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Fair power control for wireless *ad hoc* networks using game theory with pricing scheme

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Abstract: Resource allocation in wireless *ad hoc* networks is usually modelled in a non-cooperative game theoretic framework with the objective of maximising individual utility. However, the selfishness of autonomous users under such framework may lead to throughput unfairness which only benefits certain users. To alleviate this unfairness problem, the authors propose a payment-based power control scheme using game theory where each user announces a set of price coefficients that reflects different compensations paid by other users for the interference they produce. Users who generate higher interference are required to pay more by transmitting at a lower power to give other users a fairer chance of sharing the throughput. Without any incentive to play fairly, users could misbehave by broadcasting high price coefficients to force other users to transmit at a lower power. The authors treat this problem casting it into a price game which resembles a Prisoner's Dilemma game. Users who play this game iteratively will behave cooperatively and broadcast the price coefficients truthfully. Together with analytical proof, the proposed approach is shown to converge to Nash equilibrium where at this point it is able to provide a fairer throughput share among users at the expense of a slight loss in total throughput.

1 Introduction

Wireless *ad hoc* networks for the emerging pervasive computing and communication environment may comprise of a large number of autonomous users with heterogeneous quality of service (QoS) requirements. Potential applications of these networks have been envisaged in ubiquitous civilian and commercial usage, where nodes typically belong to different authorities and may not pursue a common goal. Consequently, the principle of altruism does not exist among autonomous users in such environments but they rather tend to be selfish with the intention of gaining extra share of the network resources. This could result in network partitioning and leads to performance degradation. In particular, a selfish user can benefit by: (i) acquiring a larger portion of the channel (improved throughput); (ii) reducing power consumption; (iii) obtaining a higher transmission rate and (iv) improving the QoS.

In heterogeneous wireless *ad hoc* networks, users may possess different transmit power capabilities and experience independent channel realisations. As a result, those with lower power capabilities or poor channel conditions will be starved of throughput share. This unfairness problem can be avoided using the signal-to-interference (SIR)-balancing power control (PC) scheme [1] which forces all users to achieve the target SIR and hence ensuring equal throughput share among users. However, this scheme is not applicable to networks comprising of users with different QoS requirements. Furthermore, selfish users with higher power capabilities are reluctant to adhere to the rules of this scheme because they tend to achieve a better QoS by transmitting at high power levels, thereby dominating the throughput. Hence, a PC scheme that can ensure throughput fairness in this kind of networks is desired.

By taking the selfishness of users into account, non-cooperative game theory has been shown to be very effective in examining the resource allocation problem in such networks [2–7]. Nevertheless, most of the previous approaches merely emphasise on maximising the total transmission rate [2, 3] and total throughput [4] or on minimising the total transmit power [5–7] under some constraints. The problems addressed and the proposed solutions are mainly focused on the efficiency issues which only benefit users with good channel gains or with high transmit power capabilities. The fairness issue, however, has largely been ignored.

In this paper, we propose a payment-based PC scheme under a game theoretic framework termed non-cooperative PC game with pricing (NPcGP), as an attempt to provide throughput fairness among autonomous users without the need for a central billing system. In contrast to the game proposed in [8] where a central controller is required to broadcast a common price, users in the NPcGP take the responsibility to announce prices that reflect different compensations to be paid by other users for the interference they produce. In [4], a similar approach is used but this scheme only aims to maximise the overall system performance without taking the fairness issue into account.

In the NPcGP, the users may tend to act selfishly to gain a higher payoff by broadcasting large price coefficients to be charged on others. To avoid this undesirable misbehaviour, we propose a non-cooperative price game (NPrG) to govern the users in choosing the correct price coefficients. We show that the NPrG resembles a Prisoner's Dilemma (PD) game [9] where users who play this game selfishly and iteratively in a way similar to the Iterated Prisoner's Dilemma (IPD) game [10] will sustain a cooperative outcome and achieves a higher payoff. By playing the NPrG iteratively, users will eventually cooperate and broadcast the correct price coefficients.

The rest of this paper is organised as follows. In Section 2, we outline the system model, review some relevant works and highlight the motivation of the current work. We then present the proposed NPcGP and NPrG for wireless *ad hoc* networks together with their convergence properties in Sections 3 and 4, respectively. Subsequently, the algorithm for the combined NPcGP and NPrG is presented in Section 5, where a complexity analysis of the proposed scheme is also presented. Simulation results are analysed in Section 6. This paper ends with some concluding remarks in Section 7.

2 System model and motivating example

We consider $M > 1$ randomly located pairs of quasi-stationary nodes forming distinct communication links in a shared medium power-controlled wireless *ad hoc* network. Each communicating pair consists of a dedicated sender and receiver. It is assumed that a subset of active senders, $\mathcal{S} = \{s_1, s_2, \ldots, s_N\}$ wish to communicate with another subset of active receivers, $\mathcal{R} = \{r_1, r_2, \ldots, r_N\}$ where $1 \le N \le M$ is the instantaneous number of active pairs. Without loss of generality, we use the term 'user' to denote either a sender or a receiver of a communication link in the forthcoming sections. During data transmission, the ith user transmits at a power level $p_i \le p_i^{\max}$ where $i \in \mathcal{N}$, $\mathcal{N} = \{1, 2, \ldots, N\}$ is the set of all active users and $p_i^{\max}$ is the maximum permissible transmit power for the ith user. The received power level at the ith user for a signal transmitted from the jth user is given by $G_{ij}p_j$, where $G_{ij} > 0$ is the path gain from s_j to r_i, and can be expressed as

$$G_{ij} = \frac{S_{ij}}{[(x_{\mathrm{r},i} - x_{\mathrm{s},j})^2 + (y_{\mathrm{r},i} - y_{\mathrm{s},j})^2]^{v/2}} \tag{1}$$

where S_{ij} is the attenuation factor that models the shadowing effect and $10\log_{10} S_{ij} \sim N(0, \sigma_{\mathrm{s}}^2)$, $1 \le i, j \le M$ are independently and identically distributed where $\sigma_{\mathrm{s}}^2 \sim U(4, 10)$. x and y denote the two-dimensional coordinate positions and the subscripts r and s represent the receiver and sender nodes, respectively. To model an urban non-line-of-sight propagation environment, we use $v \sim U(3, 5)$ which models the propagation pathloss. In this context, we assume the network to be geographically static in the sense that the time scale of algorithm convergence is shorter than the channel coherence time. Thus, the channel gains are fixed in one implementation of the algorithm.

Other than the signal-to-interference-plus-noise ratio (SINR), the quality of each link can also be quantified in terms of throughput which is defined as the net number of information bits received without error per unit time. The throughput [8] of the ith user can be expressed as

$$T_i = \frac{LR\omega(\gamma_i)}{M_{\mathrm{p}}} \tag{2}$$

where L and M_{p} are the number of information bits and the total number of bits in a packet, respectively, R is the transmission rate and $\omega(\gamma_i)$ is the packet successful rate (PSR). In this context, we use $\omega(\gamma_i) = (1 - \exp(-0.5\gamma_i))^{M_{\mathrm{p}}}$ which approximates the PSR for non-coherent frequency-shift keying modulation [8]. γ_i is the SINR for the ith user which is generally given by

$$\gamma_i = \frac{G_{ii}p_i}{\sum_{j=1, j\ne i}^{N} G_{ij}p_j + \sigma_i^2} = \frac{G_{ii}p_i}{I_i^{\mathrm{R}}(\boldsymbol{p}_{-i})} \tag{3}$$

where $\boldsymbol{p}_{-i}$ denotes the power vector comprising of the powers of all users except that of the ith user while $I_i^{\mathrm{R}}(\boldsymbol{p}_{-i})$ is the interference-plus-noise received by the ith user and σ_i^2 is the power of the additive white Gaussian noise at the ith receiving user.

In a competitive environment, users with higher power capabilities or better path gains could dominate the throughput share. In this scenario, a performance metric is needed to assess the unfairness incurred in the system as a result of competition. The throughput unfairness factor in [11] is adopted here and is defined as

$$\rho = \left(\frac{1}{\overline{T}}\right)\sqrt{\frac{1}{N-1}\sum_{i=1}^{N}\left(\frac{T_i}{T_i^{\max}} - \overline{T}\right)^2} \qquad (4)$$

where $T_i^{\max}$ is the maximum throughput if the ith user is the only transmitter, and $\overline{T} = (1/N)\sum_i^N T_i/T_i^{\max}$, is the normalised throughput per communication pair. The physical meaning of ρ is the normalised variance of users' throughput compared with that of the single-user case. Thus ρ provides one possible means to measure fairness, where a higher ρ indicates more unfairness in throughput sharing among the users.

2.1 Motivating example

Let $\boldsymbol{G} = \{G_{ij}\}, \forall i, j \in \mathcal{N}$ be an $N \times N$ gain matrix and consider a shared medium wireless *ad hoc* network with three users whose path gains are given by

$$\boldsymbol{G} = \begin{bmatrix} 1.00 \times 10^{-10} & 8.82 \times 10^{-13} & 3.57 \times 10^{-13} \\ 5.24 \times 10^{-13} & 9.5 \times 10^{-11} & 2.50 \times 10^{-12} \\ 7.67 \times 10^{-13} & 2.44 \times 10^{-13} & 9.9 \times 10^{-11} \end{bmatrix}$$

Consider $\sigma_i^2 = 1 \times 10^{-10}$ mW, $\forall i$ and assume that the three users possess different maximum allowable power capabilities with of $p_1^{\max} = 10$ mW, $p_2^{\max} = 10$ mW and $p_3^{\max} = 100$ mW, respectively. For throughput computation, it is assumed that $M_p = 80$ bits, $L = 64$ bits and $R = 10^4$ bits/s [8]. We examine the throughput share among the users and the maximum attainable total throughput under different PC schemes for such environment.

In Fig. 1, we present the results for the three autonomous users who achieve different throughput levels under different schemes. In the SIR-balancing PC scheme [1], all users have a fair throughput share ($\rho = 0$); however, the maximum achievable total throughput is relatively low. In contrast, it is observed that the users act selfishly in the non-cooperative PC scheme [5] in an attempt to maximise their own throughput. In this scheme, the highest total throughput can be obtained by sacrificing user 2 who achieves almost zero throughput, thereby resulting in severe unfairness ($\rho = 1.1782$). Apparently, these two schemes do not lead to satisfactory fairness and efficiency.

Instead, an alternative approach is required to provide fair throughput share while achieving a comparable total throughput. For example, if we can introduce some degree of cooperation in the PC technique such that user 3 is willing to compromise (by reducing its transmit power and hence a lower throughput), user 2 would have a fairer chance to compete for the resource. It is shown in Fig. 1*a* that if user 3 reduces its power to 10 mW, user 2 can achieve a higher throughput while the total throughput only drops slightly. More importantly, a much better throughput fairness is achieved in this case.

3 Non-cooperative game theoretic framework

In this section, the throughput unfairness problem is investigated using a non-cooperative game theoretic method. Under this framework, a utility function that takes

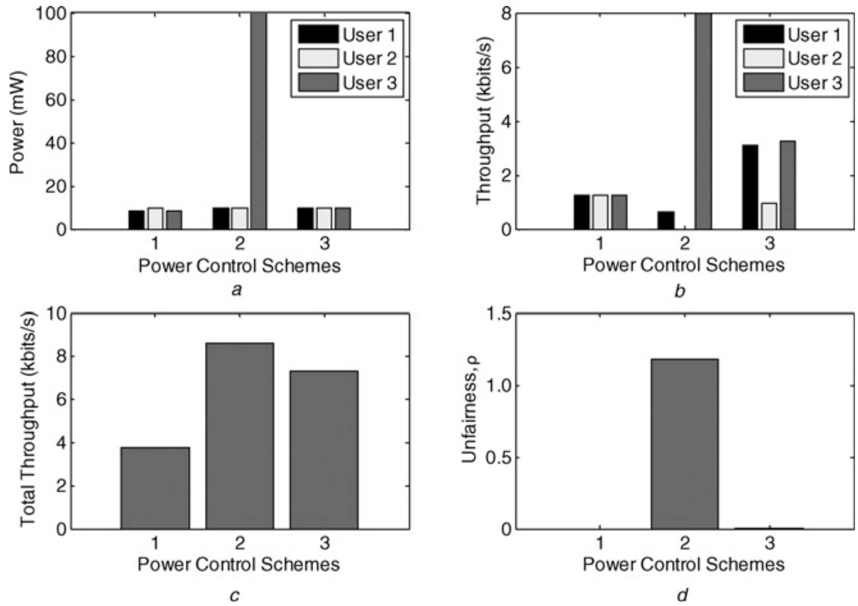

Figure 1 *Comparison of*

a Transmit power
b Throughput share
c Total throughput
d Throughput unfairness in a three-user wireless *ad hoc* network for different PC schemes: (i) SIR-balancing PC, (ii) Non-cooperative PC, (iii) Alternative PC

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both efficiency and fairness issue into consideration is required. In recent works [2–7], several notions of utility have been proposed for wireless systems in which the proposed utilities portray different objectives for the optimisation problems. In this paper where maximising spectral efficiency is also one of our goals, we adopt the logarithmic utility function used in [4] for our PC problem which captures the users' QoS as a function of SINR, that is

$$U_i^{\mathrm{Pc}}(\gamma_i(p_i, \boldsymbol{p}_{-i})) = \theta_i \log(\gamma_i) \tag{5}$$

where $\theta_i > 0$ is a user-dependent priority parameter. The function in (5) exhibits a quasi-concave characteristic which is the most desired property for a utility function in game theory [5].

3.1 Conventional non-cooperative PC Game

Consider a conventional non-cooperative PC game (NPcG) where users in the wireless *ad hoc* network do not exchange information but choose their transmit powers independently to maximise their individual utility given in (5), that is

$$\tilde{p}_i = \min(\arg\max_{p_i} (U_i^{\mathrm{Pc}}(\gamma_i(p_i, \boldsymbol{p}_{-i}))), p_i^{\max}), \ \forall i \in \mathcal{N} \tag{6}$$

where $\tilde{p}_i$ is the best response for the ith user in the maximisation problem and is upper bounded by $p_i^{\max}$. By continuously optimising the utility according to (6) for all users, a set of transmit powers, $\boldsymbol{p}^*$ can be obtained where every user is satisfied with its utility given the power selection of other users. This steady state of the network is called Nash equilibrium (NE) [8] and is defined as

$$U_i^{\mathrm{Pc}}(\gamma_i(p_i^*, \boldsymbol{p}_{-i}^*)) \geq U_i^{\mathrm{Pc}}(\gamma_i(p_i', \boldsymbol{p}_{-i}^*)), \quad \forall i \in \mathcal{N}, p_i' \in P_i \tag{7}$$

where p_i^* is the equilibrium power of the ith user, $\boldsymbol{p}_{-i}^*$ is the equilibrium power vector comprising of all elements in $\boldsymbol{p}^*$ except the ith element and $P_i = [0, p_i^{\max}]$ is a strategy profile.

From (3), we learn that U_i^{Pc} is strictly increasing with p_i, so there is no penalty for users to transmit at high power levels as long as $p_i \in P_i$. Hence, the NE for the NPcG is always $\boldsymbol{p}^* = [p_i^{\max}]_{i=1}^N$. This is similar to the scenario addressed in the motivating example where every user in the network acts selfishly by transmitting at maximum power. Eventually, this leads to serious throughput degradation for some users due to the high received interference. In order to prevent users from transmitting at maximum power all the time, we propose a NPcGP which incorporates a payment (pricing) scheme into the conventional NPcG.

3.2 Proposed NPcGP

In general, the NPcGP can be denoted as NPcGP = $[\mathcal{N}^{\mathrm{Pc}}, \{P_i\}, \{U_i^{\mathrm{net}}\}]$ where $\mathcal{N}^{\mathrm{Pc}} = \{1, 2, \ldots, N\}$ is the player or user set. $P_i = [0, p_i^{\max}]$ is a strategy profile and U_i^{net} is the net utility defined as the difference between its utility, U_i^{Pc} and payment, Ω_i. In particular, the payment to be made by the ith user is the product of the price per unit interference and the amount of interference it generates, that is

$$\Omega_i = \lambda_i(\boldsymbol{p})p_i \tag{8}$$

where $\lambda_i(\boldsymbol{p})$ is the total price per unit interference charged to the ith user and p_i is interpreted as the amount of power transmitted by the ith user. The objective of the NPcGP is to maximise the following net utility function

$$U_i^{\mathrm{net}}(p_i, \boldsymbol{p}) = U_i^{\mathrm{Pc}}(\gamma_i(p_i, \boldsymbol{p}_{-i})) - \Omega_i \tag{9}$$

Unlike the pricing scheme proposed in [8] which announces a common price to all users, we adopt an adaptive pricing scheme in which $\lambda_i(\boldsymbol{p})$ varies for different users based on their generated interference. Let $I_i^{\mathrm{F}}(p_i) = p_i \sum_{j=1, j\neq i}^N G_{ji}$ be the total interference caused by the ith user to the network, $\lambda_i(\boldsymbol{p})$ should be strictly increasing with $I_i^{\mathrm{F}}(p_i)$ so as to discourage users who cause high interference to transmit at high power. As a result, in order to maximise the net utility, users who are charged at high prices will rationally reduce their transmit power. Hence, the throughput fairness can implicitly be achieved in this context as users who are severely interfered can have a fairer chance to share the resource. The pricing policy used in this work differs from that of in [4] in which the previously proposed pricing function is a strictly decreasing function of $I_i^{\mathrm{F}}(p_i)$ where users transmit at high power are encouraged to increase their power levels continuously until NE is reached. In such formulation, fairness is not taken into consideration. In contrast, we formulate a pricing function which is exponentially proportional to $I_i^{\mathrm{F}}(p_i)$ in order to penalise users who introduce high interference to the network. The proposed pricing function is given as

$$\lambda_i(\boldsymbol{p}) = \beta - \delta \exp\left(-\mu \frac{I_i^{\mathrm{F}}(p_i)}{I_i^{\mathrm{R}}(\boldsymbol{p}_{-i})}\right) \tag{10}$$

where $\beta > 1$ is the maximum price, $\delta > 1$ is the price weight of the generated interference and $\mu > 1$ is the sensitivity of the users to interference. The motivation to formulate the pricing function exponentially in (10) will be highlighted later in the proof of existence and uniqueness of NE.

In this paper, we use a discrete-time model where time is divided into iterations and we assume that all users act only once within one iteration and their actions remain unchanged over that iteration. Let t be the current iteration and $t - 1$ indicates the previous iteration. By taking t into

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account, substituting (10) into (9) yields

$$U_i^{\text{net}}(p_i(t), \boldsymbol{p}(t-1)) = \theta_i \log\left(\frac{G_{ii}p_i(t)}{I_i^{\text{R}}(\boldsymbol{p}_{-i}(t-1))}\right) - \left(\beta - \delta \times \exp\left(-\mu \frac{I_i^{\text{F}}(p_i(t-1))}{I_i^{\text{R}}(\boldsymbol{p}_{-i}(t-1))}\right)_i\right)p_i(t) \quad (11)$$

where $p_i(t)$ and $p_i(t-1)$ are the powers of the ith user during the current and previous iterations, respectively. $\boldsymbol{p}(t-1)$ is the power vector comprising of the powers of all users from the previous iteration while $\boldsymbol{p}_{-i}(t-1)$ contains all elements in $\boldsymbol{p}(t-1)$ except the ith element. Furthermore, $I_i^{\text{F}}(p_i(t-1))$ and $I_i^{\text{R}}(\boldsymbol{p}_{-i}(t-1))$ are the total interference generated by the ith sender and received by the ith receiver during the previous iteration, respectively. For simplicity, we define $I_i^{\text{F}}(p_i(t-1))$ and $I_i^{\text{R}}(\boldsymbol{p}_{-i}(t-1))$ as follows

$$I_i^{\text{F}}(p_i(t-1)) = p_i(t-1)\sum_{j=1, j\neq i}^{N} G_{ji} \quad (12)$$

$$I_i^{\text{R}}(\boldsymbol{p}_{-i}(t-1)) = \sum_{j=1, i\neq j}^{N} G_{ij}p_j(t-1) + \sigma_i^2 \quad (13)$$

In what follows, we first show the existence and uniqueness of NE. If NE exists and is unique, the NPcGP will always converge to NE [8].

3.3 Existence and uniqueness of NE in the NPcGP

In general, NE does not necessarily exist in a game, but under certain conditions, a unique NE can be guaranteed to exist. Various mathematical approaches including graphical methods [12], super-modularity [4, 8], and quasi-concavity curve [13] have been used to study the conditions for the existence and uniqueness of NE. In this work, the graphical method adopted in [12] is used to investigate the existence and uniqueness of NE in the NPcGP.

Theorem 1: A NE exists in the NPcGP if U_i^{net} adopts the pricing function in (10).

Proof: See Appendix.

Theorem 2: The NPcGP has a unique NE if U_i^{net} adopts the pricing function in (10).

Proof: See Appendix.

By formulating the pricing function as in (10), the existence and uniqueness of NE can be ensured in the NPcGP. Moreover, it is shown in Theorems 1 and 2 that if all users update their power according to (21), the NPcGP always converges to a unique NE. From (21), every user needs the price information, $\lambda_i(\boldsymbol{p})$ to update its power where $\lambda_i(\boldsymbol{p})$ is a function of $I_i^{\text{R}}(\boldsymbol{p}_{-i})$ and $I_i^{\text{F}}(p_i)$. $I_i^{\text{R}}(\boldsymbol{p}_{-i})$ can be measured at the ith receiver and fed back to the ith sender while $I_i^{\text{F}}(p_i)$ can be determined via information exchange among users. Each user can send a pilot signal periodically at different times to each of the reachable neighbouring users who in response broadcast a set of price coefficients based on the pilot signal strengths. Intuitively, the higher the pilot signal strength, the higher will be the price coefficients broadcast by the neighbouring users. The user who sends the pilot signal shall collect the price coefficients from other users and computes the charging prices and updates its power accordingly. An example of information exchange among five users is illustrated in Fig. 2 where user A sends a pilot signal at the intended transmit power level to the neighbouring users and later receives a set of price coefficients from them.

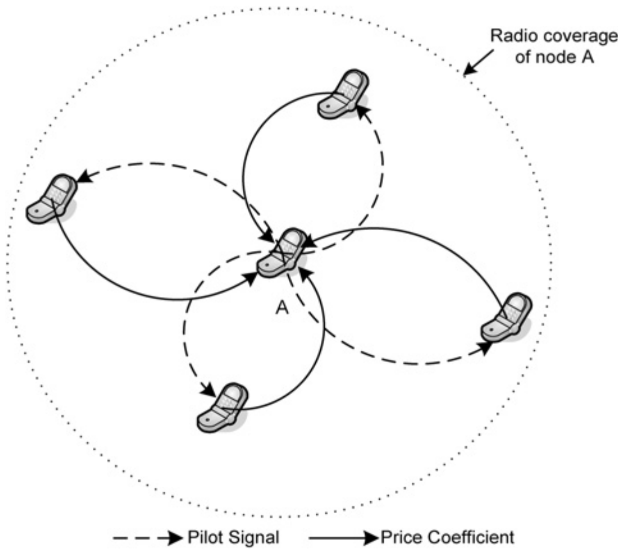

Figure 2 *Information exchange among network users*

However, since there is no rule governing the users in the NPcGP for price selection, a selfish user may attempt to reduce the interference level caused by other users by broadcasting high price coefficients and instructing them to transmit at low powers. To avoid this undesirable outcome, we propose a NPrG to administer proper price coefficients selection. The NPrG can be modelled as a scenario of the PD. We show in the next section that if the NPrG is played in a way similar to the IPD, all users will eventually cooperate and broadcast the correct price coefficients.

4 Non-cooperative price game

In this section, the NPrG is proposed as another game to govern the NPcGP users in selecting the price coefficients correctly. The price coefficients selected are broadcast and fed into the NPcGP for computing the total charging price to be imposed on every user. On the one hand, each user broadcasts a set of price coefficients to charge other users.

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On the other hand, each user also receives a set of price coefficient from other users.

In general, the NPrG can be denoted as NPrG $= [\mathcal{N}^{\text{Pr}}, \{S_i\}, \{U_i^{\text{Pr}}\}]$ where $\mathcal{N}^{\text{Pr}} = \{1, 2, \ldots, N\}$ is the player set, $S_i = [0, s_i^{\max}]^{N-1}$ is a multi-dimensional strategy profile where $s_i^{\max}$ is the maximum price coefficient and U_i^{Pr} is the utility function of the ith user which can be expressed as [4]

$$U_i^{\text{Pr}}(\boldsymbol{s}_i, \boldsymbol{S}_{-i}) = -\sum_{j=1, j\neq i}^{N} (s_i^j - G_{ij}p_j)^2 \tag{14}$$

where s_i^j is the price coefficient broadcast by the ith user to charge the jth user and $\boldsymbol{s}_i = [s_i^j]_{j=1, j\neq i}^{N} \in S_i$, is the price coefficient vector consisting of all price coefficients broadcast by the ith user to different users in the network. $\boldsymbol{S} = [\boldsymbol{s}_1, \ldots, \boldsymbol{s}_N]$ is a set containing the price coefficient vectors of all users while $\boldsymbol{S}_{-i}$ is the vector comprising of all elements in $\boldsymbol{S}$ except the ith element. $G_{ij}p_j$ is the interference produced by the jth user to the ith user and can be determined by the ith user through the measurement of pilot signal strength sent by the jth user. Based on this result, the ith user broadcasts its price coefficient accordingly to maximise (14).

The utility function in (14) is chosen such that if any user untruthfully broadcasts overly high price coefficient, a negative payoff will be obtained. Apparently, maximising the utility function in (14) corresponds to a multi-dimensional task where user strategies are represented by a vector, which can be written as

$$\tilde{\boldsymbol{s}}_i = \arg \max_{s_i^j, \forall j \in \mathcal{N}^{\text{Pr}}, j\neq i} (U_i^{\text{Pr}}(\boldsymbol{s}_i, \boldsymbol{S}_{-i})) \tag{15}$$

where $\tilde{\boldsymbol{s}}_i$ is the best response of the ith user for (15).

In order to avoid signal collisions, we assume that the users transmit their pilot signals in round robin manner over different short dedicated time slots. If we assume that the pilot signals are received at different times by the ith user, $U_i^{\text{Pr}}(\boldsymbol{s}_i, \boldsymbol{S}_{-i})$ is maximised at distinct time for every pilot signal received. Hence, (14) can be simplified to

$$U_i^{\text{Pr}}(s_i^j, \boldsymbol{S}_{-i}) = -(s_i^j - G_{ij}p_j)^2, \quad \forall j \neq i \tag{16}$$

To find the price coefficients for different pilot signals, we differentiate (16) with respect to s_i^j, equate it to zero and obtain s_i^j as follows

$$s_i^j = G_{ij}p_j, \quad \forall j \neq i \tag{17}$$

By finding the second derivative of (16), we have $\partial^2 U_i^{\text{Pr}}(s_i^j, \boldsymbol{S}_{-i})/\partial(s_i^j)^2 < 0$ which means that $U_i^{\text{Pr}}(s_i^j, \boldsymbol{S}_{-i})$ is a strictly concave function of s_i^j. Next, we analyse the existence and uniqueness of NE in the NPrG.

Theorem 3: A unique NE also exists in the NPrG if the NPcGP has a unique NE where both the NPcGP and NPrG achieve their corresponding NE at the same time.

Proof: Let $I_i^{\text{F}}(p_i) = \sum_{j=1, j\neq i}^{N} s_j^i$, we modify (21) and obtain

$$\sum_{j=1, j\neq i}^{N} s_j^i(t) = -\frac{\sum_{j=1, i\neq j}^{N} G_{ij}p_j(t-1) + \sigma_i^2}{\mu} \ln\left(\frac{\beta}{\delta} - \frac{\theta_i}{\delta p_i(t)}\right) \tag{18}$$

It has been proven in Theorems 1 and 2 that a unique NE exists for the NPcGP. In this context, β, δ, θ_i are constant, σ_i^2 is assumed to be negligible and G_{ij} are fixed in one implementation of the algorithm. Therefore $\sum_{j=1, j\neq i}^{N} s_j^i(t)$ only varies with p_i and p_j. When the NPcGP achieves NE, $p_i(t) \simeq p_i(t+1)$, $\forall i \in \mathcal{N}$ and at the same time $\sum_{j=1, j\neq i}^{N} s_j^i(t) \simeq \sum_{j=1, j\neq i}^{N} s_j^i(t+1)$, $\forall i \in \mathcal{N}$. We can conclude that the NPrG algorithm can converge to a unique fixed point if NE exists in the NPcGP. This fixed point corresponds to NE in the NPrG as $U_i^{\text{Pr}}(\boldsymbol{s}_i, \boldsymbol{S}_{-i}) = 0, \forall i \in \mathcal{N}$ (maximum value) when $\sum_{j=1, j\neq i}^{N} s_j^i(t) \simeq \sum_{j=1, j\neq i}^{N} s_j^i(t+1)$, $\forall i \in \mathcal{N}$. □

4.1 Misbehaving users in the NPrG

We next investigate the potential misbehaviours of selfish users in the NPrG who are reluctant to broadcast their price coefficients truthfully. Despite the fact that broadcasting high price coefficients in the NPrG may result in a lower payoff, no physical penalty is imposed on the misbehaving users. Consequently, one may always broadcast high price coefficients such that it receives less payoff in the NPrG but obtains a higher payoff in the NPcGP. This is an undesirable outcome as no users would adhere to rules of the NPrG. In [4], the authors propose to hardwire the algorithm into the mobile handset, making manipulation of price information difficult. In this work, we investigate the conditions that can foster cooperation among users without any hardware configuration.

In this context, we observe that the behaviours of users in the NPrG resemble that of the prisoners in the PD game [9] as summarised in Table 1, where W, X, Y, Z denote different payoffs received by a user based on its action and another

Table 1 Canonical PD payoff matrix of users A and B

		User A	
		Cooperate	Defect
user B	cooperate	(X, X)	(Y, Z)
	defect	(Z, Y)	(W, W)

Remarks: W, X, Y and Z denote different payoffs received by a user depending on its action and another user's action user's action. The following theorem helps in defining the PD game.

Theorem 4: The NPrG is a PD game if $Z > X > W > Y$.

Proof: Let us consider two users (A and B) playing the NPrG as in Table 1. We assume that user A cooperates while user B defects (uncooperative), user A will receive the lowest payoff, Y, because user A is being charged at a very high price by user B. On the contrary, user B will receive the highest payoff, Z, as user A is willing to broadcast the actual price coefficient and decrease its power, thus reducing the interference caused to user B. If both users cooperate, they will receive the same payoff, X, such that $Z > X > Y$ as both pay a reasonable price. But if both defect, they will also receive the same payoff, W, such that $W < X$, because both are being charged at a high price; W is still larger than Y as both playing defect will foster mutual interference reduction, unlike the case in which one user cooperates while another defects. Hence, we conclude that $Z > X > W > Y$ in the NPrG. □

The NE in the PD game is a Pareto-suboptimal solution [9], that is, a rational strategy that leads both users to play defection even though each user would have a higher individual payoff if they both choose to cooperate. Hence, playing the NPrG in a way similar to the PD game results in non-cooperation among users where every user will broadcast high price coefficients to over-charge others. However, in the PD game which is played repeatedly like the IPD game, each user has the opportunity to punish other users for previous non-cooperative play. Eventually, cooperation occurs as an equilibrium outcome.

Theorem 5: If two users play the NPrG more than once in succession (i.e. having the memory of at least one previous game), then rational users that repeatedly interact over indefinitely long games can sustain the cooperative outcome [9].

Proof: We assume that all users have knowledge of the actions taken by other users in the PD game, the best action is called the tit-for-tat strategy [9, 10] in which a user always follows the action of another user as this always leads to the highest payoff. If every user adopts the tit-for-tat strategy, all users will always choose to cooperate rather than to defect. □

From Theorem 5, we conclude that if the NPrG is played in a way similar to the IPD game, the users always choose to cooperate. If a user defects to broadcast high price coefficients untruthfully, it may receive a lower payoff in the NPcGP due to the punishment imposed by others, who may also defect to broadcast incorrect high price coefficients. The only condition is that every user needs to have knowledge of other users' misbehaviour. Hence, if we assume that users' misbehaviours can be detected, all users have to adhere to the rules of the NPrG in order to obtain a higher payoff in the NPcGP.

5 Algorithm for the non-cooperative power-price game and its complexity analysis

The combined NPcGP and NPrG namely non-cooperative power-price game (NPcPrG) is denoted as NPcPrG $= [\mathcal{N}^{\text{Pc}} \cup \mathcal{N}^{\text{Pr}}, \{P_i, S_i\}, \{U_i^{\text{net}}, U_i^{\text{Pr}}\}]$ where every user is a parallel player of the NPcGP and NPrG. They play the games asynchronously using the algorithm illustrated in Fig. 3.

In Fig. 3, the condition $|p_i(t) - p_i(t-1)| < \Delta_p$ must be fulfilled by all users before NE is declared. However, this condition could be accidentally achieved during the course of this game play. To avoid this from occurring, this condition is checked K times before declaring NE.

5.1 Complexity analysis

From Fig. 3, we observe that every user requires price coefficients from other users to update its power level in every iteration. Therefore the pilot signals and price coefficients need to be broadcasted iteratively, which may result in high complexity. Before introducing a possible solution for complexity reduction, we first compare the complexity of the proposed scheme with those proposed in [1, 5].

It can be seen from (21) that to implement the updates using the proposed scheme, the ith user needs to perform the following operations every iteration: (i) broadcast pilot signals

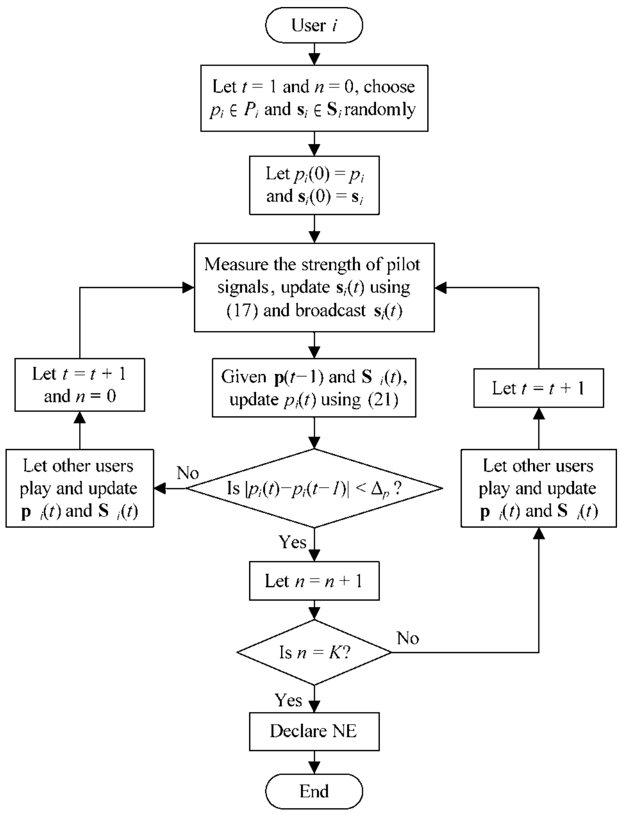

Figure 3 *Flowchart illustrating the NPcPrG algorithm*

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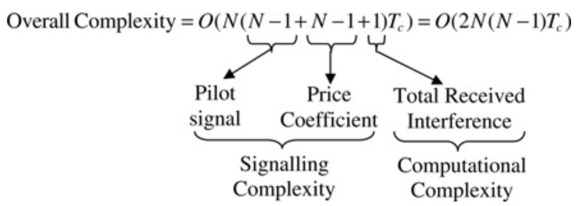
to other $N-1$ users; (ii) measure price coefficients from other $N-1$ users; (iii) measure the total received interference. Hence, the overall signalling and computational complexity of the proposed scheme is



where T_c is the number of iterations needed for the algorithm to converge. In contrast, both the SIR-balancing scheme [1] and the NPcG [5], which only require information of the total received interference to implement the updates, have an overall complexity of $O(NT_c)$. The number of iterations needed for our scheme to converge is about 20–40 depending on the pricing parameters, which approximates that of the SIR-balancing scheme depending on the step size [1].

The high complexity of our scheme is mainly due to the signalling (the game play of the NPrG) in every iteration. This signalling complexity can be reduced by allowing users to estimate the price coefficients instead of measuring them iteratively. From (17), it is observed that the price coefficient is a function of the adjacent channel gain. This gain can be measured by having each receiver periodically sending out a beacon; assuming channel reciprocity, the transmitters can measure these gains [4]. The users shall not manipulate the beacon to gain extra payoff due to the consensus reached in the NPrG. Having the memory of power level for the previous iteration with the assumption that the gain is fixed in one implementation of the algorithm, the power update for a user only necessitates a beacon broadcast and measurement of the beacons (adjacent channel gains) from $N-1$ users in the first iteration as well as the total received interference in every iteration. The broadcast of pilot signals and price coefficients can then be omitted [5] from the flowchart in Fig. 3 after the first iteration. In other words, all users merely play the NPrG in the first iteration. Hence, the overall complexity of the proposed technique can be reduced to $O(N^2 + NT_c)$, which is more feasible for wireless *ad hoc* networks.

6 Simulation results

A simulation of $M = 80$ links is considered where each user transmits isotropically with a variable transmit power, $p_i \le p_i^{max}$ where $p_i^{max} \le 0$ dBm, $\forall i = \{1, \ldots, M\}$. The sender node co-ordinates, $x_{s,i}, y_{s,i} \sim U(-100\,\text{m}, 100\,\text{m})$ and the receiver nodes positions are placed isotropically around their respective sender nodes within a common radius of $R_o = 100$ m. $N \le M$ pairs of users are activated arbitrarily with random initial power levels and price coefficients. A noise floor of -100 dBm is considered, which corresponds to a noise temperature of 300 K and a noise bandwidth of 25 MHz. Furthermore, we use $H = 80$ bits, $L = 64$ bits and $R = 10^4$ bits/s. θ_i is assumed to be unity for all users. Algorithm convergence is declared if $\Delta_p \le 10^{-5}$. Hidden nodes and exposed nodes are ignored.

The convergence of the NPcPrG has been proven mathematically in Theorems 1, 2 and 3. In order to further verify this through simulation, ten pairs of users are randomly activated to play the NPcPrG with $\beta = 1 \times 10^4$, $\delta = 1 \times 10^4$ and $\mu = 1$. Every user updates its power and price coefficients iteratively and asynchronously according to the algorithm in Fig. 3 until convergence is reached. Figs. 4*a* and *c* show that the convergence of power and

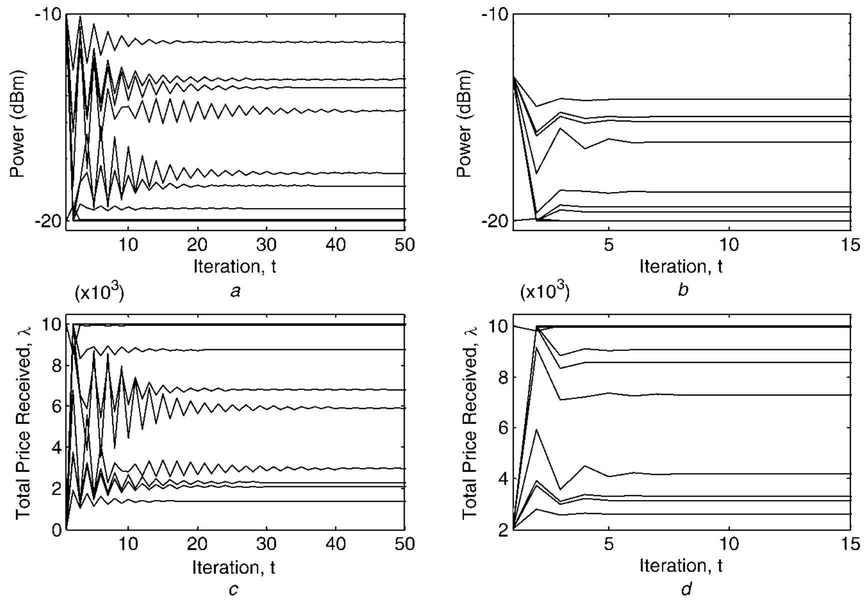


Figure 4 *Convergence of power levels and total price received for ten users where* $\beta = 1 \times 10^4$, $\delta = 1 \times 10^4$ *for sub-figures (a) and (c),* $\beta = 1 \times 10^4$, $\delta = 8 \times 10^3$ *for sub-figures (b) and (d)* μ is 1 for all sub-figures

total price received is achieved after 40 iterations. Subsequently, we keep all parameters constant but decrease δ to 8×10^3. It is noticed from Figs. 4*b* and *d* that by decreasing the price weight, the algorithm can converge faster and users who generate high interference are charged at higher prices. Moreover, the minimum price charged on a user can be deduced as $\beta - \delta$.

By using the previous simulation parameters except with $\mu = 10$, it is shown in Fig. 5 that the NPcPrG can converge faster with more users charged at the maximum price. By increasing μ, all users are forced to transmit at lower power levels as compared to the previous case because they are now more sensitive towards interference and tend to announce higher price coefficients. From Figs. 4 and 5,

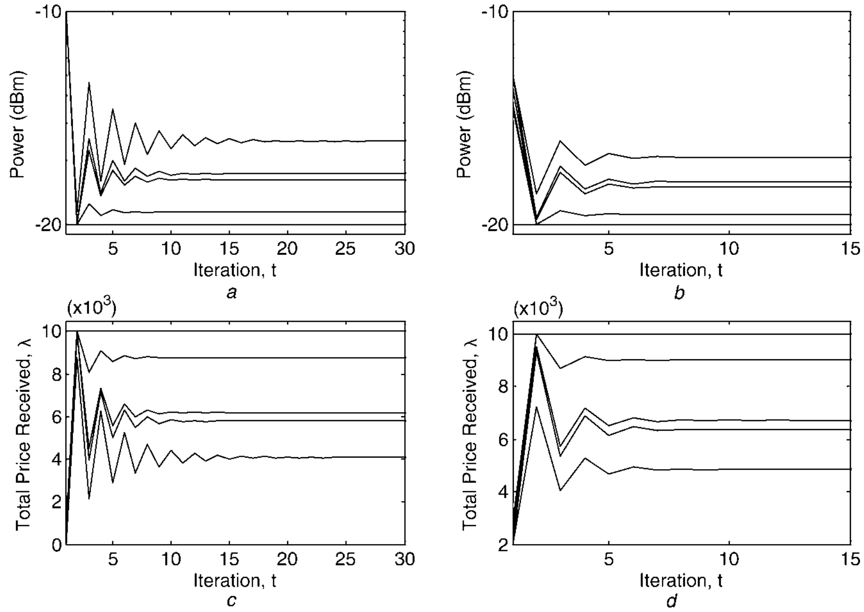


Figure 5 *Convergence of power levels and total price received for ten users where* $\beta = 1 \times 10^4$, $\delta = 1 \times 10^4$ *for sub-figures (a) and (c),* $\beta = 1 \times 10^4$, $\delta = 8 \times 10^3$ *for sub-figures (b) and (d)* μ is ten for all sub-figures

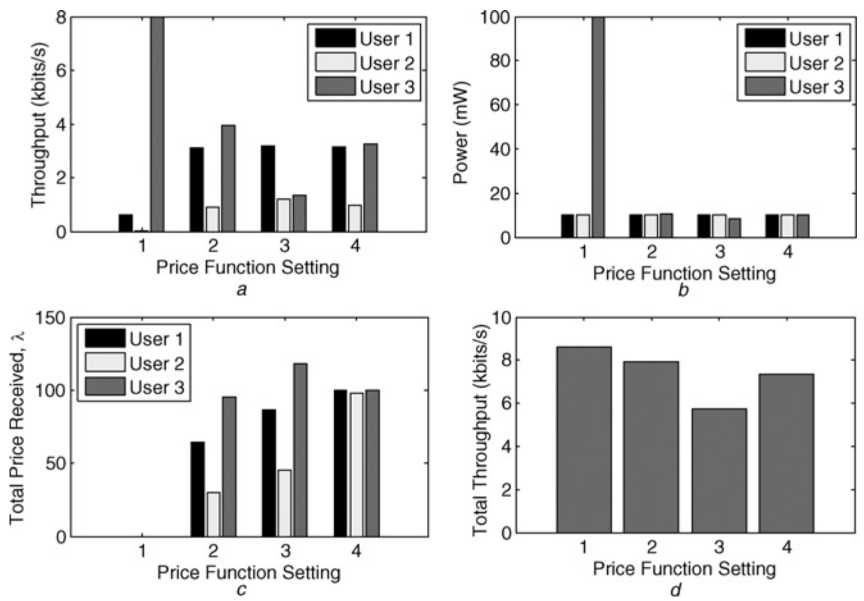


Figure 6 *Comparison of*

a Throughput
b Transmit power
c Total price received
d Total throughput for the motivating example under different parameter settings where at case (i) $\beta = \delta = \mu = 0$, (ii) $\beta = \delta = 100$, $\mu = 1$, (iii) $\beta = \delta = 130$, $\mu = 1$ and (iv) $\beta = \delta = 100$, $\mu = 10$

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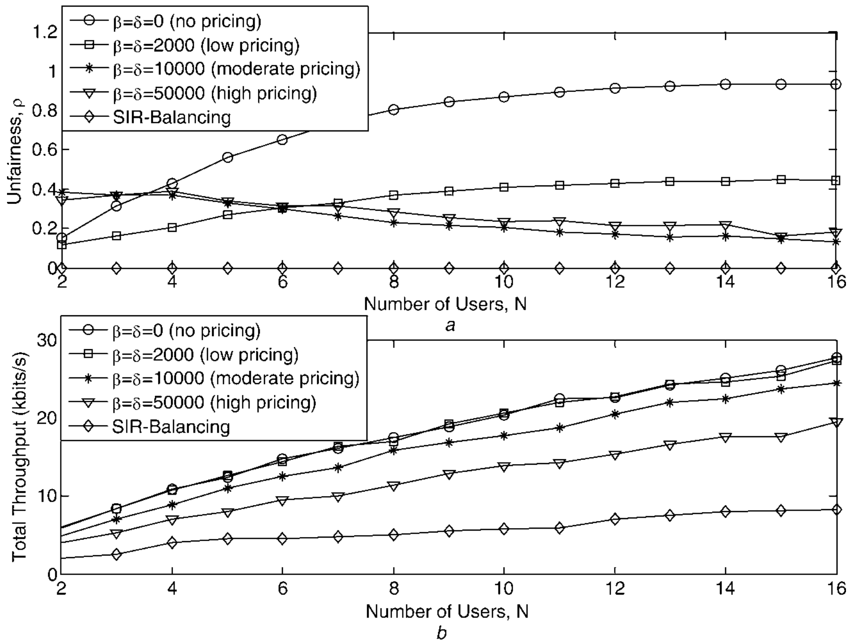

Figure 7 *Unfairness and total throughput comparison with respect to different number of users under different pricing parameter settings in a single-channel network*

it is known that β and δ determine the price range while μ reflects the price to be charged per unit interference. Hence, the NPcPrG is flexible and can be adapted to any environment by tuning β, δ and μ to obtain different desired outcomes.

To illustrate the flexibility of the NPcPrG, we apply this game to the motivating example in Section 2.1. We analyse the throughput share, transmit power, total price received among the three users and the total throughput under different parameter settings. Four different outcomes are shown in Fig. 6. For case (1), the users transmit selfishly as in [4] and only benefit user 3 while user 2 suffers from serious throughput reduction due to high interference from user 3. When a moderate price is imposed as in case (2), user 3 lowers its power thus allowing user 2 a fairer share of the throughput at the expense of a slight total throughput loss as shown in Fig. 6*d*. If a higher price is allowed as in case (3), user 3 is forced to transmit at a lower power which degrades its throughput but user 2 can achieve a higher throughput at the expense of a higher total throughput loss. Similarly, by increasing μ, all users are charged at almost equal prices because they are now more responsive towards interference.

We now investigate the unfairness issue among users and the corresponding total throughput for different number of users. The result is presented in Fig. 7, where it is shown that when no pricing is imposed, the throughput unfairness becomes more severe when the network grows larger (higher interference) but a high total throughput is attained. On the other hand, although fairness can always be ensured in the SIR-balancing scheme, however, this scheme achieves the worst total throughput as compared to the proposed technique under different pricing schemes. If a moderate pricing scheme is incorporated in the NPcPrG, the throughput fairness can be guaranteed in larger networks with a slight throughput loss, but performance degradation is observed in smaller networks. We also show that a continuous increase in price (high pricing scenario) does not help to improve the fairness but leads to higher throughput losses. Therefore the parameters in the pricing scheme should be selected appropriately based on the network environments. A detailed study on optimal selection of pricing parameters is beyond the scope of this work and will be left for future work.

7 Conclusions

In this paper, we investigated the fairness issue arising from resource allocation in wireless *ad hoc* networks. We provided important insights on how the selfishness of autonomous nodes in *ad hoc* networks can lead to throughput unfairness especially in scenarios where users possess different power capabilities. To improve throughput fairness, we propose the NPcPrG in which users are charged based on their generated interference. The game is introduced with the assumption that all users in the network follow the rules of the algorithms. Simulation results show that the proposed NPcPrG converges rapidly to a unique NE where upon reaching this point, the throughput fairness can be ensured among the users at the expense of a slight total throughput loss. The NPcPrG also provides flexibility and adaptivity to ensure fairness in different environments by tuning the parameters of the pricing policy. It is demonstrated through simulation that the proposed algorithm not only provides fair throughput share among users, but it also achieves total throughputs that are comparable to the existing scheme that ignores the fairness issue. Owing to space limitation, optimal selection of pricing parameters will be pursued in a future work.

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9 Appendix

Proof of Theorem 1: As proven in [12], a necessary condition for the existence of NE is

$$\frac{\partial U_i^{\text{net}}(p_i(t), \boldsymbol{p}(t-1))}{\partial p_i(t)} = 0, \quad \forall i \in \mathcal{N}^{Pc} \tag{19}$$

Hence, we have

$$\frac{\theta_i}{p_i(t)} - \left(\beta - \delta \exp\left(-\mu \frac{I_i^{\text{F}}(p_i(t-1))}{I_i^{\text{R}}(\boldsymbol{p}_{-i}(t-1))}\right)\right) = 0 \tag{20}$$

Substituting (12) into (20) yields

$$p_i(t) = \theta_i \left(\beta - \delta \exp\left(-\frac{\mu p_i(t-1)\sum_{j=1, j\neq i}^{N} G_{ji}}{I_i^{\text{R}}(\boldsymbol{p}_{-i}(t-1))}\right)\right)^{-1} \tag{21}$$

All players in the NPcGP update their powers according to (21) until NE is reached. If NE exists in the NPcGP, $p_i(t) = p_i(t-1) = p_i$ at the end of the power update and (21) is reduced to a transcendental equation. In order to solve a transcendental equation, a graphical method can be applied to observe if a solution exists for this equation. By rearranging (21), we obtain

$$\frac{p_i}{\beta p_i - \theta_i} = \frac{1}{\delta} \exp\left(\frac{\mu \sum_{j=1, j\neq i}^{N} G_{ji}}{I_i^{\text{R}}(\boldsymbol{p}_{-i})} p_i\right) \tag{22}$$

Let us define $\Psi_{\text{LHS}}(\xi) = \xi/(\beta\xi - \theta_i)$ for the function on the left-hand side (LHS) of (22) and $\Psi_{\text{RHS}}(\xi) = (1/\delta)\exp(\varphi\xi)$ where $\varphi = \mu(\sum_{j=1, j\neq i}^{N} G_{ji})/I_i^{\text{R}}(\boldsymbol{p}_{-i}) > 0$ for the function on the right-hand side (RHS) of (22). The solution to (22) is p_i which satisfies $\Psi_{\text{LHS}}(\xi) = \Psi_{\text{RHS}}(\xi)$. We illustrate the solution to this equation by plotting the curve of $\Psi_{\text{LHS}}(\xi)$ against ξ and $\Psi_{\text{RHS}}(\xi)$ against ξ in Fig. 8. It is observed from Fig. 8 that the curve of $\Psi_{\text{LHS}}(\xi)$ against ξ is split into two parts centred at $1/\beta$. For $\xi > 1/\beta$, $\Psi_{\text{LHS}}(\xi)$ monotonically decreases from ∞ to zero while for $\xi < 1/\beta$, $\Psi_{\text{LHS}}(\xi)$ monotonically decreases from zero to $-\infty$. On the other hand, $\Psi_{\text{RHS}}(\xi)$ increases exponentially with a vertical intercept at $1/\delta$. From Fig. 8, we know that the curve of $\Psi_{\text{RHS}}(\xi)$ against ξ does not intersect with that of $\Psi_{\text{LHS}}(\xi)$ against ξ for $\xi < 1/\beta$. However, since $\Psi_{\text{LHS}}(\xi)$ reaches ∞ at $1/\beta$ for $\xi > 1/\beta$ which is higher than the vertical intercept of $\Psi_{\text{RHS}}(\xi)$, the two curves shall intersect at one and only one point at 'a' as shown in Fig. 8, indicating that there is a unique global maximum. Hence, the existence of a NE is proven. □

Proof of Theorem 2: Let us define the update rule for all the users as $\boldsymbol{p}(t) = f(\boldsymbol{p}(t-1))$ where $\boldsymbol{p}(t) \in \Re^N$,

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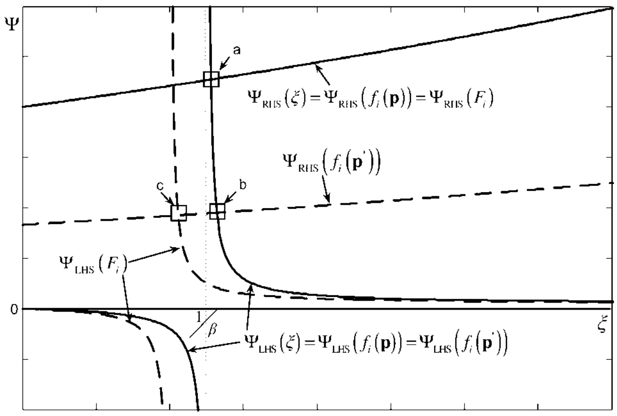

Figure 8 *Illustration of equilibrium power to prove the existence of NE, monotonicity and scalability*

$f(\bullet) \in \Re^N$ is the mapping function corresponding to the update rule. If the update rule function $f(\bullet)$ of the NPcGP algorithm satisfies three properties, namely positivity $(f(p) > 0)$, monotonicity $(p' > p \Rightarrow f(p') > f(p))$ and scalability $(f(\varepsilon p) < \varepsilon f(p), \forall \varepsilon > 1)$, then the NPcGP will converge to a fixed point which is unique [12]. From (21), since $\theta_i > 0$, positivity for $f(p)$ requires

$$\beta \geq \delta \exp\left(-\frac{\mu p_i \sum_{j=1,j\neq i}^{N} G_{ji}}{I_i^{\mathrm{R}}(p_{-i})}\right) \tag{23}$$

Since the maximum value of the exponential term is equivalent to 1, we can simplify the condition in (23) for positivity if $\beta \geq \delta$. Therefore in order to ensure positivity for $f(p)$, β should always be greater than δ. The monotonicity property can be proven with the help of Fig. 8. First, by replacing p_i in (22) with $f_i(p)$, we obtain

$$\frac{f_i(p)}{\beta f_i(p) - \theta_i} = \frac{1}{\delta}\exp\left(\frac{\mu \sum_{j=1,j\neq i}^{N} G_{ji}}{I_i^{\mathrm{R}}(p_{-i})} f_i(p)\right) = \frac{1}{\delta}\exp(\varphi f_i(p)) \tag{24}$$

Assume that $f_i(p)$ is the intersection point 'a' in Fig. 8 which is the solution to (24) satisfying $\Psi_{\mathrm{LHS}}(f_i(p)) = \Psi_{\mathrm{RHS}}(f_i(p))$. An increase in p to $p' > p$ does not affect the curve of $\Psi_{\mathrm{LHS}}(\xi)$ against ξ. However, an increase in p to $p' > p$ results in

$$p' > p \Rightarrow I_i^{\mathrm{R}}(p'_{-i}) > I_i^{\mathrm{R}}(p_{-i}) \Rightarrow \varphi(p'_{-i}) < \varphi(p_{-i}) \Rightarrow \Psi_{\mathrm{RHS}}(f_i(p')) < \Psi_{\mathrm{RHS}}(f_i(p))$$

By referring to Fig. 8, an increase in p to $p' > p$ results in the curve of $\Psi_{\mathrm{RHS}}(f_i(p'))$ against ξ being lowered down below the curve of $\Psi_{\mathrm{RHS}}(f_i(p))$ against ξ. Thus, the $f_i(p')$ that satisfies $\Psi_{\mathrm{LHS}}(f_i(p')) = \Psi_{\mathrm{RHS}}(f_i(p'))$ is the intersection point 'b' in Fig. 8, which is located at the RHS of point 'a', indicating $f_i(p') > f_i(p)$. Hence, the monotonicity property is proven. With the aid of Fig. 8, the scalability property can be proven by first showing the following inequalities

$$\sum_{i\neq j,i=1}^{N} G_{ij}\varepsilon p_j + \sigma_i^2 < \sum_{i=1,i\neq j}^{N} G_{ij}\varepsilon p_j + \varepsilon\sigma_i^2, \quad \forall \varepsilon > 1$$
$$I_i^{\mathrm{R}}(\varepsilon p_{-i}) < \varepsilon I_i^{\mathrm{R}}(p_{-i}), \quad \forall \varepsilon > 1 \tag{25}$$

Substituting p and $f(p)$ in (24) with εp and $f(\varepsilon p)$, respectively, we obtain

$$\frac{f_i(\varepsilon p)}{\beta f_i(\varepsilon p) - \theta_i} = \frac{1}{\delta}\exp\left(\frac{\mu \sum_{j=1,j\neq i}^{N} G_{ji}}{I_i^{\mathrm{R}}(\varepsilon p_{-i})} f_i(\varepsilon p)\right) \tag{26}$$

Referring to (25), we can obtain the following inequality

$$\frac{1}{\delta}\exp\left(\frac{\mu \sum_{j=1,j\neq i}^{N} G_{ji}}{I_i^{\mathrm{R}}(\varepsilon p_{-i})} f_i(\varepsilon p)\right) > \frac{1}{\delta}\exp\left(\frac{\mu \sum_{j=1,j\neq i}^{N} G_{ji}}{\varepsilon I_i^{\mathrm{R}}(p_{-i})} f_i(\varepsilon p)\right) \tag{27}$$

Using (26) and (27), the following inequality is obtained

$$\frac{\varepsilon(f_i(\varepsilon p)/\varepsilon)}{\varepsilon\beta(f_i(\alpha p)/\varepsilon) - \theta_i} > \frac{1}{\delta}\exp\left(\frac{\mu \sum_{j=1,j\neq i}^{N} G_{ji}}{I_i^{\mathrm{R}}(p_{-i})}(f_i(\varepsilon p)/\varepsilon)\right) \tag{28}$$

Let $F_i = f_i(\varepsilon p)/\varepsilon$, the inequality in (28) is transformed to

$$\frac{\varepsilon F_i}{\varepsilon\beta F_i - \theta_i} > \frac{1}{\delta}\exp(\varphi F_i) \tag{29}$$

Let F_i be the solution which satisfies $\Psi_{\mathrm{LHS}}(F_i) = \Psi_{\mathrm{RHS}}(F_i)$ for (29) and assume $\Psi_{\mathrm{LHS}}(F_i)$ against ξ and $\Psi_{\mathrm{RHS}}(f_i(p))$ against ξ share the same curve as shown in Fig. 8. On the LHS, the curve of $\Psi_{\mathrm{LHS}}(F_i)$ against ξ is lowered down below the curve of $\Psi_{\mathrm{LHS}}(f_i(p))$ against ξ due to the extra term, $\varepsilon > 1$. Hence, the solution to $\Psi_{\mathrm{LHS}}(F_i) = \Psi_{\mathrm{RHS}}(F_i)$ is point 'c' in Fig. 8 which is located at the left side of point 'a'. We conclude that

$$F_i < f_i(p) \Rightarrow f_i(\varepsilon p)/\varepsilon < f_i(p), \quad \forall \varepsilon > 1 \tag{30}$$

This proves the scalability property. Since the three properties are satisfied, NE that exists in the NPcGP is unique. □