Effect of Selfish Node Behavior on Efficient Topology Design

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Abstract—The problem of topology control is to assign per-node transmission power such that the resulting topology is energy efficient and satisfies certain global properties such as connectivity. The conventional approach to achieve these objectives is based on the fundamental assumption that nodes are socially responsible. We examine the following question: if nodes behave in a selfish manner, how does it impact the overall connectivity and energy consumption in the resulting topologies? We pose the above problem as a noncooperative game and use game-theoretic analysis to address it. We study Nash equilibrium properties of the topology control game and evaluate the efficiency of the induced topology when nodes employ a greedy best response algorithm. We show that even when the nodes have complete information about the network, the steady-state topologies are suboptimal. We propose a modified algorithm based on a better response dynamic and show that this algorithm is guaranteed to converge to energy-efficient and connected topologies. Moreover, the node transmit power levels are more evenly distributed, and the network performance is comparable to that obtained from centralized algorithms.

Index Terms—Game theory, selfish nodes, topology control, network connectivity, power efficiency, ad hoc networks.

1 Introduction

1.1 Preliminaries

With widespread proliferation of mobile, portable, communication, and computing devices, continuing advancements in technology, and increasing demand for ubiquitous connectivity, there is an overwhelming interest in ad hoc network research. These networks consist of autonomous, independent, and heterogeneous devices, which communicate wirelessly in a multihop manner without a fixed infrastructure.

Energy efficiency is one of the most crucial requirements in ad hoc networks [1]. Nodes are equipped with radios, memory, and processors, all of which are often powered by a battery. Hence, it is imperative that every node be energy efficient: this not only increases the node's own operational lifetime but also contributes to an overall increase in the network performance. Thus, energy is a limiting factor for desirable network performance.

Ad hoc networks are typically communication oriented; therefore, their performance can be improved by developing energy-efficient protocols. Transmitting with low power is one way of increasing energy efficiency.[1] *Topology Control*—the study of how to assign per-node transmission power level so as to achieve certain network-wide goals—is one such design technique used to enhance global network performance. Here, an ad hoc network is often abstracted as a graph consisting of a set of vertices representing wireless devices (nodes) and an edge set containing all potential communication links between these nodes. (We describe a formal model in Section 3.) A topology control (TC) algorithm then dynamically assigns per-node transmission powers that determine an optimal set of neighbors for each node; the resulting set of feasible communications forms a transmission digraph. The purpose of TC is to generate graphs that are "efficient" and satisfy certain properties such as connectivity and energy efficiency: attributes related to global network performance. By reducing the transmission power level of nodes, usually to much smaller values compared to their maximum levels, TC helps build energy-efficient networks and thereby prolong network lifetimes. For an elaborate survey on the subject of TC, we refer the reader to [3] and [4].

1. Low transmission power also promotes spatial reuse, potentially leading to higher end-to-end network throughput [2].

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1.2 Motivation

Most protocols and design paradigms for ad hoc networks are based on the fundamental assumption that nodes cooperate in order to simply establish a network, let alone to achieve better network performance. These networks are inherently distributed and controlled by end users. From a user's perspective, it is difficult to justify the cooperative assumption because nodes are either competing for network resources (for example, bandwidth) or conserving their own limited resources (for example, battery lifetime). For instance, why would a node choose to forward packets along its next-hop interface and drain its battery resource when it has no incentive for doing so?[2] In this scenario,

2. From a broader perspective, it may be in the best interest of nodes to cooperate with each other and do well; however, each node faces a temptation to "defect" and increase its payoff at the expense of other nodes and network performance in general. Such problems are classified as social dilemma games [5].

nodes may behave in exactly the opposite way: conserve their resources and act in their self-interest. In some sense, the problem is further exacerbated by network heterogeneity, where user objectives may conflict. These issues pose serious questions regarding the applicability of cooperative algorithms for ad hoc networks and may render the solution they provide infeasible.

In pursuit of conserving energy, nodes in an ad hoc network can be *selfish* and may act in their self-interest. This is further substantiated by the lack of a centralized controller (such as a base station in cellular systems) in these systems to enforce node cooperation. Taking such selfish node behavior into consideration, we model the interactions between nodes as a *game* and analyze the TC problem as a noncooperative game.

Several TC algorithms (for example, [6], [7], [8], and [9]) have been proposed to create power-efficient topologies. These algorithms are based on the underlying assumption that communicating agents are altruistic, and they collectively optimize power to achieve the desired global objective of network connectivity. While these assumptions may hold in some applications where nodes are controlled by a single administrative entity, they may not hold in commercial applications or in competing environs. We relax these assumptions and formulate the problem as a noncooperative game, where each node selfishly maximizes its individual utility. Modeling systems based on selfish algorithms have been shown to work well and improve the performance of ad hoc networks [10]. The focus of this work is on *developing a distributed algorithm to create topologies that are globally energy efficient.* This work is a follow-up to [11], where the authors guarantee local energy efficiency of the steady-state topologies but not global efficiency. While centralized algorithms that create energy-efficient networks exist [8], the task of developing a distributed algorithm for the same is nontrivial. This becomes even more challenging when the network consists of selfishly motivated nodes; there is little work that encompasses all these aspects of the problem. Nonetheless, the problem is of great practical importance and has also been underscored in [12], which provides an excellent motivation to consider selfish behavior in topology design.

1.3 Contribution

We consider the TC game and the *Max-Improvement Algorithm* (MIA)[3] developed in [11]. The utility function of the game specifies that nodes have enough incentive 1) to establish and maintain connectivity with a sufficient number of neighbors and 2) to ensure that the network does not partition. In [11], it has been shown that this TC game is a potential game. The potential game formulation guarantees the existence of a Nash equilibrium (NE) [13], as well as the convergence of MIA to an NE [11] (because the action space is compact). The TC game admits many NEs; which NE topologies emerge depend on the order in which nodes update their strategies under MIA. We prove that only a subset of these NE topologies is desirable from an energy-efficiency standpoint. Specifically (as shown in Section 5), every NE is locally energy efficient, but only a subset of these is globally energy efficient.

We propose a δ-*Improvement Algorithm* (DIA), where each node makes small decrements in its power level if the change improves its utility; otherwise, the node reverts to its previous power level. First, we prove that under DIA, the induced topologies are energy efficient and preserve network connectivity. Second, a main drawback of MIA is that, being greedy, it leads to a biased steady-state power-level distribution. Following DIA, the transmit power distribution is much fairer than that produced by MIA. We point out that the issue of whether or not a given power-level allocation is fair has received little attention in the domain of TC problems. In general, there may be a fundamental conflict between an efficient allocation and a fair allocation.

1.4 Organization

The rest of the paper is structured as follows: Section 2 overviews key concepts of game theory and potential games as applicable to our problem. Section 3 presents our network model, assumptions, and definitions used throughout this paper. Section 4 formalizes the TC algorithms that account for selfish node behavior. Section 5 analyzes our game-theoretic model and algorithms and discusses the results of this paper and their implications. Section 6 validates our model through simulations. Section 7 briefly reviews the related work. Section 8 presents concluding remarks.

2 Game Theory and Potential Games

In this section, we present a brief overview of important elements and notations of noncooperative strategic-form game theory. For a rigorous treatment of these and other topics in game theory, refer to [14]. Here, we specifically focus on potential games.

The main object of game-theoretic study is the *game*, which is a formal model of an interactive decision-making situation. A strategic non-cooperative game $\Gamma = \langle N, A, u \rangle$ has three components:

1. Player set $N : N = \{1, 2, \ldots, n\}$, where n is the number of players in the game.
2. Action set $A : a \in A = \times_{i=1}^{n} A_i$ is the space of all action vectors (tuple), where each component a_i of the vector a belongs to the set A_i, the set of actions of player i. Often, we denote an action profile $a = (a_i, a_{-i})$, where a_i is player i's action, and a_{-i} denotes the actions of the other $n-1$ players. Similarly, $A_{-i} = \times_{j \neq i} A_j$ is used to denote the set of action profiles for all players except i.
3. For each player $i \in N$, utility function $u_i : A \rightarrow \mathbb{R}$ models his or her preferences over action profiles. $u = (u_1, \ldots, u_n) : A \rightarrow \mathbb{R}^n$ denotes the vector of such utility functions.

NE is the most prevalent and an important equilibrium concept in noncooperative strategic-form game theory. This solution concept is defined as a stable point because no

3. This was called the Best Response algorithm in [11]. However, since the algorithm is initialized to the maximum power topology, it is a specific instance of a general best response scheme.

player has any incentive to unilaterally change his or her action from it.

Definition 1. *An action profile* $a^* = (a_i^*, a_{-i}^*)$ *is an* NE *if* $\forall i \in N$ *and* $\forall a_i \in A_i$

$$u_i(a^*) \geq u_i(a_i, a_{-i}^*). \tag{1}$$

A game may possess a large number of NEs or none at all. Some classes of games are known to possess at least one NE.

Definition 2. *A strategic game* $\Gamma = \langle N, A, u \rangle$ *is an* ordinal potential game *(OPG) if there exists a function* $V : A \rightarrow \mathbb{R}$ *such that* $\forall i \in N$, $\forall a_{-i} \in A_{-i}$, *and for all* a_i, $b_i \in A_i$

$$V(a_i, a_{-i}) - V(b_i, a_{-i}) > 0 \Leftrightarrow u_i(a_i, a_{-i}) - u_i(b_i, a_{-i}) > 0. \tag{2}$$

V is called the ordinal potential function (OPF) of Γ.

In essence, an OPG requires payoffs that exhibit the same "directional" behavior, when that individual unilaterally deviates.

Potential games with compact action spaces are known to possess at least one NE in pure strategies [13]. The following lemma due to [13] establishes how NEs of the game can be identified:

Lemma 1. *Let* Γ *be an OPG and* V *be its corresponding OPF. If* $a \in A$ *maximizes* V, *then it is an NE.*

Thus, potential maximizers form a subset of the NE of a potential game. If we can identify potential functions for a game, we can immediately identify some NE of the game by solving for the potential maximizers.

Potential games also exhibit certain convergence properties that we make use of in our algorithms (see Section 5). Rather than delving into these technical details, we simply refer to [13] and [15] for a primer. We will revisit and address the issue of convergence to NE in the following sections.

3 Framework and Assumptions

3.1 Network Model

The wireless medium is subject to losses like fading and multipath effects; therefore, it is desirable to have link-level acknowledgments for packets received. Link bidirectionality is also crucial for proper functioning of MAC protocols such as 802.11 [16]. Hence, we assume that links in our ad hoc network model must be bidirectional in order to be useful. Also, our focus is on single-channel networks. As a consequence, wireless channels are characterized by interference between nearby transmissions. We suppose that a MAC protocol ensures the temporal separation of conflicting transmissions and disregard interference in our model. In our current ongoing research, we explicitly model interference and design efficient interference-aware topologies using coloring techniques [17].

For our model, let the network consist of heterogeneous nodes embedded in a 2D planar region $\mathbb{R}^2$. Each node may have different maximal power $p_{i,\max}$, allowing asymmetries in the network. It is then convenient to represent an ad hoc network as a graph $H = (N, E, \Omega)$ consisting of a set of nodes $N = \{1, \ldots, n\}$ and an edge set $E \subseteq N^2 = N \times N$. An edge between any two nodes represents an abstraction of the communication link between them. Let $\Omega = [\omega_{ij}]$ be a matrix of edge weights with the weight function $\omega : E \rightarrow \mathbb{R}^+$, where $\omega(i,j)$ is the power required to close a link $ij \equiv (i,j) \in E$. We leave $\omega(i,j)$ unspecified since the exact threshold is a function of channel attenuation and internodal separation; as such, our model is generalized to accommodate varying channel characteristics. Following an adjustable power model, each node can adapt its transmission power appropriately and select a set of neighbors. The transmit level determines the transmission range of a node; a necessary (but not sufficient) condition for node i to hear node j is that it be within the range of node j. In other words, the transmission level $p : N \rightarrow \mathbb{R}^+$ such that $p(i) = p_i \geq \omega(i,j)$ determines the subset of edges $E' \subseteq E$ that are supported. Likewise, a bidirectional link ij exists if the power setting at i is sufficient to meet the signal-to-noise ratio (SNR) threshold[4] at receiver j, in the absence of any interferer, and vice versa. Thus, given Ω, a bidirectional link (i,j) exists if and only if $\min\{p_i, p_j\} \geq \max\{\omega(i,j), \omega(j,i)\}$. The collection of all such bidirectional links results in a subgraph $G = (N, E')$ of H, called a *transmission graph*, that contains edges (i,j) if j is present in i's transmit range and vice versa. With a slight abuse of notation, we use G to represent the set of all possible graphs generated by various power assignments p; and $g(p)$ is a typical element in G.

More precisely, for each node i, define a link state variable l_{ij} as

$$l_{ij}(p_i) = \begin{cases} 1 & \text{iff } p_i \geq \omega(i,j), \\ 0 & \text{otherwise.} \end{cases} \tag{3}$$

When node i broadcasts with a transmission power p_i, it forms a neighborhood containing every node that is within its transmission range. Due to the broadcast nature of the wireless medium, each node can obtain its neighborhood information by broadcasting "hello" beacon messages at a certain power level and by gathering the ACK replies. The hello messages should at least include the node's identification, current transmission power, and maximal transmission power. Let $\mathcal{N}_i(p_i) = \{j \mid l_{ij}(p_i) \cdot l_{ji}(p_j) = 1\}$ be the set of (direct) neighbors of node i. The collection of such neighborhoods forms a topology on N. In other words, the joint transmit power-level profile $\mathbf{p} = (p_1, \ldots, p_n)$ induces a network, given by

$$g(\mathbf{p}) = \{ij \mid l_{ij}(p_i) \cdot l_{ji}(p_j) = 1; i \neq j \in N\}. \tag{4}$$

We denote the above network, in short, by g_p. Also, note that the inclusions $ij \in g_p$, $j \in \mathcal{N}_i$, and $i \in \mathcal{N}_j$ are all equivalent. If every node i transmits at $p_{i,\max}$, we call the induced topology $g_{\max}$ a *maximum power network.* Because our model acknowledges only bidirectional links, g_p is connected if and only if there exists a bidirected path—a

4. We assume that when a node j receives a message from node i, it knows the power and SNR with which the message was received. The received power is much lower than the transmit power, following the model of radio signal attenuation in space. Using this information, $\omega(i,j)$ can be estimated.

collection of contiguous bidirectional links—between every node pair $i, j \in N$.

Assumption 1. $g_{\max}$ *is a connected network.*

The objective of our distributed TC algorithm is then to derive a subgraph g_p of $g_{\max}$ that is energy efficient and preserves the connectivity of $g_{\max}$.

In the literature, energy efficiency has been defined in different ways: minimizing the maximum transmission power, minimizing the sum of radii, or maximizing energy stretch factor (for a definition, see [4]). Throughout this paper, we use the following definitions of energy efficiency:

Definition 3. *A connected network* g_p *is said to be* locally energy efficient *if no node can reduce its transmit power level without disconnecting the network.*

Definition 4. *A connected network* g_p *is said to be* minmax energy efficient *if* $\max_{i \in N} p_i$ *is minimized.*

Definition 5. *A connected network* g_p *is said to be* globally energy efficient *if* $\sum_{i \in N} p_i$ *is minimized.*

3.2 Game Mapping

Here, we formally describe the TC process as a normal-form game. Individual radios form the player set $N = \{1, 2, \dots, n\}$ of the game. Each radio can autonomously set its transmit power level $p_i \in [0, p_{i,\max}]$. The individual power levels can be collected into a power vector $\mathbf{p} = (p_1, p_2, \dots, p_n)$, which forms the action space A for the game. The power vector induces a topology g_p, which is a collection of feasible links, as defined by (3) and (4).

Let $G = \{g_p \mid \mathbf{p} \in \times_{i=1}^{n} [0, p_{i,\max}]\}$ denote the collection of all possible networks that can be generated by power vectors $\mathbf{p}$. Note that for all $g_p \in G$, $g_p \subseteq g_{\max}$. Each node perceives a trade-off between the benefit it derives from a connected topology g_p and the cost it incurs in establishing g_p. A utility function captures these trade-offs and maps the power vector to a payoff for each node. For every $i \in N$, the utility function u_i is expressed by

$$u_i(\mathbf{p}) = \varphi_i(g(\mathbf{p})) - \chi_i(p_i). \tag{5}$$

Here, $\varphi_i : G \to \mathbb{R}$ represents the benefit node i derives from network g, and χ_i is the cost incurred. In the context of network connectivity, each node perceives a benefit in being connected and, therefore, in being able to establish communication sessions with other nodes in the network. The specific utility function we adopt is discussed in Section 4.

4 Topology Control Algorithms

We propose two TC algorithms for wireless ad hoc network formation in the presence of self-interested nodes: MIA and DIA. Both algorithms consist of three phases: an initialization phase, an adaptation phase, and an update phase. The two algorithms primarily differ in how the adaptation phase is implemented. In MIA, nodes adapt their transmit levels according to a "greedy" best response process. Under DIA, nodes adapt their transmit levels according to a "restrained" better response process.

Given these preliminaries, we formalize the initialization, adaptation, and update phases as follows:

1. (Initialization) Each node i transmits at its maximum power level $p_{i,\max}$ and discovers its neighborhood $\mathcal{N}_i(p_{i,\max})$; the induced topology is $g(\mathbf{p}_{\max}) = g_{\max}$.
2. (Adaptation) Node i, selected via some sequential order, improves its utility (given by (5)) by adjusting its power setting from $p_{i,\max}$—according to a best or better response adaptation process—to a value $p_i \le p_{i,\max}$.
3. (Update) The neighborhood of i $\mathcal{N}_i(p_i)$ is recomputed, and the induced topology $g(p_i, p_{-i})$ is updated for the new power setting.
4. Repeat steps 2 and 3 until no node revises its power setting in a given round.

We now elaborate on each of these phases.

4.1 Initialization Phase

Every node initializes its power setting to $p_{i,\max}$. Each node then discovers its neighborhood by broadcasting neighbor request messages at $p_{i,\max}$ and collecting the responses provided by the receivers at $p_{j,\max}$. Upon successful reception of ACKs from each responding neighbor j, node i sets its link state variable l_{ij} to 1 according to (3). The collection of all such individual neighborhoods defines the initial topology $g_{\max}$.

4.2 Adaptation Phase

In this phase, each node is chosen from a permutation—round-robin or random—to determine its transmission power. All nodes execute either MIA or DIA during the course of game. We emphasize that only one node adapts its power setting at a time.[5] If a node alters its power setting, other nodes are made aware of this adaptation through control messages. In Section 5, we discuss the outcome of the TC game when these update strategies are implemented.

4.2.1 MIA Adaptation

Each iteration of the game can be viewed as a normal-form game, wherein every node chooses to maximize its utility in that iteration. This iterative process allows the network topology to evolve dynamically. In a best-response-based algorithm, whenever a node has an opportunity to revise its power setting, it chooses a transmit level that maximizes its utility (5), given the transmit levels of all other nodes, according to

$$\hat{p}_i = \arg\max_{q_i \in A_i} u_i(q_i, p_{-i}). \tag{6}$$

4.2.2 DIA Adaptation

For the ease of exposition, we discretize the action space. Intuitively, it is sufficient to search for the optimum action over those power values that correspond to the power

5. This is reasonably justified because, in a practical setting, the probability of any two nodes updating their strategies at the same time instant is zero. To realize this restriction, one can imagine nodes embedded by a random timer; nodes update their strategies whenever the timer goes off. Alternately, a token passing scheme, as part of the protocol, can also serve the purpose.

thresholds entries of Ω. This requires each node to maintain per-neighbor power levels and may necessitate modifications at the MAC layer. Instead of introducing additional complications, we form a modified set $\tilde{A}$ that consists of a finite number of power levels, common for all nodes. We envision the network-interface-card (NIC) hardware to only be capable of power control in such discrete steps.

For each node $i \in N$, define a modified action set as

$$\tilde{A}_i = \left\{ p_{\max} = p^{(0)}, p^{(1)}, \ldots, p^{(\zeta)} = p_{\min} \right\}, \quad (7)$$

where $\tilde{A}_i$ is an ordered set, that is, $p^{(k)} < p^{(k-1)}$. (In Section 5, we show that maintaining network connectivity is always a better response strategy; therefore, $\exists \zeta$ (and thus a $p_{\min} \neq 0$) such that $p_i \geq p^{(\zeta)} \forall i$ is a necessary (though not sufficient) condition to ensure connectivity.) One way to construct $\tilde{A}_i$ is to let the transmit level of all nodes be initialized to the $p_{\max}$ that guarantees connectivity with sufficiently high probability [18] and decrement power in steps of a predefined step size δ. (In Section 5, we show that when a sufficiently small δ is chosen, DIA converges to an energy-efficient state. Because A is a compact set, such a $\delta > 0$ (as a function of node density) can always be chosen).

Under DIA, each node i chooses a power level one level lower[6] than its current level if the chosen action gives a better payoff than its current action. Otherwise, the node reverts to the power level it was currently transmitting at. More concisely, let $p_i^{(k)}$ be the current level at which node i is transmitting, $k = 0, 1, \ldots, \zeta - 1$. Given the transmit level of all other nodes, each node chooses to transmit next at a level given by

$$\tilde{p}_i = \underset{q_i \in \left\{ p_i^{(k+1)}, p_i^{(k)} \right\}}{\arg\max} \; u_i(q_i, p_{-i}). \quad (8)$$

In some sense, nodes are more aggressive when following MIA; whereas nodes following a DIA adaptation process are more restrained when improving their payoffs. These contrasting selfish behaviors lead to significantly different steady-state outcomes. In the context of potential games, these two simple adaptive processes are assured to converge; the latter goes one step further and aligns node-centric objectives to network-level goals (we formally prove this in the Section 5).

4.3 Update Phase

The nodes' choice of power level in each iteration redefines its neighborhood; this, in turn, modifies the overall topology. Once a particular node adapts its power level to the current topology, it broadcasts its current power-level information. Upon receiving these control messages, other nodes update their respective link state tables. In turn, these nodes respond to the topology change by choosing an appropriate power level.

If none of the nodes update their power-level setting from its current level, the algorithm is said to have to converged to a steady state (NE). For arbitrary games, convergence is to an NE not assured. However, since the TC game we consider is a potential game, the network is assured of converging to an NE steady state when nodes selfishly update their power settings in a sequential manner (see the proofs of Proposition 1 and Lemma 2).

5 A Topology Control Game

Consider a multihop network constituting of independent and autonomous nodes that adapt their transmit power levels according to their connectivity and energy consumption preferences. Such adaptations could potentially affect the performance of other nodes and thereby influence their decisions. This kind of an interactive and distributed power control process impacts the topology of the network. In the context of this paper, the above interactive process defines our TC game.

We address the problem of designing energy-efficient topologies that preserve network connectivity in the presence of complex interactions among nodes in a network. A network designer may prefer to minimize the total power consumption (global energy efficiency) of the network or minimize the maximum power consumption of a node (minmax energy efficiency) in the network and seek to design an efficient topology. On the other hand, individual nodes may choose to reduce their own power consumption, regardless of the network performance. More often than not, such myopic behavior may lead to an undesirable equilibrium from a network perspective. This inherent conflict can sometimes be reconciled if the system designer's objective function (social welfare function) is a potential function for the game. A potential game also offers strong convergence properties of simple dynamic processes such as MIA and DIA, described in Section 4.

5.1 Utility Function

We consider the same normal-form game model as in [11]. Using the general utility function given by (5), a specific utility for each node is given by

$$\bar{u}_i(\mathbf{p}) = M f_i(\mathbf{p}) - p_i. \quad (9)$$

Here, $f_i(\mathbf{p})$ is the number of the nodes that can be reached (possibly over multiple hops) by node i via bidirectional links and paths.[7] Naturally, f is nondecreasing, that is, $f_j(p_i, p_{-i}) \geq f_j(q_i, p_{-i}), \;\; \forall j \in N \text{ and } q_i < p_i$. The scalar benefit multiplier M signifies the value each node places on being connected to other nodes; we assume that $M = \max\{p_{i,max} | i \in N\}$.

Network connectivity is a basic requirement in TC as it provides the means for nodes to establish communication sessions with their destinations. The benefit component in (9) signifies the reachability of a node. It implicitly assumes that each node has some traffic for every other node in the network. This is a reasonable assumption because traffic load and selection of destinations are typically not

6. Given the ordering of $\tilde{A}_i$, note that if the current power is $p_i^{(k)}$, the node makes a switch to $p_i^{(k+1)}$ at the next opportunity.

7. In other words, a node places the same "value" whether it can reach another node in one hop or in 10 hops. From a connectivity standpoint, this assumption is reasonable since at the topology formation level, we only need to know whether there exists a path to any given destination. In reality though, we may prefer shorter paths to longer ones depending on the QoS (for example, minimum latency) requirement of the traffic, which may alter the benefit structure.

available during topology formation. This necessitates that the underlying topology be connected.

Connectivity is a function of the transmit power of all nodes in the network. Each node chooses a transmit level based on its objective function and not for the objectives of other nodes. However, we make a slight distinction here and emphasize that once a node decides the power level to transmit at, it continues to forward packets at its chosen transmit power. The validity of node cooperation for packet forwarding in ad hoc networks is a research thread in itself; we refer interested readers to [19], [20], [21], and [22].

The cost component in (9) suggests that transmission power is the primary source of energy consumption. Transmission costs may include energy consumed in sourcing or in forwarding packets in a given session between two consecutive executions of the TC protocol. We ignore all additional energy consumed when receiving, storing, and processing packets. It is important to underscore that we make these assumptions to keep the utility function simple; the essence of the problem is nonetheless still preserved.

Consider the case of neglecting the reception power, which may be unrealistic in certain applications. From a game-theoretic viewpoint, the present cost function can be easily extended and modified to incorporate the received power as well. The number of incoming edges in the topology that terminate at i specifies which other nodes can be heard by i. According to (3), an incoming edge to i from j is defined by the power level of j; thus, nodes in general have little control on their received cost. In the semantics of game theory, the received cost component of each node i can be modeled by $C_i(p_{-i})$, a "dummy" function that depends on the power levels of all nodes except i. The addition of a dummy function does not alter the potential game property of the TC game. Consequently, the subsequent analysis of the TC game such as its convergence properties and the efficiency of NE topologies, discussed in the following sections, are unaffected. Nonetheless, the topologies that minimize the total cost (that is, the sum of data transmission and reception powers) may, in fact, be different from those that minimize transmission power alone. As we shall see in Section 6, the topologies produced by DIA are quite sparse with very few extraneous unidirectional edges on the average; thus, we believe that the reception costs will be comparable to those in optimal topologies. Additionally, the received cost can further be reduced by decoding a few header bits and turning off the receiver for the rest of the transmission period in case the transmission was intended for some other receiver.

More aggressive energy consumption models can be used to create energy-efficient networks. For instance, a protocol where nodes turn their radio off and go to "sleep" mode if their participation is not mandated by the network can significantly save energy. Likewise, a node may choose to selectively forward packets in order to conserve energy. The study of such energy models is beyond the scope of this paper; we refer the readers to [23] and [21] for further discussions.

A quick note before we move on: our utility function given in (9) is quite generic and works even without the knowledge of exact node locations, so long as the threshold power levels ω required to establish links are estimated accurately. Certainly, many other utility functions can be used to model the specific systems under study. An example, for instance, is one in which each node views benefit from covering a given area (instead of connecting to a certain number of nodes as considered in (9)). Such a construct models applications such as sensor networks well but requires the knowledge of node locations in assessing the sensor field coverage. For instance, two nodes that are within the proximity of each other do not add to the individual coverage areas of each node because both nodes more or less "observe" the same information. However, if the two nodes are distant from each other, each node, with location information of the other, can improve its utility by connecting to the other node and thereby increasing the coverage area.

We next study the implications of the utility function given in (9) and analyze the TC game properties.

5.2 Game-Theoretic Analysis

We begin by showing that the game $\bar{\Gamma} = \langle N, A, \bar{u} \rangle$ with the objective function of each node given by (9) is a potential game.

Theorem 1. *The game $\bar{\Gamma} = \langle N, A, \bar{u} \rangle$, where the individual utilities are given by (9), is an OPG. An OPF is given by*

$$V(\mathbf{p}) = M \sum_{i \in N} f_i(\mathbf{p}) - \sum_{i \in N} p_i. \tag{10}$$

Proof. We prove by applying the asserted OPG in (10). First, we have

$$\begin{aligned} \Delta \bar{u}_i &= \bar{u}_i(p_i, p_{-i}) - \bar{u}_i(q_i, p_{-i}) \\ &= M[f_i(p_i, p_{-i}) - f_i(q_i, p_{-i})] - (p_i - q_i). \end{aligned} \tag{11}$$

Similarly

$$\begin{aligned} \Delta V &= V(p_i, p_{-i}) - V(q_i, p_{-i}) \\ &= M[f_i(p_i, p_{-i}) - f_i(q_i, p_{-i})] - (p_i - q_i) \\ &\quad + M\left[\sum_{j \in N; j \neq i} \{f_j(p_i, p_{-i}) - f_j(q_i, p_{-i})\}\right]. \end{aligned}$$

Thus, we have

$$\Delta V = \Delta \bar{u}_i + M\left[\sum_{j \in N; j \neq i} \{f_j(p_i, p_{-i}) - f_j(q_i, p_{-i})\}\right]. \tag{12}$$

Since $f_i(\mathbf{p})$ is monotonic and $M \geq p_{i,\max} \forall i$, it follows from (11) that

$$\Delta \bar{u}_i = \begin{cases} \geq 0 & \text{if } p_i > q_i \text{ and } f_i(p) > f_i(q_i, p_{-i}), \\ \leq 0 & \text{if } p_i < q_i \text{ and } f_i(p) < f_i(q_i, p_{-i}), \\ < 0 & \text{if } p_i > q_i \text{ and } f_i(p) = f_i(q_i, p_{-i}), \\ > 0 & \text{if } p_i < q_i \text{ and } f_i(p) = f_i(q_i, p_{-i}). \end{cases} \tag{13}$$

The sign of the second term in (12) is the same as the sign of $\Delta \bar{u}_i$ for the first two cases of (13). For the last two cases of (13), the second term in (12) is zero, because the connectivity profile of every node remains unchanged;

therefore, $\Delta V = \Delta\bar{u}_i$. In general, $\mathrm{sgn}(\Delta V) = \mathrm{sgn}(\Delta\bar{u}_i) \Rightarrow V$ is an OPF, and $\bar{\Gamma}$ is an OPG. □

One of the overaching consequences of being a potential game is the possible relationship between a potential function and a social welfare function. In the context of our TC game, the social welfare function is the energy-efficiency metric. Alternately, the potential maximizing NE of the TC game can be interpreted as the optimal power assignment vectors, that is, steady-state topologies that are globally energy efficient.

Theorem 2. *For the game $\bar{\Gamma} = \langle N, A, \bar{u}\rangle$, the class of global potential maximizers coincide exactly with the class of topologies that are globally energy efficient.*

Proof. Let $\mathbf{p}$ belong to the set of potential maximizers. For a given $\mathbf{p}$, we show that $g(\mathbf{p})$ is connected and globally energy efficient. We prove this by contradiction:

Case 1. Say $g(\mathbf{p})$ is not connected. Then, $f_i(\mathbf{p}) = k_i < n$, the number of nodes in the network, $\forall i$. In other words, $k_i \leq n-1$. Since $\mathbf{p}$ is a potential maximizer, $V(\mathbf{p})$ must be greater than the value $V(\mathbf{p}^*)$ generated by another (connected) network, say, $g(\mathbf{p}^*)$. Note that since $g(\mathbf{p}^*)$ is connected, $f_i(\mathbf{p}^*) = n$, $\forall i$, and $V(\mathbf{p}^*) = M \cdot n^2 - (\sum_{i\in N} p_i^*)$. In other words

$$\begin{aligned} V(\mathbf{p}) = M\left(\sum_{i\in N} k_i\right) - \left(\sum_{i\in N} p_i\right) > M\cdot n^2 - \left(\sum_{i\in N} p_i^*\right) \\ \Rightarrow M\left(n^2 - \sum_{i\in N} k_i\right) < \left(\sum_{i\in N} p_i^* - \sum_{i\in N} p_i\right). \end{aligned} \tag{14}$$

Since $k_i \leq n-1$, the left-hand side of (14), $M(n^2 - \sum_{i\in N} k_i) \geq M(n^2 - n\cdot(n-1)) = n\cdot M \geq n \cdot p_{i,max}$. On the other hand, the right-hand side of (14), $(\sum_{i\in N} p_i^* - \sum_{i\in N} p_i) \leq n\cdot p_{i,max}$. Thus, (14) is a contradiction. Hence, $g(\mathbf{p})$ is always connected when $\mathbf{p}$ is a potential maximizer.

Case 2. Now, suppose $g(\mathbf{p})$ is connected, but p_i is not minimum for some i. This implies that $(\sum_{i\in N} p_i) > (\sum_{i\in N} p_i^*)$. However, since $\mathbf{p}$ is the potential maximizer, $V(\mathbf{p}) = M\cdot n^2 - (\sum_{i\in N} p_i) > M\cdot n^2 - (\sum_{i\in N} p_i^*) \Rightarrow (\sum_{i\in N} p_i) < (\sum_{i\in N} p_i^*)$, a contradiction to our assumption.

Combining Cases 1 and 2, we conclude that $g(\mathbf{p})$ is always globally energy efficient when $\mathbf{p}$ is a potential maximizer.

We now prove the reverse direction. Let $g(\mathbf{p})$ be globally energy efficient. We show that $\mathbf{p}$ is a potential maximizer.

Since g is globally energy efficient, $\forall i$, $f_i(\mathbf{p}) = n$, and $(\sum_{i\in N} p_i)$ is minimal. Thus, $V(\mathbf{p}) = M\cdot n^2 - (\sum_{i\in N} p_i)$ is maximal. Thus, $\mathbf{p}$ is indeed a potential maximizer.

Thus, we conclude that the network $g(\mathbf{p})$, resulting from the game $\bar{\Gamma}$, is globally energy efficient if and only if $\mathbf{p}$ is a potential maximizer of (10). □

5.3 Analysis of the MIA

An immediate upshot of Theorem 1 is that both MIA and DIA are guaranteed to converge to an NE [13]. Consider the MIA: in every round, each node plays a best response to the power setting of other nodes. This defines a sequence of action profiles, where contiguous action vectors differ in exactly one element. Using the finite-improvement-path property of potential games, it can be shown that this sequence always converges to an NE profile. Besides, the topology induced by the NE has some desirable properties, as shown in the following proposition:

Proposition 1. *MIA converges to an NE of the game $\bar{\Gamma}$ that is locally energy efficient and preserves the connectivity of $g_{\max}$.*

Proof. From Theorem 1, we have that $\bar{\Gamma}$ is an OPG. From [13], it follows that MIA will converge to an NE. However, we are interested in only those NE that preserve connectivity in the final topology. Recall that the input to MIA is the topology $g_{\max}$, with every node transmitting at $p_{i,\max}$. The best response for each node is to reduce its transmission power (and maximize its utility) to a value p_i so that the resulting topology remains connected. We prove this by contradiction. Suppose node i maximizes its utility at $q_i < p_i$, given p_{-i}, and the network is not connected. This implies that $u_i(q_i, p_{-i}) = M\cdot k_i - q_i > M\cdot n - p_i$, where $k_i < n$, the total number of nodes in the network. This implies that $M\cdot(n-k_i) < p_i - q_i$, an impossible inequality, because the term on the left-hand side is larger than $p_{i,\max}$ and the term on the right-hand side is smaller than $p_{i,\max}$. Thus, in every round, the topology is always connected.

Since the topology is always connected at every stage of the iteration, $\forall i$, $f_i(\mathbf{p}) = n$, a constant. The utility maximization problem now becomes a power minimization problem. Thus, the final steady-state topology is also locally energy efficient. □

5.4 Analysis of the DIA

We have shown that MIA is guaranteed to converge to locally efficient topologies, by Proposition 1. Under the dynamics of this process, any initial state $\mathbf{p}_{\max}$ forms the basin of the attraction and the system converges to the local maxima of the potential function.

Theorem 2 identifies the existence of globally energy-efficient states; thus, if the global maxima of the potential function are the attractors of a dynamical system, convergence to efficient topologies can be assured. Recall that the outcome of MIA depends on the order in which nodes take turn in updating their updating their actions. Unfortunately, the problem of finding the optimal order, and consequently the problem of minimizing the total sum power in a network, is an NP-hard problem [24]. Hence, one needs to resort to developing efficient heuristics to closely approximate a global solution at best. We instead adopt an alternate approach and develop a DIA that is guaranteed converge to minmax energy-efficient topologies (that minimize the maximum power of a node in the network).

In the DIA process, each node selects a power setting with a higher payoff than its current payoff. Given that each node transmits at $p_{\max}$ and the induced $g_{\max}$ is connected at the start of the algorithm, any $p_i < p_{\max}$ that preserves connectivity is sufficient to improve i's payoff. As described in Section 4.2.2, each user adapts by decrementing his or her transmission power, albeit one level at a time, as long as it improves his or her payoff; otherwise, the user continues transmitting at his or her current level. In order to guarantee the convergence of DIA to the minmax energy-efficient states, the step size δ—the amount by which power levels are decremented in each step—should be sufficiently small.

Assumption 2. *The step size δ is chosen so that at most one connection (link) is dropped from the network when the powers are adapted from $\mathbf{p}^{(k)}$ to $\mathbf{p}^{(k+1)}$, where $p^{(k)}$, $p^{(k+1)} \in \tilde{A}_i$ from (7).*

Similar to MIA, this DIA dynamic also specifies an improvement path—a sequence of improving action profiles. The improvement path is finite, and as a result, the DIA dynamic converges to an NE [13].

The following result is the cornerstone of this paper: we show that when nodes employ DIA, the process converges to an NE that induces a minmax energy-efficient topology. The proof of this theorem is based on a minimum spanning tree (MST) property. Recall from Section 3.1 that we adopt a network model where the edge weights of the underlying graph are the power thresholds. Taking into account the wireless broadcast property, we first define a power-based MST (PMST) as follows:

Definition 6. *A graph g is a PMST if it contains the MST, as well as any additional edges induced by the wireless broadcast property.*

We first present the following two lemmas, which are essential in proving our main result:

Lemma 2. *Consider the game $\bar{\Gamma} = \langle N, A, \bar{u} \rangle$, where nodes employ DIA under Assumption 2. Starting with an initial topology $g_{\max}$ induced by the power vector $\mathbf{p}_{\max}$, the algorithm converges to a subgraph g_{dia} of the PMST.*

Proof. The proof is by induction. For the ease of presentation, we suppose, without loss of generality, that $g_{\max}$ is a complete network. Also, let the power thresholds $\omega(i,j)$ be identical to the euclidean distance d_{ij} between the corresponding nodes (therefore, Ω is symmetric). Consider a $g_{\max}$ comprising of three nodes: A, B, and C. Suppose $d_{AB} > d_{AC} > d_{BC}$ is the relationship between euclidean distances. Based on Assumption 2, nodes start at power level d_{AB} and keep decreasing their power in steps of δ until d_{AC}. At this point, nodes A and C will not reduce their power any further; otherwise, the network would disconnect, and the nodes' payoff would decrease.[8] Because $p_A = d_{AC}$ and $d_{AB} > d_{AC}$, link AB is severed as a result. Thus, DIA converges to a topology containing links AC and BC, the two shortest bidirectional links needed to connect the three nodes.

Now, consider a fully connected topology with four nodes: A, B, C, and D. Let $d_{DA} > d_{DC} > d_{DB} > d_{AB} > d_{AC} > d_{BC}$. All nodes keep decreasing their power from d_{DA} until d_{DB}. Node D now has only a single link DB that is bidirectional. The problem then reduces to a three-node topology as before. Thus, the algorithm converges to a topology containing the three shortest bidirectional links AC, BC, and BD (and possibly some extraneous unidirectional links as well).

8. According to the argument in Proposition 1, which also applies for a DIA dynamic, network connectivity is preserved at every stage of the game.

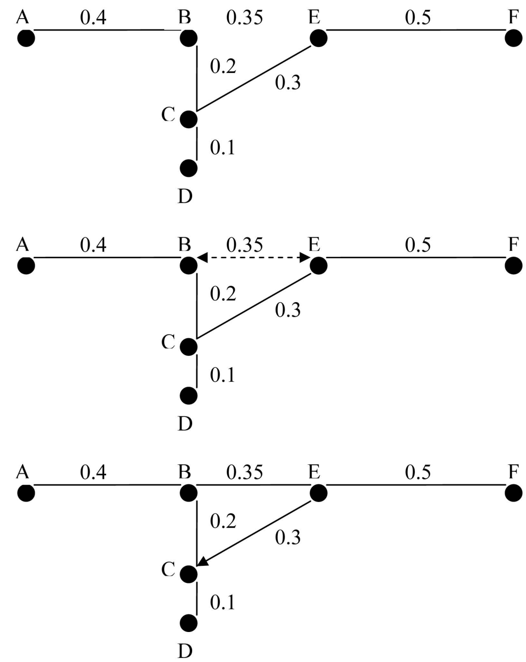

Fig. 1. (a) The MST. (b) The induced PMST. (c) The g_{dia} derived using DIA.

The above line of reasoning can be generalized to any arbitrary network of size n. Therefore, the algorithm always hits a state that consists of the shortest $n-1$ bidirectional links needed to maintain connectivity. Note that at this point, the network is a PMST by definition.

If the PMST contains a bidirected cycle (a cycle with all bidirectional links), at least one node in the cycle may still reduce its power level further and still maintain connectivity. Otherwise, the PMST contains exactly all the bidirectional links of MST. In either case, the steady-state topology g_{dia} is a subgraph of PMST (the subgraph may not be proper). This completes the proof. □

For a visual illustration on the difference between MST, PMST, and g_{dia}, see Fig. 1. Note that because the bidirected link BE in the PMST is incidental, node C can lower its power level further.

Lemma 3. *MST minimizes maximum power of any given node in the network.*

Proof. The main idea behind the proof is the fact that MST minimizes the maximum edge weight of the network. We show this by contradiction.

Let us assume, on the contrary, that there exists another tree T that minimizes the maximum edge weight. Let $e_t^{\max} = \arg\max_{ij \in T} \omega(ij)$ and $e_{mst}^{\max} = \arg\max_{ij \in MST} \omega(ij)$, where ω is the edge-weight function. By our contradiction, $\omega(e_t^{\max}) < \omega(e_{mst}^{\max})$. Introduce a cut—and partition the nodes into two sets N_1 and N_2—in MST by removing $e_{mst}^{\max}$ from the graph. Since T is a tree, we can find an edge $\tilde{e} \in T$ to join N_1 and N_2 and create an new tree $\tilde{T}$. Because $e_t^{\max}$ is the edge in T with the maximum weight, we have $\omega(\tilde{e}) \leq \omega(e_t^{\max}) < \omega(e_{mst}^{\max})$. Since $\tilde{T}$ is essentially created from the MST, $\sum_{e \in \tilde{T}} \omega(e) < \sum_{e \in MST} \omega(e)$; we obtain a contradiction. Therefore, MST is indeed the tree with the minimum maximum edge weight.

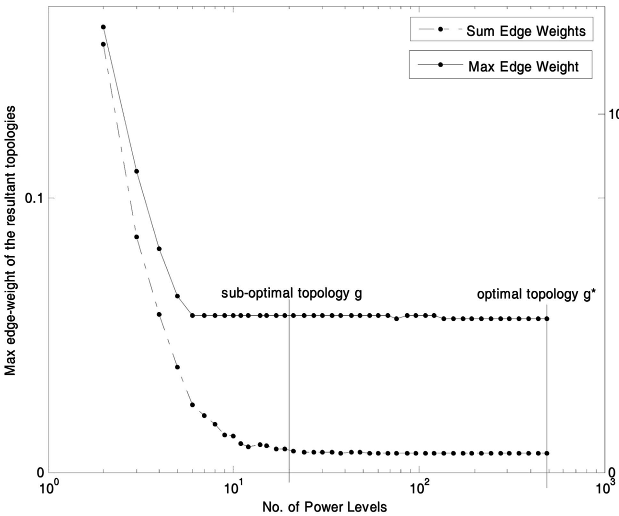

Fig. 2. *Impact of δ on the steady-state outcome.* The higher the number of power levels in the search space, the closer the margin between optimal and suboptimal convergent states.

The edge $e_{mst}^{\max}$ with the maximum weight determines the node with the maximum power. Therefore, it follows that MST minimizes the maximum power of any node in the network. □

Using the above two lemmas, the following main theorem of the paper is an immediate consequence.

Theorem 3. *DIA converges to a minmax energy-efficient topology—one that minimizes the maximum power of any given node.*

Proof. We know that PMST contains MST and all the additional induced edges. Because none of the induced edges increase the maximum edge weight of the graph, PMST preserves Lemma 3. From Lemma 2, the steady-state topology g_{dia} is a subgraph of PMST; therefore, every edge in g_{dia} is contained in PMST. It follows immediately that Lemma 3 still holds for g_{dia}. Hence, the result follows. □

We have shown that if a sufficiently small δ is chosen, DIA converges to the minmax energy-efficient topologies. As a general rule, δ decreases with increasing network density. Because δ specifies the number of power levels in the search space, it requires fine granularity in power adaptations in order to converge to efficient topologies; in real applications, using such small δ can be prohibitive. In Fig. 2, we consider a random topology with a density of 30 $\mathrm{nodes/unit}^2$ and quantify the impact of various δ values on the efficiency of the steady-state network. Minmax energy-efficient topology g^* is identified using a δ^* value that satisfies Assumption 2; then, the maximum edge weight and sum of edge weights of g^* are computed. Using this optimal δ^* as the reference, several different δ values are chosen leading to this optimal value. For each δ, the DIA converges to (a possibly) different steady state g. We compare the energy-efficient metrics (max edge weight and sum of edge weights) for these suboptimal topologies to those of the optimal topology g^*. For the sake of clarity, we plot the number of power levels in the search space $\tilde{A}$ in log scale along the x-axis in Fig. 2.

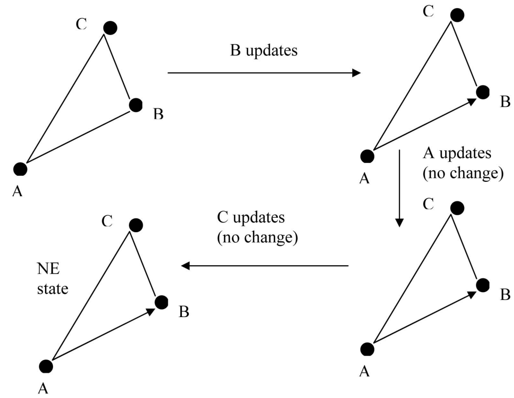

Fig. 3. *Illustrating the suboptimality of MIA.* Choose benefit factor $M = \omega(A, C)$. Node B updates first and chooses power $\omega(B, C)$; this necessitates A and C to not lower their power in order to preserve connectivity.

From the figure, we note that as the size of the search space increases (that is, as δ decreases), the resulting steady-state topologies approach the optimal topology configuration. As an engineering trade-off, one can choose a suboptimal topology with performance comparable to that of the optimal one while reducing the algorithm complexity. In the context of Fig. 2, choosing a δ that corresponds to 20 power levels (size of $\tilde{A}$) reduces the search space to a more practical value while still generating a good approximation of the optimal solution, which requires a search space of 500 power levels.

5.5 Comparative Discussion

The difference between MIA and DIA can perhaps be best explained by a simple example. Consider a three-node topology consisting of nodes A, B, and C; for the sake of illustration, assume identical and symmetric channel states. The dynamics of the game when nodes employ MIA is shown in Fig. 3. We note that different steady-state outcomes emerge depending on the order in which nodes update their actions. For instance, if the order is $\{C, A, B\}$ or $\{A, C, B\}$ instead of $\{B, A, C\}$ as in Fig. 3, then the outcome would be a topology containing links AB and BC (same as that obtained from DIA).

The dynamics of the game when nodes employ DIA is shown in Fig. 4. Unlike in MIA, the outcome of DIA is a unique PMST, regardless of the order in which players update their power setting.

In all the discussions above, we assume that nodes are "programmed" to follow the rules specified by DIA or MIA. Both DIA and MIA are selfish algorithms, each at two extremes on the "selfishness scale"; MIA is extremely selfish, allowing nodes to minimize power consumption in one shot, whereas DIA is more moderate, mitigating the first mover advantage by restricting the amounts by which each node can reduce its power. The DIA we developed is essentially a protocol for selfish nodes that, if they follow, is assured of converging to efficient NE states. The algorithm, although conservative, is certainly true to the noncooperative theory and adheres to the rationality

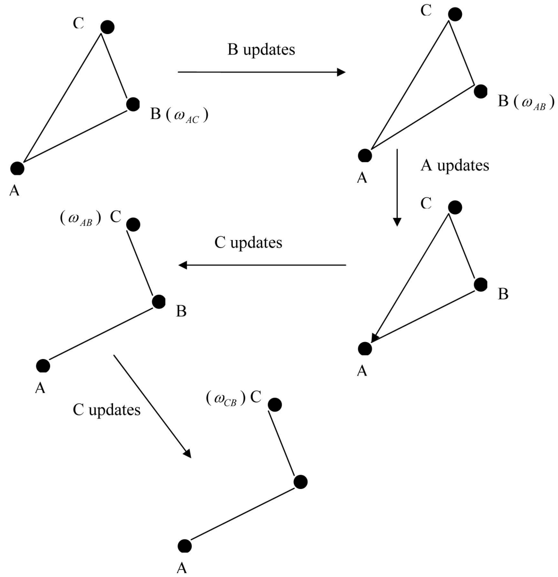

Fig. 4. *DIA dynamics.* In round 1, A, B, and C decrement their power level to $\omega(A, B)$. In round 2, only C minimizes its power further, to $\omega(C, B)$. Power levels in the parenthesis indicate a change from its previous state.

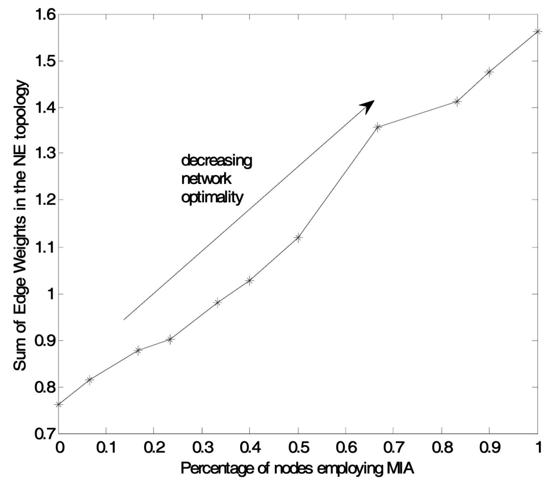

Fig. 5. Variation in the performance of NE topology with a fraction of nodes employing MIA (and remaining nodes employing DIA).

principle. Given this, it is nonetheless worthwhile investigating the outcomes when nodes disobey the selfish rules. Specifically, we study what NE states are likely to emerge in systems where nodes are selfish but not programmed to behave strictly according to some selfish algorithm (like MIA or DIA). In such systems, some nodes may behave more selfishly than others (perhaps because the more selfish nodes have stricter energy conservation requirements).

We simulate one version of the above scenario by considering a noncooperative network in which a certain percentage (q) of selfish nodes employs MIA and the remaining employs DIA. Observe that when $q = 0$ percent, the steady-state topologies are minmax efficient (by Theorem 3), whereas when $q = 100$ percent, the topologies are locally efficient (by Proposition 1). For any other value of q, we expect that the resultant NE topologies are efficient in some degree between local efficiency and minmax efficiency. This result is corroborated in Fig. 5. The figure displays the loss in network efficiency due to the greedy nature of MIA. DIA overcomes this first-mover advantage inherent in MIA, and thus, the NE topologies are more efficient as q decreases.

5.5.1 Fairness and Pareto Optimality

MIA converges to one of the many NE of the game $\bar{\Gamma}$; which NE state emerges depends on the order in which nodes update their power. While all NE states satisfy Proposition 1, the power assignment vectors that define these states may be substantially different. The "greedy" nature of the algorithm immediately suggests that the nodes that update their actions earlier, in a given round, choose the minimum power necessary to preserve connectivity. Consequently, the nodes that update later are forced to transmit at a higher power in order to maintain connectivity (recall that maintaining connectivity is always a best response for all nodes). Thus, the "first-mover advantage" inherent in the MIA results in a biased and unfair power allocations. To a certain extent, updating in a randomized ordering alleviates this bias in power assignment.

In DIA, power levels are more evenly distributed across all nodes. In some sense, the node with maximum power is the "weakest link" of the network; therefore, minimizing the maximum power prolongs the network operability and maximizes the network lifetime.[9] The distribution of steady-state power levels is comparable to that obtained from a centralized algorithm such as [8], [9]. We conjecture that the loss of network performance due to the presence of selfish nodes in the network is significantly small; in other words, the price of anarchy is close to one.

Definition 7. *A power assignment vector* $\hat{\mathbf{p}} \in A$ *is Pareto optimal if*

$$u(\hat{\mathbf{p}}) \succeq u(\mathbf{p}) \forall \mathbf{p} \in A, \tag{15}$$

where $\succeq$ *implies strict equality in at least one element of vector* u.

In other words, it is impossible to increase the utility of a player without decreasing the utility of some other player.

Theorem 4. *Any algorithm that starts at* $g_{\max}$ *and implements a selfish strategy—such as MIA or DIA—converges to a Pareto-optimal NE. Alternately, every NE that preserves network connectivity is Pareto optimal.*

Proof. Any selfish algorithm that starts at $g_{\max}$ converges to an (locally efficient) NE that preserves the connectivity of $g_{\max}$; see the proof of Proposition 1. First, by Definition 3, no node can reduce its power any lower; otherwise, the network would be disconnected and hence violate Proposition 1. Second, no m-node (where $m \geq 2$) reduction in power levels can preserve the network connectivity either. This is because if some node reduces its power (and therefore disconnects the network), some other node must increase its power to reconnect the

9. Network lifetime is defined as the time span between the start of a network to the death of the first node.

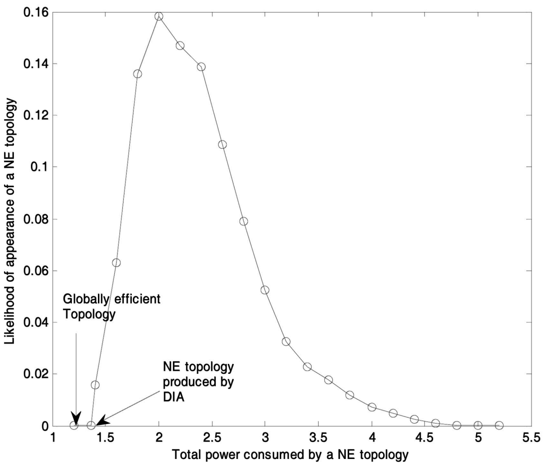

Fig. 6. Illustrating the efficiency of g_{dia} among 26,000 other efficient topologies.

network. It follows that the new configuration is not Pareto optimal. □

As a corollary to Theorem 4, observe that the NE topology T^* obtained by DIA is Pareto efficient: Suppose, on the contrary, that there exists another topology T that is Pareto efficient. This implies that every edge in T has a lower or equal weight ω than the edge weights in T^*. This suggests that the $\sum_{e \in T} \omega(e) < \sum_{e \in T^*} \omega(e)$; we obtain a contradiction because T^* is a subgraph of PMST.

Proposition 2. *For any random topology, the steady-state power assignment vector under DIA is unique.*

Proof. Note that MST (and therefore PMST) is unique if the edge weights are distinct because the edges can be uniquely ordered by their weights. Thus, the result follows immediately. □

From the Pareto efficiency and uniqueness of NE, it can be deduced that power allocation $\mathbf{p}^*$ under DIA is lifetime optimal: every other power distribution that preserves connectivity results in a lower network lifetime; hence, no node can reduce its power without degrading the network performance and, thereby, its own performance. To get a feel for the performance of the topology that results from DIA, we generate NE topologies for a 30-node topology, using various permutations of the order in which nodes update their power settings under the MIA, and compare against the topology generated by DIA. To demonstrate this, we plot a distribution of the total power consumed by an arbitrary NE state in Fig. 6. The figure corroborates the fact that DIA performs much better than an average NE state generated by any other selfish algorithm; in addition, the plot also suggests that g_{dia} performs significantly close to the globally efficient topology.

5.5.2 Convergence and Overhead Issues

As in any engineering algorithm, there is a trade-off between the efficiency and convergence rate of the algorithm. While the topologies that emerge from MIA are only locally (and not globally) efficient, the algorithm convergence speed is linear in network size.

Proposition 3. *For the TC game given by $\bar{\Gamma}$, MIA converges at a rate $O(n)$, where n is the number of nodes in the topology. More specifically, the algorithm converges in exactly n steps.*

Proof. As shown in the proof of Proposition 1, at each step, the best response for each node is to choose the minimum power level required to remain connected—no node can reduce its power level any lower and still get a higher payoff. After the first round (when every player has updated his or her strategy), the payoff of each player i is given by $u_i(\mathbf{p}^*) = M.n - p_i^*$. In the second round, no player i can choose a power level $p_i < p_i^*$ and still be connected. If this was possible, then p_i^* would not be the best response of player i in the first iteration. Thus, after n steps, MIA converges to the NE given by $\mathbf{p}^* = \{(p_1^*, \ldots, p_n^*) | f_i(\mathbf{p}^*) = n \ \forall i\}$. □

The following proposition formalizes the convergence speed of DIA. The step size δ of the algorithm should be sufficiently small to assure the convergence to the minmax energy-efficient NE. On the other hand, the small step size also reduces the rate of convergence significantly. The choice of δ depends on the internode separation or, more generally, is a function of the network size.

Proposition 4. *For the TC game given by $\bar{\Gamma}$, DIA converges at a rate $O(n^2)$, where n is the number of nodes in the topology.*

Proof. The initial topology $g_{\max}$, induced by $\mathbf{p}_{\max}$, is at most a complete graph and therefore contains at most $n(n-1)/2$ bidirected edges. According to Assumption 2, in each iteration of DIA, exactly one edge is severed when nodes revise their power levels. Consider the extreme case: the node j that chooses minimum power (at the end of the algorithm) is located at the periphery of the topology. In this case, the algorithm converges only after j chooses its minimum power. This means that j severs all its links except the smallest one. Under this scenario, the algorithm traverses through the maximum number of iterations, that is, through $n(n-1)/2 - 1$ steps. Therefore, the convergence rate of the DIA algorithm is $O(n^2)$. Also, note that in each iteration, one edge is severed; therefore, one update is required in terms of message complexity. Therefore, the maximum number of updates required is $n(n-1)/2 - 1$. For the example topology shown in Fig. 4, DIA requires two updates (powers are first reduced to $\omega(A, B)$, and then, C reduces its power level further to $\omega(C, B)$), and the convergence is achieved in two rounds. □

Both MIA and DIA, although distributed, require large control overhead; each node, as it makes power adaptations, needs to know whether or not it is connected to all other nodes in the network. The worst case message complexity is on the order of $O(n^2)$, where n is the number of nodes in the network. Having such information helps in the convergence to maxmin efficient topologies, as shown in Theorem 3; this, however, comes at a cost (overhead). Analyzing this trade-off between the cost of control information and the steady-state network optimality is beyond the scope of this paper (see [25] for a formal analysis); nevertheless, the problem is important and warrants some discussion on how the overhead cost can be reduced.

One way of reducing the overhead cost of DIA is by designing its local counterpart. Consider, for example, Local-DIA (LDIA), a localized version of DIA, which is a more practical algorithm than DIA in terms of its implementation feasibility. A possible utility function that can be conceived is one in which utilities are functions of neighborhood connectivity and not the entire network connectivity as in DIA. The present utility function (9) can be modified to a localized one as

$$\bar{u}_i^{(k)}(\mathbf{p}) = M f_i^{(k)}(\mathbf{p}) - p_i, \tag{16}$$

where $f_i^{(k)}$ is the number of nodes within i's k-neighborhood, that is, the nodes that can be reached in at most k hops from i (ideally k must be as low as possible).

In LDIA, each node observes its current k-neighborhood and strives to maintain connectivity with every node in its k-neighborhood while making power adaptations. (Note that maintaining the k-neighborhood is always a dominant action for each node, by Proposition 1.) Observe that as node i reduces its power level, it may remove a node from the k-neighborhood of some other node j; this happens if i drops a connection with one of its current one-hop neighbors that belongs to the k-neighborhood of j. Unless i broadcasts its new neighborhood, the nodes in the k-neighborhood of i may be unaware of the changes in their respective k-neighborhoods. To avoid this, it is critical, from an implementation viewpoint, that each node broadcast its neighborhood table every time there is a change in its one-hop neighborhood. Note that it is sufficient for nodes to send updates only to those nodes that are within their k-neighborhood and not to all nodes in the network (as done in DIA). It is also sufficient for each node to broadcast its one-hop neighbor table and not the entire k-hop neighbor information. By propagating the control updates only to small neighborhoods, LDIA greatly reduces the overhead cost. Besides, this idea of k-hop neighborhood prevents the overhead cost from growing with the network size. We are currently studying this modification of DIA in our ongoing work [25].

6 Simulation Results

In this section, we present simulation results to demonstrate the validity of our results. In the simulation study, nodes are placed randomly on a 2D plane in a $[-1, 1] \times [-1, 1]$ grid. We assume omnidirectional antenna gain patterns with a path loss exponent of two (however, the results hold for any exponent); the power required to support a link ij is $\omega(i, j) = p_{ij} = d_{ij}^2$, where d_{ij} is the euclidean distance between nodes i and j (again, all results hold even when Ω is asymmetric). The simulation is implemented in C++ and GUI in Gnuplot. Nodes are chosen to update their transmission level in a round-robin manner. Under MIA, each node carries out the best response update scheme given by (6). Under DIA, each node carries out the better response update scheme given by (8).

Consider the initial state topology $g_{\max}$, containing 75 nodes, with each node transmitting at $p_{\max} = 2$ units. We

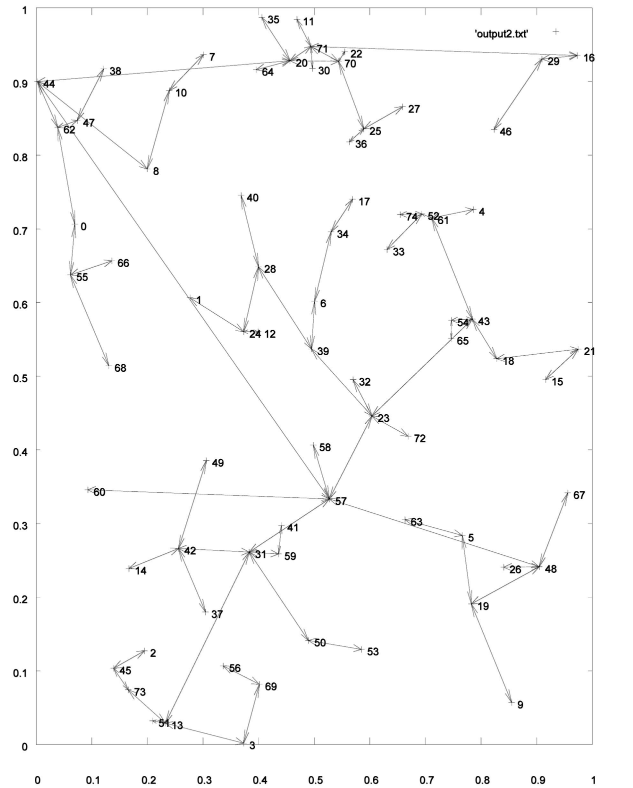

Fig. 7. *Output of MIA.* A steady-state topology that emerges when nodes implement MIA (Average power $= 0.041$ units and Maximum power $= 0.596$ units).

let $g_{\max}$ be the input to the two TC algorithms, MIA and DIA. A possible steady state when nodes implement MIA is shown in Fig. 7. The topology is much sparser as nodes operate at power levels significantly lower than their maximum levels. The steady-state topology, when nodes implement DIA, is shown in Fig. 8. As expected, the topology is much sparser than that produced by MIA. This topology is a subgraph of PMST and contains a few induced cycles.[10] As evident from the figures, both MIA and DIA preserve network connectivity as there exists a bidirectional path between any two nodes; besides, DIA produces a minmax topology: no other topology configuration can reduce the maximum power of any node in the network.

7 Related Work

Broadly, our work belongs to the body of research that addresses the impact of selfish node behavior on network performance. It is generally perceived that even if nodes act selfishly, some amount of cooperation is required to sustain an autonomous ad hoc network (see [20] and references contained therein). The crux of the problem is how to stimulate the nodes to cooperate—by using reputation-based or pricing-based frameworks—when they are driven by self-interested objectives. The need for cooperation is a fundamental problem, which manifests in various forms at all layers of the protocol stack in a communication system [26].

10. For the sake of clarity, we suppressed the unidirectional links from Figs. 7 and 8 and depicted only bidirectional links.

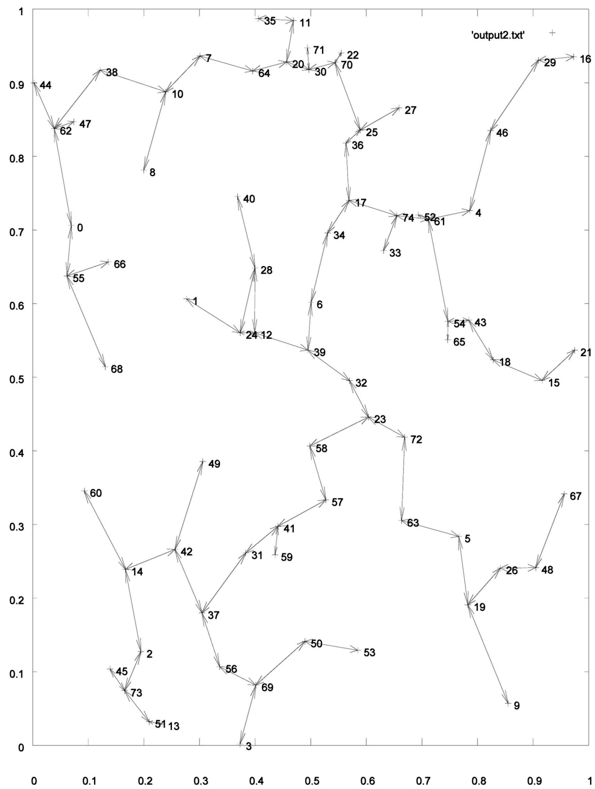

Fig. 8. *Output of DIA.* The steady-state topology that emerges when nodes implement DIA (Average power $= 0.001$ units and Maximum power $= 0.023$ units).

The research efforts to address the problem of TC in the presence of selfish nodes are fairly recent. Game theory and mechanism design are the commonly used approaches to address this problem. We now discuss the related work that uses these two approaches.

Eidenbenz et al. are the first to pose the TC problem as a noncooperative game and study connectivity properties [27]. Much of their work is devoted to the analysis of algorithmic complexity in finding an NE, when it exists, and deriving bounds on the price of anarchy. In [11], the authors formulate TC games as potential games. Potential games guarantee the existence of at least one NE. In addition, if the nodes employ a best response algorithm to choose an appropriate power setting, convergence to these equilibria is also guaranteed.

In [27], the existence of NE is not guaranteed. Furthermore, the authors do not provide the energy-efficiency characteristics of the topologies that emerge. In [11], the steady-state topologies that emerge are locally efficient but are not necessarily globally efficient. In contrast, we prove the existence and present convergence results pertaining to a global NE.

Mechanism design seeks to achieve global efficiency by aligning the selfish objectives of individual users with the socially desirable outcome. In the context of TC, mechanism design is employed to provide the adequate incentives to individual users so that they maximize their objective function when the network minimizes total energy consumption, subject to connectivity constraints. This approach has been adopted in [28] and [12] by engineering a payment system that leads selfish nodes to forward packets for others. The utility function proposed in [12] requires that each node declare the per-edge price that it intends to charge in exchange for forwarding packets.

The approach of assigning prices on a per-link basis does not account for the wireless broadcast advantage. The cost incurred by a node when forwarding packets along a link is a function of the transmit power required to establish that link; therefore, a node incurs uniform energy costs in maintaining links to each of its one-hop neighbors, all accessible at the same power level. In our model, we evaluate costs as a function of the transmit power and do not assume any link-based charges.

8 Conclusion

Nodes in an ad hoc network have restricted communication radius and limited battery capacity. This forces the nodes to rely on intermediate nodes, not only to extend their reach, but also to conserve their energy consumption. This gives rise to conflicting dynamics in the network, where nodes try to selfishly maximize their own performance.

We show that a particular instance of TC games can be viewed as a potential game. Using potential game theory, we show that the game $\bar{\Gamma}$ admits many locally efficient NE, a subset of which are also globally efficient. We develop two algorithms that deal with selfish nodes: MIA and DIA. MIA converges to topologies that preserve network connectivity but are inefficient from an energy consumption standpoint. In contrast, DIA algorithm guarantees convergence to minmax efficient and connected topologies.

In developing the game models, we assume that information about the existence of bidirectional paths, between nodes that are beyond each other's transmission range, is available from the network layer. We are currently studying the impact of modifying the utility function (5) so that each node views benefit from only its neighborhood and establishing a localized TC algorithm. We speculate that this modification will still retain the core results.

The present work considers relatively static topologies, where the TC algorithm converges faster than the changes in the network due to node mobility. Analyzing TC games in the presence of network dynamics is a natural extension of this work and a subject of future work. We believe that repeated games form an appropriate basis for game-theoretic models of dynamic TC. Repeated games allow nodes to choose actions that improve their expected payoffs that take histories into consideration. Because the network configuration changes in each time slot, optimizing individual performance by considering a history of past topology states, instead of just the most recent topology state (as in a potential game), is likely to converge faster and to better NE topologies.

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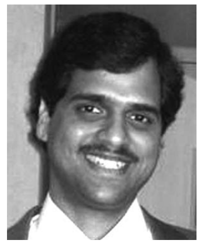

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