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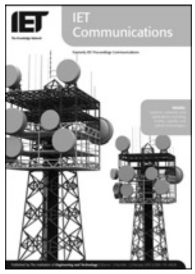

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Game theoretic approach for channel assignment and power control with no-internal-regret learning in wireless *ad hoc* networks

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Abstract: In wireless *ad hoc* networks, co-channel interference can be suppressed effectively through proper integration of channel assignment (CA) and power control (PC) techniques. Unlike centralised cellular networks where CA and PC can be coordinated by base stations, the integration of CA and PC into infrastructureless wireless *ad hoc* networks where no global information is available is more technically challenging. The authors model the CA and PC problems as a non-cooperative game, in which all wireless users jointly pick an optimal channel and power level to minimise a joint cost function. To prove the existence and uniqueness of Nash equilibrium (NE) in the proposed non-cooperative CA and PC game (NCPG), the authors break the NCPG into a CA subgame and a PC subgame. It is shown that if NE exists in these two subgames, the existence of NE in the NCPG is ensured. Nonetheless, due to unpredictable network topology and diverse system conditions in wireless *ad hoc* networks, the NCPG may encounter the 'ping-pong' effect that renders NE unattainable. By incorporating a call-dropping strategy and no-internal-regret learning into the NCPG, an iterative and distributed algorithm that ensures convergence to NE is proposed. It is shown through simulation results that the proposed approach leads to convergence and results in significant improvements in power preservation and system capacity as compared with the popular distributed dynamic CA technique incorporated with PC.

1 Introduction

Driven by the increasing demands for wireless broadband services, future wireless systems are expected to witness a rapid growth in high-data-rate applications with very diverse quality of service (QoS) requirements. However, in interference-limited wireless *ad hoc* networks, channel bandwidth and power are usually scarce. Furthermore, co-channel interference (CCI) [1], which is caused by excessive power usage and improper frequency planning, is detrimental to network system capacity and QoS. Hence, the main challenge in designing wireless *ad hoc* networks is to use network resources efficiently while suppressing CCI.

An effective solution to manage network resources and mitigate CCI is to apply a power control (PC) [1] technique that allows each user to transmit sufficient power to achieve the required QoS without causing excessive interference to other users. Another approach is to perform channel assignment (CA) [2] which ensures that the nodes sharing the same spectrum are widely separated. It is also possible to achieve better system performance by integrating CA and PC. Some combined CA with PC algorithms for cellular systems have been proposed in [2–4] where CA and PC are jointly managed by base stations. Such algorithms rely on cooperation among different nodes with a third-party coordination by the base station, thus making the integration easy to be implemented.

On the contrary, one of the distinctive features of wireless *ad hoc* networks is the lack of a central controller resulting in each node making its own decisions independently. Hence, the principle of altruism does not exist in wireless *ad hoc* networks; instead every user tends to exploit network resources selfishly without contributing to the network [5]. In particular, the usual assumption of spontaneous willingness to cooperate is unrealistic for autonomous users, especially when there are conflicting interests among the users [6]. Without coordination among users, the integration of CA and PC becomes very challenging. Thus, an appropriate approach to characterise the distributed, non-cooperative and self-organising behaviour of *ad hoc* networks is needed to integrate CA and PC effectively. Having taken note of the selfishness of users, it is believed that non-cooperative game theory can serve as a suitable tool to study this problem.

Recently, there has been growing interest in applying game theory to study PC and CA in wireless networks [6–10]. In [7], PC is modelled as a non-cooperative game in which users choose their transmit power to maximise a utility function defined as the ratio of throughput to transmit power. Subsequently, the work in [8] includes pricing in the game to obtain more efficient solutions. In [6], game theory is used to study multi-carrier CDMA (MC-CDMA) systems where the conditions for the existence and uniqueness of Nash equilibrium (NE) are analysed. An excellent overview of PC using game theory can be found in [5]. In addition to the PC game, studies pertinent to the CA game can be found in [9, 10].

Despite the intensive research activities in both the CA and PC, little research has been done to integrate these two techniques together using game theoretic approach. In this paper, by formulating a joint cost function, we combine the CA and PC into a single game termed non-cooperative CA and PC game (NCPG). This combined game differs from previous works in the following aspects: 1) we focus on *ad hoc* networks instead of cellular networks; 2) we consider both CA and PC in a multi-channel network; 3) we formulate a cost function rather than a utility function. In this model, network users selfishly and locally select a channel and power level to minimise a joint cost function at a periodic interval. In other words, the NCPG is played iteratively with the aim of achieving NE where upon reaching this point, every user can transmit at an optimal power level on the best channel while achieving the required QoS. Nevertheless, unpredictable network topology and diverse system conditions may cause the NCPG to enter into a 'ping-pong' (PP) state where NE is unattainable. In an attempt to ensure the existence of NE in the NCPG, we break the NCPG into a CA subgame (CASG) and a PC subgame (PCSG) to investigate the existence of their respective NE. It is shown that if NE exists in these two subgames, the existence of NE in the NCPG is ensured. To guarantee convergence to NE in the NCPG, we apply a call-dropping technique [1] and the no-internal-regret learning algorithm (NIRLA) [10] to obtain another new algorithm.

The rest of this paper is organised as follows. In Section 2, we outline the system model and integrate the CA and PC problems into a game theoretic framework. In this section, we present the proposed NCPG, its cost function and the solution. The conditions along with additional measures to ensure the existence of NE in the NCPG are discussed in Section 3. In Section 4, a distributed algorithm that leads to NE for the NCPG is formulated. Simulation results are presented in Section 5. We end the paper with some concluding remarks in Section 6.

2 System model

We consider $M > 1$ randomly located pairs of stationary nodes forming communication links in a frequency-reused and power-controlled wireless *ad hoc* environment where the transmitted powers are continuously tunable and the communication links need to share transmission among K orthogonal channels. Each communicating pair consists of a dedicated sender and receiver. Without loss of generality, we assume a subset of active sender, $\mathcal{S} = \{s_1, s_2, \ldots, s_N\}$ transmitting packets to another subset of active receivers, $\mathcal{R} = \{r_1, r_2, \ldots, r_N\}$ where $1 \le N \le M$ is the instantaneous number of active pairs. In this work, we focus on the forward link of a frequency division duplex and frequency division multiple access (FDD/FDMA) system, in which the ith sender, s_i transmits to the ith receiver, r_i on one of the channels in the channel set, $\mathcal{K} = \{1, 2, \ldots, K\}$ where $i \in \mathcal{N}$, $\mathcal{N} = \{1, 2, \ldots, N\}$ is the set for all active users. The reverse-link case can also be treated similarly. s_i transmits at power level $p_i \le p_{\max}$ where $p_{\max}$ is the upper-bound imposed by the physical limits of the mobile device. The received power level at r_i for a signal transmitted by s_j is given by $G_{ij}p_j$, where $G_{ij} > 0$ is the path gain from s_j to r_i, which can be expressed as

$$G_{ij} = \frac{S_{ij}}{\left[\left(x_{\mathrm{r},i} - x_{\mathrm{s},j}\right)^2 + \left(y_{\mathrm{r},i} - y_{\mathrm{s},j}\right)^2\right]^{\alpha/2}} \qquad (1)$$

where S_{ij} is the attenuation factor that models the shadowing effect and $10\log_{10} S_{ij} \sim N(0, \sigma_s^2)$, $1 \le i, j \le M$ are independent and identically distributed where $\sigma_s \sim U(4, 10)$ [11]. x and y denote the two-dimensional coordinate positions and the subscripts r and s represent the receiver and sender nodes, respectively. α models the propagation pathloss. Accordingly, the signal to interference-plus-noise ratio (SINR), γ_i^k obtained by r_i on the kth channel where $k \in \mathcal{K}$ is given by

$$\gamma_i^k = \frac{G_{ii}p_i^k}{\sum_{j=1, j\neq i}^{N} G_{ij}p_j^k + \sigma^2} = \frac{G_{ii}p_i^k}{I_i^k\left(\boldsymbol{p}_{-i}^k\right)} \qquad (2)$$

where σ^2 is the power of the additive white Gaussian noise (AWGN) at r_i. $\boldsymbol{p}_{-i}^k$ denotes the power vector comprising of the powers of all users on the kth channel except that of

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the ith user. $I_i^k(\boldsymbol{p}_{-i}^k)$ is the interference plus noise received by the ith user.

To maintain reliable connection between s_i and r_i on the kth channel, the SINR should be above some threshold, $\bar{\gamma}_i$ which corresponds to a QoS requirement such as the bit error rate. For simplicity, we assume identical $\bar{\gamma}_i$ for all users and denote this minimal SINR as $\bar{\gamma}$. Following the above argument, we have the requirement

$$\gamma_i^k \geq \bar{\gamma} \forall\, i \tag{3}$$

From the perspective of a selfish user, it is immaterial whether other users can meet $\bar{\gamma}$. What a selfish user concerns is that it can utilise the channel to transmit above $\bar{\gamma}$ at an optimal power level, disregarding of its impact on other transmissions. For this reason, the framework of non-cooperative game theory is well-suited to analysing and solving the joint PC and CA problem. In the following subsections, we model the CA and PC problem as a non-cooperative game and formulate an appropriate cost function. We then study the conditions to achieve NE in the proposed game.

2.1 Non-cooperative channel assignment and power control game

The CA and PC problems are modelled as a non-cooperative game termed NCPG in which the players are the network users. The strategy to play the game corresponds to channel acquisition and the choice of power level. Instead of utility function, level of satisfaction for each players is quantified in a cost function. The outcome of the game results in NE where all players are satisfied with the cost they need to pay in order to utilise the channel.

Formally, we define the NCPG as follows.

Definition 1: Let NCPG $= [\mathcal{N}, \{P_i^K\}, \{J_i(\cdot)\}]$ denote the proposed non-cooperative game where $\mathcal{N} = \{1, 2, \ldots, N\}$ is the player set, and N is the total number of players in the NCPG (equivalent to the number of active users in the wireless ad hoc network), $P_i^K = [0, p_{\max}]^K$ is the multi-dimensional strategy set for the ith user over K orthogonal channels where $i \in \mathcal{N}$ and $J_i(\cdot)$ are the cost function for the ith user.

Let $P_i = [0, p_{\max}]$ be a strategy set for the ith user on one of the channels. We define the power vector $\boldsymbol{p}^k = [p_1^{k_1}, p_2^{k_2}, \ldots, p_N^{k_N}]$ as the outcome of the game in terms of the selected power levels and the selected channels for all users where $p_i^{k_i} \in P_i$ is the action of the ith user. Each users select its own action $p_i^{k_i}$ to minimise the joint cost function $J_i(\cdot)$. In general, the NCPG can be expressed as

$$\left(\tilde{k}_i, \tilde{p}_i\right) = \arg \min_{k_i \in K, p_i^{k_i} \in P_i} J_i\left(p_i^{k_i}, \boldsymbol{p}_{-i}^{k_i}\right) \forall\, i \in \mathcal{N} \tag{4}$$

where $\tilde{k}_i$ and $\tilde{p}_i$ are the best responses [8] of the ith user for the NCPG while $\boldsymbol{p}_{-i}^{k_i}$ is the power vector containing the powers of all users on the selected channel k_i except that of the ith user.

2.2 Cost function for the NCPG

In a wireless *ad hoc* network with shared transmission medium, the signal of one node may interfere with that of other nodes. Typically, a selfish user with no energy constraint would tend to achieve a high SINR at the receiver irrespective of the fact that it may cause excessive interference to other users [12]. Eventually all nodes will transmit at their maximum power level because power is strictly increasing with the SINR. Owing to the fact that mobile devices are energy-constrained, there is a trade-off between attaining high SINR and maintaining low power utilisation. Therefore the desired cost function should take the SINR and power into account. Formally, the cost function is defined as follows.

Definition 2: For any strategy profile $p_i^{k_i} \in P_i$, a function that assigns a numerical value to the elements of the strategy set $P_i(J_i{:}P_i \to \mathbb{R})$ is a cost function, if for all $a, b \in P_i$, a is preferred over b if and only if $J_i(a) < J_i(b)$.

In the NCPG, CA and PC are interrelated through a joint cost function. We associate with the ith user the cost function $J_i(p_i^{k_i}, \boldsymbol{p}_{-i}^{k_i})$ and extend the single-channel cost function in [13] into a multi-channel one as

$$J_i(p_i^{k_i}, \boldsymbol{p}_{-i}^{k_i}) = b_i(\bar{\gamma} - \gamma_i^{k_i})^2 + c_i p_i^{k_i} \tag{5}$$

By expanding $\gamma_i^{k_i}$, we have

$$J_i\left(p_i^{k_i}, \boldsymbol{p}_{-i}^{k_i}\right) = b_i\left(\bar{\gamma} - \frac{G_{ii} p_i^{k_i}}{\sum_{j=1, i\neq j}^{N} G_{ij} p_j^{k_j} \chi\left(k_i, k_j\right) + \sigma^2}\right)^2 + c_i p_i^{k_i} \tag{6}$$

where the parameter b_i and c_i are non-negative weighting factors while $\chi(k_i, k_j)$ is the interference function [9] such that

$$\chi(k_i, k_j) = \begin{cases} 1 & \text{if } k_i = k_j \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

The cost function in (5) captures the trade-off between SINR and power. By adjusting the weighting factors, b_i and c_i, different levels of emphasis on the SINR and power can be obtained. If $b_i/c_i > 1$, we place more emphasis on the SINR while if $b_i/c_i < 1$, power usage is the main consideration. In order to strictly attain $\bar{\gamma}$, it is necessary to choose $b_i/c_i \gg 1$. In the latter section, we will discuss the selection of b_i and c_i. To better appreciate this multi-channel cost function, we present an example cost function for a two-channel system under fixed interference as shown in Fig. 1. Apparently, channel 1 which needs a lower cost than channel 2 will be

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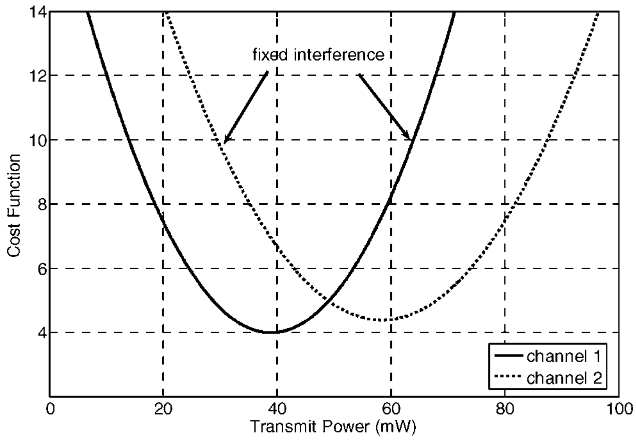

Figure 1 *Cost function against users' transmission power under fixed interference on two different channels*

picked (i.e. the minimal point of the plot for using channel 1 is less than that of using channel 2).

2.3 Nash equilibrium for the NCPG

The channel acquisition and power selection that optimise individual cost function depend on the actions of all other users in the network. Thus, we can characterise a set of powers on their selected channels where the users are satisfied with the cost they pay given the power and channel selection of other users. Such state is known as Nash equilibrium (NE) [8]. Likewise, all users are in NE if each user makes the best decision, taking into account the decisions of others. Here, NE is the optimal criterion for the NCPG in which no user can gain any extra benefits by changing only its own strategy unilaterally [6]. We begin by characterising the cost minimisation of a single user when other users' transmit powers are fixed and show that the cost can be minimised when a user opts to transmit on the channel with the lowest received interference.

Proposition 1: With all other users' transmit powers fixed on their own channel, the ith user chooses its channel and transmits power to minimise (5), i.e. $p_i^{k_i} = (p_i^*)^{k_i^*}$ if and only if $k_i = k_i^*$ where $k_i^* = \arg\min_{k_i \in \mathcal{K}} I_i^{k_i}(\boldsymbol{p}_{-i}^{k_i})$ and $p_i = p_i^*$ where $p_i^* = \arg\min_{p_i \in P_i} J_i(p_i^{k_i^*}, \boldsymbol{p}_{-i}^{k_i^*})$ for the range $I_i^{k_i}(\boldsymbol{p}_{-i}^{k_i}) < c_i G_{ii}\bar{\gamma}/b_i$.

Proof: We show that (5) is minimised when $k_i = k_i^*$ such that $k_i^* = \arg\min_{k_i \in \mathcal{K}} I_i^{k_i}(\boldsymbol{p}_{-i}^{k_i})$. On the basis of (2) and (5), we can rewrite (5) as

$$J_i\left(p_i^{k_i}, \boldsymbol{p}_{-i}^{k_i}\right) = b_i\left(\bar{\gamma} - \frac{G_{ii}p_i^{k_i}}{I_i^{k_i}\left(\boldsymbol{p}_{-i}^{k_i}\right)}\right)^2 + c_i p_i^{k_i} \tag{8}$$

To determine the transmit power on a particular channel which gives the lowest cost, we differentiate (8) with respect to $p_i^{k_i}$, set it to zero and solve it. The power level which minimises (5) is given as

$$p_i^{k_i} = \frac{\bar{\gamma}}{G_{ii}} I_i^{k_i}\left(\boldsymbol{p}_{-i}^{k_i}\right) - \frac{c_i}{2b_i G_{ii}^2}\left(I_i^{k_i}\left(\boldsymbol{p}_{-i}^{k_i}\right)\right)^2 \tag{9}$$

From (9), it can be shown that $p_i^{k_i}$ increases monotonically with $I_i^{k_i}(\boldsymbol{p}_{-i}^{k_i})$ for the range $I_i^{k_i}(\boldsymbol{p}_{-i}^{k_i}) < (b_i G_{ii}\bar{\gamma})/c_i$. Meanwhile, if $I_i^m(\boldsymbol{p}_{-i}^m) < I_i^n(\boldsymbol{p}_{-i}^n)$, for a fixed $\bar{\gamma}$ we have $p_i^m < p_i^n$ where $m, n \in \mathcal{K}$. If we choose $b_i/c_i \gg 1$ to ensure that $\gamma_i^{k_i}$ strictly converges to $\bar{\gamma}$, then the first term of (5) will approach zero and $J_i(p_i^{k_i}, \boldsymbol{p}_{-i}^{k_i})$ only relies on $p_i^{k_i}$. Thus we have $J_i^m(p_i^m, \boldsymbol{p}_{-i}^m) < J_i^n(p_i^n, \boldsymbol{p}_{-i}^n)$ if $p_i^m < p_i^n$ or $I_i^m(\boldsymbol{p}_{-i}^m) < I_i^n(\boldsymbol{p}_{-i}^n)$. To minimise (5), the ith user always picks the channel with the lowest interference. □

If all users select their power levels and channels according to Proposition 1, a power vector, $(\boldsymbol{p}^*)^{\boldsymbol{k}^*} = \left[(p_1^*)^{k_1^*}, (p_2^*)^{k_2^*}, \ldots, (p_N^*)^{k_N^*}\right]$ which characterises NE can be obtained. Next we can define NE for the NCPG in the following [6, 8].

Definition 3: $(\boldsymbol{p}^*)^{\boldsymbol{k}^*}$ is a NE of the NCPG if and only if $J_i((p_i^*)^{k_i^*}(\boldsymbol{p}_{-i}^*)^{\boldsymbol{k}^*}) \le J_i(p_i^{k_i}(\boldsymbol{p}_{-i}^*)^{\boldsymbol{k}^*})$ for all $p_i^{k_i} \in P_i$ and for all $i \in \mathcal{N}$ where $(\boldsymbol{p}_i^*)^{\boldsymbol{k}^*}$ consists of all elements of $(\boldsymbol{p}^*)^{\boldsymbol{k}^*}$ except the ith element.

3 Existence and uniqueness of nash equilibrium

In contrast to cellular networks, unpredictable network topology and diverse system conditions in wireless *ad hoc* network make accurate prediction of CCI very difficult. For this reason, it is more challenging to apply game theory to wireless *ad hoc* networks, as there are multiple possible outcomes of NE that is, no NE, single NE or multiple NEs. In this section, we study the conditions which guarantee the existence and convergence to NE in the NCPG. The following theorem is useful to identify NE in the NCPG.

Theorem 1: If NE exists in both the PCSG and the CASG of the NCPG, NE always exists in the NCPG.

Proof: $(\boldsymbol{p}^*)^{\boldsymbol{k}^*}$ which is NE in the NCPG comprises of NE^{PC}, $\boldsymbol{p}^*$ (NE for the PCSG) and NE^{CA}, $\boldsymbol{k}^*$ (NE for the CASG). Instead of simultaneously selecting channel and power level, Proposition 1 suggests to play the CASG and subsequently the PCSG which also minimise (5). Hence the NE achieved by playing the CASG and PCSG is equivalent to NE obtained by playing the NCPG. Therefore if $\boldsymbol{k}^*$ exists in the CASG and $\boldsymbol{p}^*$ also exists for all users sharing the same channel in the PCSG, $(\boldsymbol{p}^*)^{\boldsymbol{k}^*}$ can be ensured in the NCPG. □

To ensure the existence of NE in the NCPG, we first need to ensure the attainment of NE^{PC} and NE^{CA} in the PCSG

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and the CASG, respectively. In what follows, we derive the condition for the existence of NE^{PC} in the PCSG.

3.1 Existence of nash equilibrium in the PCSG

In the PCSG, NE^{PC} cannot exist in an infeasible system in which $\bar{\gamma}$ has to be attained at all cost. This is similar to the conventional PC strategies which cannot converge to a fixed point whenever the solution is infeasible [11, 12]. In this context, we use the term 'feasible channel' instead of feasible system to describe a channel in which all users are able to achieve $\bar{\gamma}$ simultaneously. A feasible system is defined differently as a system having at least one feasible channel for all users. On an infeasible channel, however, every user blindly increases its power in the PCSG to obtain an unachievable $\bar{\gamma}$. Nonetheless, the PCSG must reach NE^{PC} in which every user on the channel can strictly achieve $\bar{\gamma}$. For this purpose, we can always drop one or more users from an infeasible channel to make the channel feasible for the remaining users.

Proposition 2: In the PCSG, NE^{PC} can only exist on a feasible channel where all users on the channel can strictly achieve $\bar{\gamma}$ simultaneously. NE^{PC} can also exist on an infeasible channel if call dropping strategy is used to drop one or more users to make the channel feasible for the remaining users.

Proof: See Appendix.

An optimal strategy for choosing which users to be turned off and their turn-off time is required in the PCSG. In consistency with our assumption on the users' selfishness characteristic, individual mobile users may choose to stop transmitting rather than continuing to expend power unnecessarily to obtain an unattainable QoS. In this work where wireless *ad hoc* communications are considered, a call-dropping decision is made by the user itself, unlike other centralised call-dropping strategies [1, 11] in which the decision is made by the base station or access point. We name this call-dropping scheme as autonomous call-dropping (ACD) strategy. Next, we need to find the bounds beyond which the users will self-terminate from the network. For this purpose, we define the following feasibility bounds.

Proposition 3: A channel is considered feasible for a user if the user's received interference levels on this channel are within the following feasibility bounds

$$0 \le I_i^k(\boldsymbol{p}_{-i}^k) \le \frac{2b_i G_{ii}\bar{\gamma}}{c_i} \tag{10}$$

Proof: See Appendix.

After defining the feasibility bound in (10), we need to determine if the PCSG converges to any NE^{PC} within this feasibility bound. We define the convergence bound in the following proposition.

Proposition 4: The PCSG converges to a unique NE^{PC}, provided that the following condition is fulfilled

$$I_i^k(\boldsymbol{p}_{-i}^k) < \frac{c_i G_{ii}\bar{\gamma}}{b_i} \tag{11}$$

Proof: It is proven in [13] that if a fixed point from (9) satisfies the following properties: 1) positivity $(f(p_i^k) > 0)$; 2) monotonicity $(p_i^k > (p_i^k)' \Rightarrow f(p_i^k) > f((p_i^k)'))$; 3) scalability $(f(\alpha p_i^k) > \alpha f(p_i^k) \forall \alpha > 1)$, then the PCSG converges to a unique fixed point. By referring to [13], a fixed point from (9) satisfies the properties under condition (11). □

From (10) and (11), we notice that if the received interference of a user falls within $c_i G_{ii}\bar{\gamma}/b_i \le I_i^k(\boldsymbol{p}_{-i}^k) \le 2c_i G_{ii}\bar{\gamma}/b_i$, a feasible solution exists but the subgame may diverge from the solution. In order to ensure convergence, we use a more stringent convergence bound in (11) rather than the feasibility bound in (10).

In a high traffic system in which more users are admitted, convergence problem may occur in a feasible channel as the channel may not be able to accommodate all the admitted users. To solve the convergence problem, we implement the ACD strategy in which the power of a user is set to zero momentarily when its detected interference exceeds the convergence bound in (11). The user will self-terminate from the network if it has no chance to retransmit [its received interference does not drop within the bound in (11)] before NE is declared.

3.2 Existence of Nash equilibrium in the CASG

If all users in a multi-channel system can find at least one feasible channel, the system is said to be feasible. If NE^{CA} is achieved, the best feasible channels are assigned to all users in a feasible system. If the system is infeasible to a user (all channels in the system are infeasible to the user), the user may play the PCSG by blindly increasing its power on the infeasible channel to achieve the unattainable $\bar{\gamma}$. During the course of power alteration, the user may notice that the cost of accessing another channel is lower and may switch to that channel accordingly. Since the channel is also infeasible, it will switch back to the previous channel after some power adjustment. This phenomenon, in which a user keeps switching between two channels, is known as the ping-pong effect (PPE).

Definition 4: A PPE is the divergence of the CASG where a user keeps changing its strategy in the CASG by switching between two channels continuously.

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Having incorporated the ACD strategy to ensure system feasibility, we investigate next the possible occurrence of the PPE in a feasible system and introduce a countermeasure. First, this detrimental effect occurs in an infeasible system where the user cannot find any feasible channel. Second, even in a feasible system, the PPE may still occur when

$$I_i^m(\boldsymbol{p}_{-i}^m) > I_i^n(\boldsymbol{p}_{-i}^n) \tag{12}$$

$$I_i^m(\boldsymbol{p}_{-i}^m) < I_i^n((\boldsymbol{p}_{-i}^*)^n) \tag{13}$$

where $m, n \in \mathcal{K}$ and $m \neq n$. Let $(\boldsymbol{p}^*)^n$ be the NE^{PC} on the nth channel. Hence $(\boldsymbol{p}_{-i}^*)^n$ is the power vector containing all elements of $(\boldsymbol{p}^*)^n$ except the ith element. Under condition (12), a user picks the nth channel rather than the mth channel based on Proposition 1. On reaching NE^{PC} (the user increases its power to achieve $\bar{\gamma}$ and other users will also do so, hence the received interference increases accordingly), if a user discovers that the cost of accessing the mth channel is lower than that of the current nth channel as indicated in (13), it then switches to the mth channel. This phenomenon recurs and the user keeps switching between two channels and results in the PPE.

To eliminate the PPE in feasible system, we propose to use the NIRLA [10, 14] in the CASG. In general, the NIRLA can be used to determine the probabilistic strategies for all users by observing their play history. In the CASG, the implicated user can learn during the PP process and pick the best channel with some weight. By incorporating the NIRLA, the CASG can converge to a fixed point and achieve correlated equilibrium (CE) [14].

Definition 5: A CE is a joint distribution Π over the set of user strategies that has the property that if, before taking an action, each user receives a recommendation such that the recommendations are drawn randomly according to the joint distribution of Π, then no users have any incentive to divert from the recommendation, provided that all other users follow theirs. A CE is an NE if and only if it is a product measure.

Theorem 2: If the game is played repeatedly many times such that every user plays according to a certain regret-minimisation strategy, then the empirical frequencies of play converge to a set of CE.

Proof: Proof of Theorem 2 can be found in [14].

Proposition 5: By using the regret-minimisation strategy in the CASG, the game play always converges to CE.

Proof: Proven in Theorem 2.

In what follows, we discuss the regret minimisation strategy. In the CASG where $K = 2$, the regret felt by the ith player to pick the mth channel at a discrete time instance t is formulated as the difference between the cost of accessing two different channels, say channels m and n corresponding to the strategies $p_i^{m,t}, p_i^{n,t} \in P_i$ where $m, n \in \mathcal{K}$ and $m \neq n$.

$$R^t(p_i^{m,t} \to p_i^{n,t}) = 1_i^t\left(J_i^m(p_i^{m,t}, \boldsymbol{p}_{-i}^{m,t}) - J_i^n(p_i^{n,t}, \boldsymbol{p}_{-i}^{n,t})\right) \tag{14}$$

where 1_i^t is the indicator function which has a value of 1 if accessing the mth channel at discrete time instance t and a value of 0 otherwise. The cumulative regret is the summation of regret from $p_i^{m,t}$ to $p_i^{n,t}$ through $T+1$ time instances.

$$\text{CR}^{\text{T}}(p_i^{m,t} \to p_i^{n,t}) = \sum_{t=0}^{\text{T}} R^t(p_i^{m,t} \to p_i^{n,t}) \tag{15}$$

The internal regret is then defined as

$$\text{IR}^{\text{T}}(p_i^{m,t} \to p_i^{n,t}) = \max\left\{\text{CR}^{\text{T}}(p_i^{m,t} \to p_i^{n,t}), 0\right\} \tag{16}$$

Given the above definition, the NIRLA updates the components of the weight factor, namely $w^{t+1}(p_i^{m,t})$ and $w^{t+1}(p_i^{n,t})$ which reflect the cumulative feeling of regret as

$$w^{t+1}(p_i^{m,t}) = \frac{\text{IR}^t(p_i^{n,t} \to p_i^{m,t})}{\text{IR}^t(p_i^{m,t} \to p_i^{n,t}) + \text{IR}^t(p_i^{n,t} \to p_i^{m,t})} \tag{17}$$

and

$$w^{t+1}(p_i^{n,t}) = \frac{\text{IR}^t(p_i^{m,t} \to p_i^{n,t})}{\text{IR}^t(p_i^{m,t} \to p_i^{n,t}) + \text{IR}^t(p_i^{n,t} \to p_i^{m,t})} \tag{18}$$

If the regret for having played strategy $p_i^{n,t}$ rather than strategy $p_i^{m,t}$ is significant, then the algorithm updates the weight such that the probability of playing strategy $p_i^{m,t}$ is increased.

4 Distributed algorithms for the NCPG and the NCPGL

In this section, we present an iterative, distributed and asynchronous algorithm for the NCPG. To ensure convergence of the NCPG, the ACD strategy and the NIRLA are incorporated into the NCPG to obtain another new algorithm, known as NCPG with learning (NCPGL). Let $\boldsymbol{p}^{\mathbf{k}}(n-1)$ be the power vector comprising of the power levels of all users on their selected channels at the previous iteration and $\boldsymbol{p}^{\mathbf{k}}(n-1)$ is always known by every user. Both the NCPG and NCPGL algorithms are illustrated using a flow chart in Fig. 2.

It is noteworthy that the proposed algorithms are purely distributed, thus making them very attractive for deployment in *ad hoc* network environments. Both PC and CA can be accomplished using only the information of each user's received interference. If the interference level of

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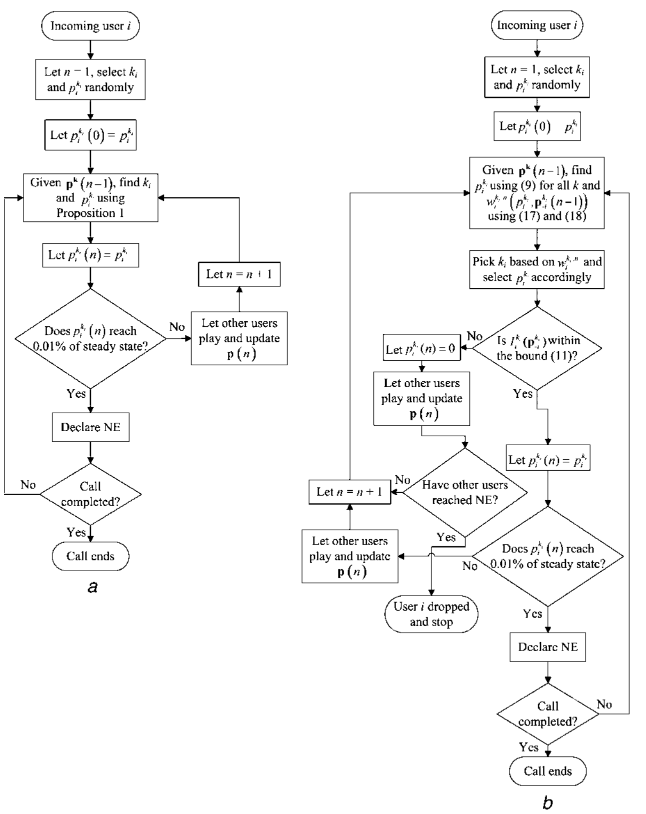

Figure 2 *Flow chart illustrating the NCPG and the NCPGL algorithms*

a NCPG algorithm
b NCPGL algorithm

a user on its selected channel is beyond the bound in (11), the user will stop transmitting rather than attempting to obtain an unachievable SINR. This does not mean that the user has been dropped from the network; it implies, however, that the user opts to set power momentarily to zero to save cost while keep observing. The user will retransmit again if its detected interference has dropped below a certain threshold. In some cases, we notice that some users set their power to zero temporarily to facilitate channel rearrangement so that they can start transmitting again after proper interference management has been done. The user will self-terminate from the network if it still has no chance to transmit at the required QoS after NE has been declared.

5 Simulation results

A simulation of $M = 40$ pairs of stationary nodes (transmitters or receivers) forming 40 simplex links is considered in a $K = 2$ system. Isotropic transmissions with variable transmit powers where $p_{\max} = 20$ dBm are considered. The sender node co-ordinates, $x_{s,i}, y_{s,i} \sim U(-250\text{ m}, 250\text{ m})$ for all $i = \{1, \ldots, M\}$. The receiver node positions are placed isotropically around their respective sender nodes within a common radius of $R_o = 100$ m. The initial power levels and channels of active senders are selected randomly. To model an urban non-line-of-sight (NLOS) propagation environment, we use $\alpha \sim U(3, 5)$ and $\sigma_s \sim U(4, 10)$ [11]. Besides, $\bar{\gamma} = 10$ dB [15] is used and $b_i/c_i = 500$ is computed using (27). A noise floor of -100 dBm is considered, which corresponds to a receiver noise temperature of 300 K and a noise bandwidth of 25 MHz. Convergence of the NCPG and NCPGL is defined as reaching within 0.01% of the steady-state values in power update [13]. We assume that convergence of the algorithms can be reached before a new call arrives. Fast fading and interference from adjacent channels are ignored.

We first simulate the NCPG algorithm in a feasible system to verify Proposition 4. To ensure system feasibility, $N = 8$ random links which are widely separated with one another are activated from the simulated M links to play the NCPG asynchronously. This implies that $I_i^{k_i}\left(\boldsymbol{p}_{-i}^{k_i}\right)$ is always within the convergence bound in (11). In Figs. 3*a* and 3*b*, joint strategies (power selection and channel acquisition) for all users (senders) with respect to the number of iterations in the game are presented. While playing the game, every user picks the optimal channel according to Proposition 1 and updates its power level using (9). During the first ten iterations, the users are acquiring channels as shown in Fig. 3*b*. Once all users have acquired their optimal channel, they start updating their power to achieve $\bar{\gamma}$ on the selected channels while simultaneously observe if other channels with lower cost can be found for possible channel switching. Convergence of the NCPG algorithm is declared after 22 iterations. In Fig. 3*c*, it is shown that all users can attain $\bar{\gamma}$ simultaneously with a deviation <1% after NE is reached.

The PPE caused by system infeasibility is illustrated next. In this simulation, $N = 8$ random links which are located close to each other where at least one of the users has its $I_i^{k_i}(\boldsymbol{p}_{-i}^{k_i})$ exceeding the feasibility bound in (10) are activated to play the NCPG in a manner similar to the previous example. Figs. 4*a* and 4*b* show the power

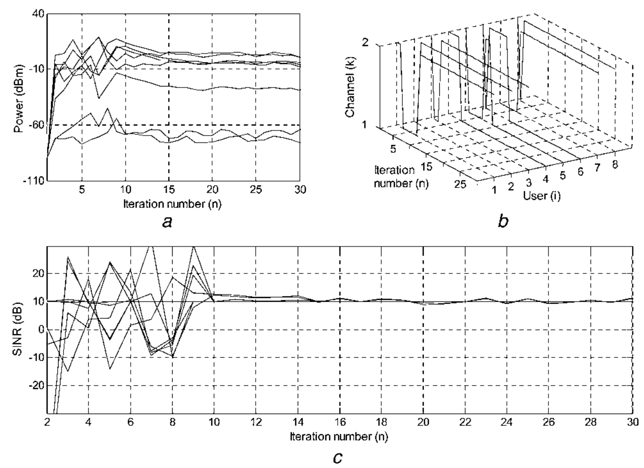

Figure 3 *Convergence of the NCPG in a feasible system for N = 8 and K = 2*

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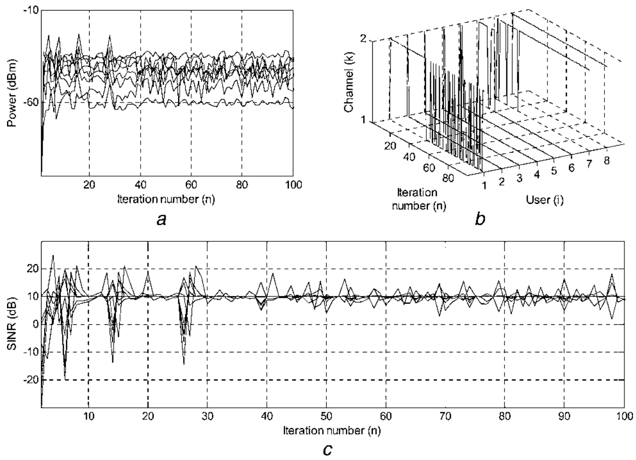

Figure 4 *Divergence of the NCPG in an infeasible system for N = 8 and K = 2*

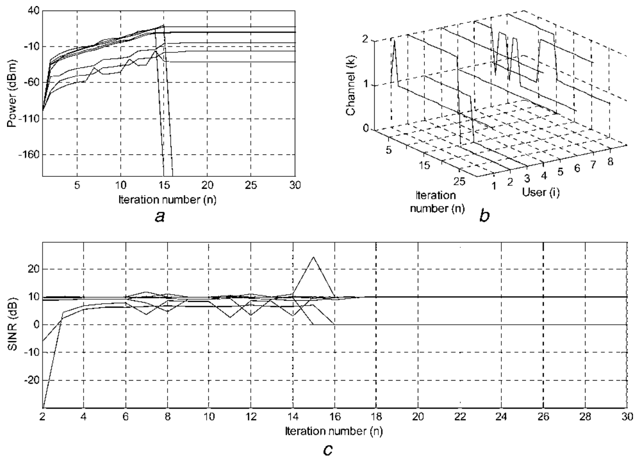

Figure 5 *Convergence of the NCPGL in an infeasible system for N = 8 and K = 2*

alterations and channel switchings of the users. The NCPG diverges because the power levels of the users fluctuate beyond 0.01% of their steady-state values. In Fig. 4*b*, it is observed that user 1 enters the PP state and keeps switching between channels 1 and 2 continuously because a feasible channel cannot be found. Consequently, user 1 blindly adjusts its power level in both channels to obtain the unachievable $\bar{\gamma}$, resulting in fluctuation around $\bar{\gamma}$ as shown in Fig. 4*c*.

By using the same simulation model considered previously, we now investigate the NCPGL and the corresponding results are shown in Fig. 5. Accordingly, the users can now learn the PP process and pick the channels with the weights computed according to (17) and (18). As observed from Figs. 5*a* and 5*b*, users 2 and 3 temporarily set their power to zero and select no channel (denoted by channel 0) because their $I_i^{k_i}(\boldsymbol{p}_{-i}^{k_i})$ has exceeded the convergence bound in (11). After the two users stop transmitting, the overall system has become feasible for the remaining users which play the NCPGL iteratively until NE is attained after 20 iterations. Eventually, users 2 and 3 self-terminate from the network as they have no chance to retransmit before NE is declared. As indicated in Fig. 5*c*, all users are able to achieve $\bar{\gamma}$ simultaneously. Hence, by introducing the ACD strategy and the NIRLA, convergence to a unique NE is ensured. Interestingly, by comparing Figs. 3 and 5, we notice that in the NCPG, the power levels after convergence is declared fluctuating around the steady-state values, whereas in the NCPGL, the power levels converge to constant values. This scenario happens because the simulation environment for the NCPG is governed by the noise because of low power levels selection (noise-limited), whereas the simulation environment for the NCPGL is mainly dominated by interference due to high power levels selection (interference-limited). The convergence speed and

Table 1 Convergence speed and final channel selection of different users for playing the NCPG and NCPGL in the environment with and without PPE

User index	Without PPE		With PPE	
	NCPG n_{conv}, k_{conv}	NPCGL n_{conv}, k_{conv}	NCPG n_{conv}, k_{conv}	NCPGL n_{conv}, k_{conv}
1	20, 2	20, 2	∞^{PPE}	18, 1
2	20, 2	20, 2	∞^{pow}	dropped
3	20, 2	20, 2	∞^{pow}	dropped
4	22, 1	22, 1	∞^{pow}	18, 2
5	22, 1	22, 1	∞^{pow}	18, 1
6	22, 1	22, 1	∞^{pow}	18, 1
7	20, 2	20, 2	∞^{pow}	18, 2
8	20, 2	20, 2	∞^{pow}	18, 1

Remarks: n_{conv} denotes the number of iterations required for the transmit power to converge within ± 2 dB over five consecutive iterations. k_{conv} denotes the channel number selected at convergence. ∞^{PPE} and ∞^{pow} denote non-convergence because of PPE and power fluctuation, respectively

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the channel selection of different users playing the NCPG and NCPGL in the environment with and without PPE are summarised in Table 1.

The distributed dynamic CA with PC (DDCA/PC) technique proposed in [15] is also compared with the NCPG and NCPGL in terms of power utility. In this simulation, N is varied from 3 to 10 and the total powers utilised by different number of users are recorded. For a fair comparison, we assume a feasible system which accommodates all links activated and no call dropping is considered. In Fig. 6, it is seen that both the NCPG and NCPGL reach NE and exhibit similar performances. It is noticed that users always take the same strategies in both the NCPG and NCPGL if the PPE does not occur. Furthermore, the proposed NCPGL algorithm is also proven to be more power efficient than the DDCA/PC technique in which ~12 dB of power gain is achieved.

Finally, the overall system performance is investigated under different traffic loads. Communicating links are activated arbitrarily with link durations and inter-arrival times which are exponentially distributed with mean node activity between 0.015 and 0.15 Erlangs [15]. Each point shown in Fig. 7 has been determined from ~60 000 node activations. The PPE occurs randomly and the NCPG will drop random user(s) till NE is achieved while the NCPGL has no convergence problem. In Fig. 7, the probability of unsuccessful call for the NCPG, NCPGL and DDCA/PC are compared with respect to different loads offered, λ in the system with $K = 2$, 3, 4 and 5. The probability of unsuccessful call is defined as $P_U(\lambda) = 1 - (1 - P_B(\lambda))(1 - P_D(\lambda))$ where $P_B(\lambda)$ is the probability of call blocking and $P_D(\lambda)$ is the probability call dropping [16]. Fig. 7 shows that the NCPGL achieves the lowest $P_U(\lambda)$ under different traffic loads as compared with the NCPG and the

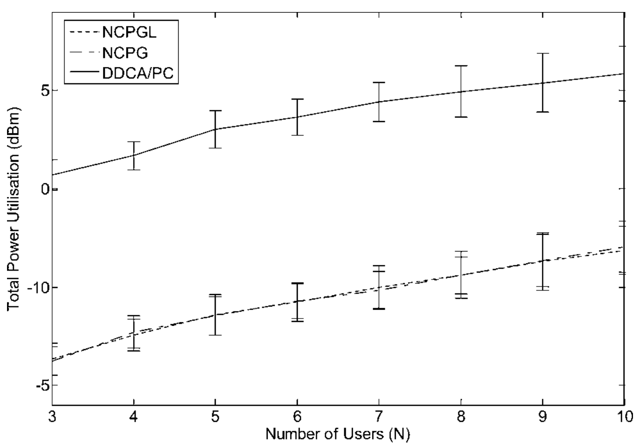

Figure 6 *Total power utilisation against total number of users for $K = 2$*

The error bars indicate 95% confidence interval

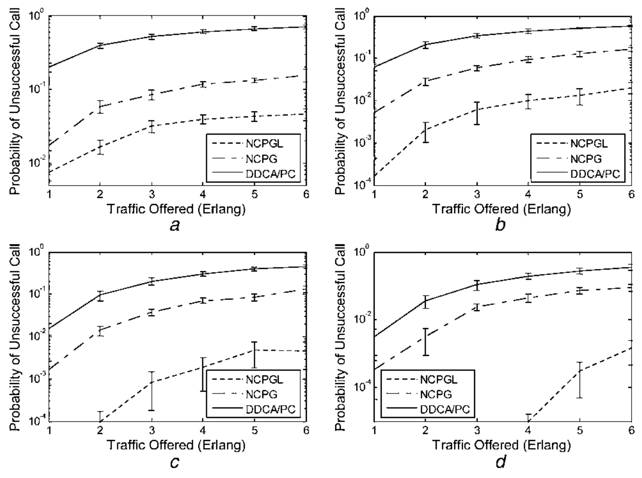

Figure 7 *Probability of unsuccessful call against traffic offered for the NCPGL, the NCPG and the DDCA/PC in different systems with $K = 2$, 3, 4 and 5*

The error bars indicate 95% confidence interval

a $K = 2$
b $K = 3$
c $K = 4$
d $K = 5$

DDCA/PC. As K increases, the performance improvement of the NCPGL becomes more significant. By using non-cooperative game theory and the NIRLA, the overall system capacity is improved dramatically as channel reassignment [16] can be easily incorporated. Besides, the NCPGL which guarantees convergence has also eliminated the potential instability (avalanche effect) [16] that arises due to the incorporation of channel reassignment into the DDCA/PC. Furthermore, users playing the NCPGL can share the spectrum more efficiently than those playing the NCPG as the NCPGL players can learn during the game play and pick the best channels based on the play history [9].

6 Conclusion

In this paper, we have integrated CA and PC into wireless *ad hoc* networks using non-cooperative game theory. This game-theoretic approach leads to a more efficient integrated solution as it well-characterises the distributed, non-cooperative and self-organising behaviour of wireless *ad hoc* networks. The proposed NCPG algorithm can be easily implemented in a distributed manner and requires only knowledge of the received interference. Nevertheless, the PPE may arise in the NCPG due to unpredictable network topology and diverse system conditions. This problem is solved by incorporating the ACD strategy and the NIRLA into the NCPG to obtain the NCPGL algorithm. The NCPGL algorithm achieves significant improvements in terms of total power utility and overall system capacity as compared with the existing DDCA/PC technique. Instead of dropping users who cannot attain the required QoS, the NCPGL allows the users to choose among themselves who should be dropped based on their own valuation. An interesting avenue for further research is to introduce some degree of cooperation among the wireless users such that channel selection, channel reallocation and power control can be performed more efficiently at the expense of higher system complexity.

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9 Appendix

Proof of Propostion 2: According to the implicit function theorem, the Jacobian matrix must be non-singular at the point of existence [17]. From (2), we have $I_i^k(p_{-i}^k) = \sum_{i\neq j,j=1}^{N} G_{ij}p_j^k + \sigma^2$ (we use superscript k instead of k_i because we consider a single-channel case in which all users transmit on the kth channel) and by substituting $I_i^k(p_{-i}^k) = \sum_{i\neq j,j=1}^{N} G_{ij}p_j^k + \sigma^2$ into (9), the algebraic equation of the considered system is given by

$$F_i = -p_i^k + \frac{\bar{\gamma}}{G_{ii}}\left(\sum_{i\neq j,j=1}^{N} G_{ij}p_j^k + \sigma^2\right) - \frac{c_i}{2b_iG_{ii}^2}\left(\sum_{i\neq j,j=1}^{N} G_{ij}p_j^k + \sigma^2\right)^2 = 0 \qquad (19)$$

If $F_1, F_2, \ldots, F_N$ are differentiable functions, we have a

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Jacobian matrix as

$$J_F(p_1, p_2, \ldots, p_N) = \begin{pmatrix} \frac{\partial F_1}{\partial p_1^k} & \frac{\partial F_1}{\partial p_2^k} & \cdots & \frac{\partial F_1}{\partial p_N^k} \\ \frac{\partial F_2}{\partial p_1^k} & \frac{\partial F_2}{\partial p_2^k} & \cdots & \frac{\partial F_2}{\partial p_N^k} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial F_N}{\partial p_1^k} & \frac{\partial F_N}{\partial p_2^k} & \cdots & \frac{\partial F_N}{\partial p_N^k} \end{pmatrix}$$

$$= \begin{pmatrix} -1 & \frac{G_{12}}{G_{11}}\left(\bar{\gamma} - \frac{c_1}{b_1\gamma_2^k}\right) & \cdots & \frac{G_{1N}}{G_{11}}\left(\bar{\gamma} - \frac{c_1}{b_1\gamma_N^k}\right) \\ \frac{G_{21}}{G_{22}}\left(\bar{\gamma} - \frac{c_2}{b_2\gamma_1^k}\right) & -1 & \cdots & \frac{G_{2N}}{G_{22}}\left(\bar{\gamma} - \frac{c_2}{b_2\gamma_N^k}\right) \\ \vdots & \vdots & \ddots & \vdots \\ \frac{G_{N1}}{G_{NN}}\left(\bar{\gamma} - \frac{c_N}{b_N\gamma_1^k}\right) & \frac{G_{N2}}{G_{NN}}\left(\bar{\gamma} - \frac{c_N}{b_N\gamma_2^k}\right) & \cdots & -1 \end{pmatrix} \quad (20)$$

The non-singularity of the Jacobian matrix (20) can be preserved if the terms outside the main diagonal are small [17]. It is shown that these terms mainly depend on G_{ij}. From (2), we know that γ_i^k is also given in terms of G_{ij} while b_i/c_i is constant. Hence, if G_{ij} are large enough to cause singularity in (20), we can turn off the implicated user(s), for example, the nth user is turned off by setting $G_{ij} = 0 \forall i = n \cup j = n$. Physically, the users stop transmitting at their current frequency channel. This ensures that the terms outside the main diagonal are always small enough, and thus the solution of Nash algebraic equations can always exist. □

Proof of Propostion 3: We impose the power bound of $0 \le p_i^k \le p_{\max}$. If the power selected is outside this bound, it is no longer a practical solution for the subgame and NE^{PC} is not achievable. Accordingly, using (9) and the lower power bound $p_i^{k_i} > 0$, we define

$$\frac{\bar{\gamma}}{G_{ii}} I_i^k(\boldsymbol{p}_{-i}^k) - \frac{c_i}{2b_i G_{ii}^2}(I_i^k(\boldsymbol{p}_{-i}^k))^2 \ge 0 \quad (21)$$

Rearranging terms yields the upper bound of the interference for the ith user on the kth channel as

$$I_i^k(\boldsymbol{p}_{-i}^k) \le \frac{2b_i G_{ii}\bar{\gamma}}{c_i} \quad (22)$$

Similarly, using (9) and the upper power bound $p_i^k \le p_{\max}$ we can find another range of interference as

$$\frac{\bar{\gamma}}{G_{ii}} I_i^k(\boldsymbol{p}_{-i}^k) - \frac{c_i}{2b_i G_{ii}^2}(I_i^k(\boldsymbol{p}_{-i}^k))^2 \le p_{\max} \quad (23)$$

By solving (15), another range of $I_i^{k_i}(\boldsymbol{p}_{-i}^{k_i})$ other than (14) is given as follows

$$I_i^k(\boldsymbol{p}_{-i}^k) \ge \frac{b_i G_{ii}\bar{\gamma}}{c_i} + \sqrt{\frac{b_i^2 G_{ii}^2 \bar{\gamma}^2 - 2b_i c_i G_{ii}^2 p_{\max}}{c_i^2}}$$

$$\text{or} \quad I_i^k(\boldsymbol{p}_{-i}^k) \le \frac{b_i G_{ii}\bar{\gamma}}{c_i} - \sqrt{\frac{b_i^2 G_{ii}^2 \bar{\gamma}^2 - 2b_i c_i G_{ii}^2 p_{\max}}{c_i^2}} \quad (24)$$

From (22) and (24) we obtain the feasibility bounds in which if all users' received interference falls within this range on any channel, they can achieve $\bar{\gamma}$ on that channel. The infeasible region in between is denoted as

$$\frac{b_i G_{ii}\bar{\gamma}}{c_i} - \sqrt{\frac{b_i^2 G_{ii}^2 \bar{\gamma}^2 - 2b_i c_i G_{ii}^2 p_{\max}}{c_i^2}} < I_i^k(\boldsymbol{p}_{-i}^k) < \frac{b_i G_{ii}\bar{\gamma}}{c_i} + \sqrt{\frac{b_i^2 G_{ii}^2 \bar{\gamma}^2 - 2b_i c_i G_{ii}^2 p_{\max}}{c_i^2}} \quad (25)$$

Infeasible region in (24) can be minimised by adjusting b_i/c_i so that

$$\sqrt{\frac{b_i^2 G_{ii}^2 \bar{\gamma}^2 - 2b_i c_i G_{ii}^2 p_{\max}}{c_i^2}} = 0 \quad (26)$$

By solving (26), we have

$$\frac{c_i}{b_i} \le \frac{(\bar{\gamma})^2}{2p_{\max}} \quad (27)$$

(27) will be used later to determine the weighting factor b_i and c_i. Substituting (26) in to (24), feasibility bounds in (10) can be obtained. □

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