step 2 until a fixed point is reached.

Convergence: This utility maximization process (steps 1-2) can be viewed as an extension of the standard "cyclic coordinate" optimization procedure [20] and iteratively each adaptation iteratively increases the potential function. It is hence assured to converge. However, the algorithm could converge to points which are not the NE.

Figure 10 illustrates the convergence of the joint power and WA algorithm. The transmit power level of a user node is allowed to vary from 100mW to 500mW. Each iteration represents 10 waveform and power adaptations by a user (steps 2.a and 2.b are repeated 10 times). It is seen that the algorithm substantially decreases the interference in the network. The figure also compares the joint power and WA algorithm with the pure WA algorithm (where the user's transmit power levels were fixed at 100mW). Multiple runs illustrate the convergence of the algorithms from random initial choice of waveforms. It is observed that the combined waveform and power algorithm leads to better solutions (lesser interference in the network) than the pure WA algorithm. This has also been observed through simulations in different network scenarios and different fixed transmit power levels for the users in the pure WA algorithm. However, it is to be noted, that the joint power and WA algorithm takes a larger number of iterations
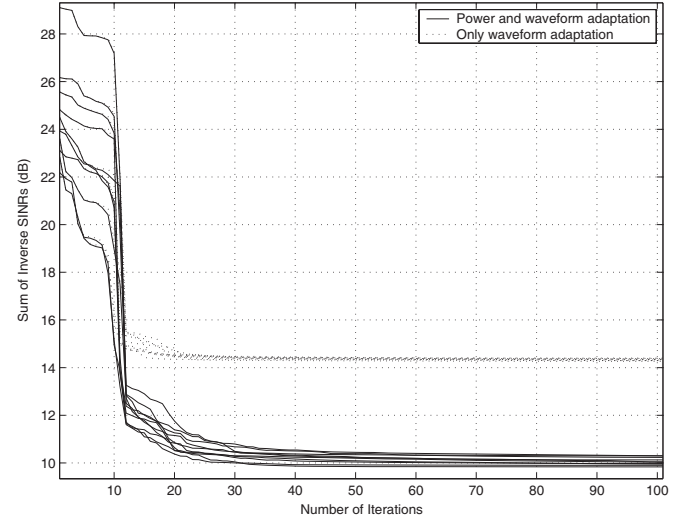

Fig. 10. Convergence of the SISINR power and WA algorithm in a network with 12 users sharing 6 signal dimensions from multiple random initial sequences. $P_U = 500$mW and $P_L = 100$mW. Performance of the algorithm is also compared with that of a SISINR pure WA algorithm.

than the pure WA algorithm to converge.

VIII. CONCLUSIONS AND FUTURE WORK

A waveform adaptation framework based on potential game theory to construct convergent interference avoidance algorithms in networks with multiple distributed receivers (as in ad hoc networks) was developed in this paper. This is motivated by the fact that direct extensions of myopic greedy IA algorithms do not always lead to convergence in these networks. The proposed algorithms lead to solutions that are desirable from a network perspective. Reduced feedback implementations based on properties of potential games were then developed for the IA algorithms. Finally several variations of IA algorithms based on the framework were also designed. These include IA with respect to legacy systems and joint power and WA algorithms for IA. Note that the framework presented in the paper can also be utilized to construct adaptations algorithms with other desirable network performance measures.

An alternate approach to develop distributed adaptation strategies for these networks is to develop games in which users try to achieve a feasible target performance (and not maximize their performance). Target-performance-based WA algorithms for distributed networks are presented in [14]. However, the identification of a feasible target performance regions in ad hoc networks (de-centralized networks) remains an open problem. This is the subject of future work.

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