Core Capacity of Wireless Ad Hoc Networks*

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Abstract
In this paper, we model energy-limited wireless ad hoc networks as non-transferable-utility cooperative network flow games. We define the "core capacity region" of a wireless ad hoc network to be the set of Pareto-optimal utility vectors that cannot be blocked by any proper subset of the node set. We show that the core capacity region is non-empty under the linear utility model. Under the many-to-one traffic model in which all the nodes have traffic demands for a single base station, we show that the only utility vector in the core capacity region is the one achieved by the cellular uplink topology. Under the one-to-one traffic model in which each node generates a traffic demand for another randomly picked node, we demonstrate by simulation the growth of core sum capacity as a function of the number of nodes.

Keywords
cooperative, core capacity, energy, game theory, wireless ad hoc network, wireless network

INTRODUCTION

Wireless ad hoc networks present a tremendous opportunity to extend the Internet into the wireless domain far beyond what can be accomplished today by wireless LANs. Ad hoc networks have the potential to span a wide area without incurring the cost of setting up base stations. However, their deployment hinges critically on the efficient use of the energy supplies of the portable devices that comprise the network.

Because a node in an ad hoc network is both a host and a router, it has conflicting objectives of saving energy for its own transmissions and relaying the traffic of other nodes so that the network does not fall apart. This dual host-router status of a node is a significant challenge in the deployment of large-scale ad hoc networks. For example, the owners of laptops in a conference room want a protocol that enables communication between their laptops but do not want their batteries to be drained by the transit traffic from a larger ad hoc network.

Our main motivation for this paper is the question: "Can wireless ad hoc networks grow into a single grand wireless network in the future?" We answer this question by developing a framework in which we incorporate the incentives of users into the definition of network capacity. First, we model each node as an autonomous, rational agent and define its utility as a function that is monotonically increasing in the number of bits that it sends as a source and in the number of bits that it receives as a destination. In this paper, we restrict our attention to the linear utility model in which the utility of a node is a positive linear function of these variables. Second, we formulate the energy-limited wireless ad hoc network as a multi-commodity non-transferable-utility (NTU) cooperative network flow game between these agents. Third, based on the notion of the "core" of a NTU cooperative game [10], we formulate the "core capacity region" of a wireless ad hoc network as the set of Pareto-optimal utility vectors that all of the agents can achieve together such that no proper subset (a.k.a. "coalition") of the agents can achieve a strictly better utility for each of its members than in this Pareto-optimal utility vector.

The game theoretic formulations of network problems so far have predominantly been non-cooperative [7],[8],[13]. The few papers [1],[9],[15] that applied cooperative game theory to communication networks used the Nash bargaining solution. However, the Nash bargaining solution ignores the possibility of coalitions, the formation of which constitute the main threat to the attainment of a global wireless network. The concept of core capacity region addresses the formation of coalitions effectively and also contrasts sharply with the heuristic developed in [4].

In this paper, we propose the non-emptiness of the core capacity region as a necessary condition for the growth of wireless ad hoc networks. In [11], Owen proved that the core of transferable-utility (TU) linear programming games is non-empty. In [5],[6] the authors applied this result to TU network flow games. Our main result in this paper is that the core capacity region, which is the core of a NTU network flow game, is non-empty. This means that no matter how large the wireless ad hoc network grows, there exists a Pareto-optimal utility vector that the grand network can achieve such that no coalition has an incentive to withdraw from the grand network.

The rest of the paper is organized as follows: In the first section, we cover the fundamental ideas of cooperative game theory and draw the relationship between these ideas and ad hoc networks. In the second section, we describe our energy-limited network model and define an energy-limited network game. In the third section, we introduce the core capacity region as a central concept in this development and prove that the core capacity region is non-empty. In the fourth section, we apply our framework to two representative traffic models. In the last section, we discuss the implications of this work for the deployment of ad hoc networks in the future.

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COOPERATIVE GAME THEORY

Since an important part of our framework is based on cooperative game theory, we present in this section the concepts and results from this theory that we will use in subsequent sections.

An "autonomous" agent (or user) is an agent that has the freedom to choose as it wishes, between alternatives with which it is presented. In this paper, we model the nodes in a network as autonomous agents. This model is well-suited to many realistic situations in which each node is controlled by a single person (such as the owner of a portable phone). Placing constraints on the nodes' actions can be effected easily by placing constraints on the decision space of autonomous users; hence, no generality is lost in this treatment.

A game is a construct with agents in which each agent is presented with a set of alternatives and makes a decision. For each specification of the vector of decisions of these agents, each agent receives a payoff or utility. A "rational" agent is an autonomous agent that chooses the alternative that maximizes its own utility over its entire set of alternatives. In this paper, we model the nodes in a network as rational, autonomous agents, and we assume that the number of nodes is finite.

A coalition is a subset of the agents that agree to cooperate with each other and agree not to cooperate with any other agent. An example of a coalition in networks would be the proprietary wireless network of a company.

A transferable utility (TU) game is a game in which the worth of a coalition can be completely described by a single real non-negative number, such as the sum of the utilities of the members of a coalition. In contrast, a non-transferable utility (NTU) game is a game in which the payoff to a coalition requires for its description the utility achieved by each agent. We model energy-limited networks as NTU games because a single non-negative real number cannot describe completely how well-off each node is; therefore, the utility levels of individual nodes must be represented separately as in a NTU game.

A NTU cooperative game (in coalitional form) is a NTU game in which agents can form coalitions and each coalition can attain a well-defined set of utility vectors that are feasible for its members. We formalize these concepts by the following definition:

Definition 1 (NTU cooperative game) :
A NTU cooperative game $(\aleph, \nu)$ is a mapping $\nu : \mathcal{P}(\aleph) \rightarrow \Phi(\mathcal{R}_+^{\mathrm{N}})$, where $\aleph$ is the set of agents, N is the cardinality of $\aleph$, $\mathcal{P}(\aleph)$ is the power set of $\aleph$, and $\Phi(\mathcal{R}_+^{\mathrm{N}})$ is the set of measurable subsets of $\mathcal{R}_+^{\mathrm{N}}$.

NETWORK MODEL

In this section, we first state our assumptions on the set-up of the network. Then, we describe the multicommodity flow-based network model. Finally, we define an energy-limited NTU network flow game based on this model.

In our framework, a set of nodes, $\aleph$, is assumed to be randomly deployed over a deployment region [12]. Each node i (A) sees a wireless channel to every other node, (B) can transmit at high enough power for its transmission to reach any other node, (C) can transmit with the minimum transmit power necessary to reach any other node, (D) is allowed to relay its traffic via any other node, and (E) has a finite supply of energy E_i. Each wireless link ij has an associated energy-per-bit c_{ij} required to transmit along that link.

We say that a node m has a demand for node n if m wishes to send information to n. We refer to the traffic of m for n on the network as the "commodity" (m, n) and the amount of traffic transmitted end-to-end from m to n as the "demand" of node m for node n. A "traffic model" is a specification of the set of source-destination node pairs that have a positive demand and is described by the matrix Γ where $\Gamma_{mn} = 1$ if node m has a demand for n and $\Gamma_{mn} = 0$ otherwise.

Definition 2 (Energy-limited Network) :
An energy-limited network G is an ordered quadruple $(\aleph, C, \mathbf{E}, \Gamma)$ where $\aleph$ is the node set, $C \in \mathcal{R}_+^{\mathrm{N\times N}}$ is the "energy-per-bit cost matrix", $\mathbf{E} \in \mathcal{R}_+^{\mathrm{N}}$ is the vector of node energies, and $\Gamma \in \mathcal{B}^{\mathrm{N\times N}}$ is the "traffic matrix".

Definition 3 (Energy-limited Network Model) :
An energy-limited network model M_G on an energy-limited network G is the set of the following variables and constraints: $d^{(m,n)}$ denotes the demand of node m for node n, $x_{ij}^{(m,n)}$ the flow of commodity (m, n) on the arc ij, and u_i the utility of node i.

Flow Constraints:

Inflow: $\forall m \in \aleph$, $\forall i \in \aleph$:

$$\sum_{j\in\aleph\setminus\{i\}} x_{ji}^{(m,i)} = d^{(m,i)} \tag{1}$$

Outflow: $\forall n \in \aleph$, $\forall i \in \aleph$:

$$\sum_{k\in\aleph\setminus\{i\}} x_{ik}^{(i,n)} = d^{(i,n)} \tag{2}$$

Balance: $\forall l \in \aleph, l \neq i$, $\forall p \in \aleph, p \neq i$, $\forall i \in \aleph$:

$$\sum_{j\in\aleph\setminus\{i\}} x_{ji}^{(l,p)} = \sum_{k\in\aleph\setminus\{i\}} x_{ik}^{(l,p)} \tag{3}$$

No Self-Shipment: $\forall j \in \aleph$, $\forall m \in \aleph$, $\forall i \in \aleph$:

$$x_{ij}^{(m,i)} = 0 \tag{4}$$

No Self-Demand: $\forall i \in \aleph$:

$$d^{(i,i)} = 0 \tag{5}$$

Node Energy Constraint: $\forall i \in \aleph$:

$$\sum_{m\in\aleph}\sum_{n\in\aleph}\sum_{k\in\aleph} c_{ik} x_{ik}^{(m,n)} \leq E_i \tag{6}$$

Traffic Model Constraint: $\forall (m, n)$ for which $\Gamma_{mn} = 0$:

$$d^{(m,n)} = 0 \tag{7}$$

Node Utility Model: $\forall i \in \aleph$:

$$u_i = \sum_{m \in \aleph \setminus \{i\}} \alpha_i^{(m)} d^{(m,i)} + \sum_{n \in \aleph \setminus \{i\}} \beta_i^{(n)} d^{(i,n)} \qquad (8)$$

for non-negative parameters $\alpha_i^{(m)}, \beta_i^{(n)}$ defined $\forall (i,m,n) \in \aleph \times \aleph \times \aleph$.

Let $\mathbf{x} \in \mathcal{R}_+^{N \times N \times N \times N}$ be the network flow variables, $\mathbf{d} \in \mathcal{R}_+^{N \times N}$ the traffic demand variables, and $\mathbf{u} \in \mathcal{R}_+^{N}$ the utility variables, expressed in matrix form. An ordered triple $(\mathbf{x}, \mathbf{d}, \mathbf{u})$ is said to be "feasible in M_G" if it satisfies the above constraints. We will also call such a flow $\mathbf{x}$ feasible in M_G in this case. (Note that $\mathbf{d}$ and $\mathbf{u}$ are free variables.) A utility vector $\mathbf{u}$ is said to be feasible in M_G if there exists a $(\mathbf{x}, \mathbf{d}, \mathbf{u})$ that is feasible in M_G for some $(\mathbf{x}, \mathbf{d})$. The set of all utility vectors feasible in M_G is defined to be the "feasible set" of M_G.

Based on the above network model, we define the "coalitional network model". The main idea is that we want to represent a network formed only among those nodes that have formed a coalition.

Definition 4 (Coalitional Network Model) :
A subset of $\aleph$ is called a "coalition". A coalitional network model M_G^S for coalition S, defined on the network model M_G, is the network model M_G with the additional constraint $x_{ij}^{(m,n)} = 0 \; \forall (i,j,m,n) \notin S \times S \times S \times S$.

Finally, we define the "energy-limited network game" which is a cooperative game based on the entire set of coalitional network models. This game associates with each coalition, the set of utility vectors that this coalition can achieve for its members.

Definition 5 (Energy-limited network game) :
An energy-limited network game is a NTU cooperative game in which each node is an agent, and the mapping ν is defined on an energy-limited network model M_G and its set of coalitional network models as $\nu(S) = \{\mathbf{u} \in \mathcal{R}_+^N \mid \mathbf{u} \text{ is feasible in } M_G^S\}$ for every $S \subset \aleph$.

CORE CAPACITY REGION

In this section, we develop the notion of the core capacity region of an energy-limited wireless ad hoc network. In communication theory so far, the capacity of a multiuser system has been defined based on the notion of Pareto-optimality. The main idea in our definition of the core capacity region is that Pareto-optimality does not suffice; we must carve out all of the Pareto-optimal solutions that do not make sense from the perspectives of coalitions. What remains behind is the core capacity region. We propose the non-emptiness of the core capacity region as a necessary condition for the growth of wireless ad hoc networks. In this section, we show that the core capacity region is non-empty under the linear utility model.

Definition 6 (Core) :
The core of a NTU cooperative game ν is defined as the set $\{\mathbf{u} \in \nu(\aleph) \mid \text{for any coalition } S \text{ and any } \mathbf{u}' \in \mathcal{R}_+^N, \mathbf{u}'_i > \mathbf{u}_i \; \forall i \in S \Rightarrow \mathbf{u}' \notin \nu(S)\}$.

Definition 7 (Core Capacity Region) :
The core capacity region of an energy-limited network is defined as the core of the corresponding energy-limited network game. Explicitly, the core capacity region is the set $\{\mathbf{u} \in \mathcal{R}_+^N \mid \mathbf{u} \text{ is feasible, and for any coalition } S \text{ and any utility vector } \mathbf{u}' \in \mathcal{R}_+^N, \mathbf{u}'_i > \mathbf{u}_i \; \forall i \in S \Rightarrow \mathbf{u}' \text{ is not feasible in } M_G^S\}$.

A utility vector $\mathbf{u} \in \nu(\aleph)$ is said to be "blocked" by coalition S if there exists a $\mathbf{u}' \in \nu(S)$ such that $\mathbf{u}'_i > \mathbf{u}_i \; \forall i \in S$.

Definition 8 (Pareto-optimality) :
A utility vector $\mathbf{u} \in \mathcal{R}_+^N$ is said to be Pareto-optimal in a coalitional network model M_G^S if there exists no $\mathbf{u}' \in \mathcal{R}_+^N$ that is feasible in M_G^S such that $\mathbf{u}'_i \geq \mathbf{u}_i \; \forall i \in S$ and $\mathbf{u}'_j > \mathbf{u}_j$ for some $j \in S$.

In order to show that the core capacity region is non-empty, we will utilize the concept of a balanced NTU game from cooperative game theory:

Definition 9 (Balanced family) :
A balanced family $\mathcal{B}$ of coalitions in a NTU cooperative game is a collection of non-empty coalitions of $\aleph$ such that for each coalition $S' \in \mathcal{B}$ there exists a scalar $\delta^{(S')}$ with the property that $0 \leq \delta^{(S')} \leq 1$ and $\forall i \in \aleph$, $\sum_{S' \in \mathcal{B}_i} \delta^{(S')} = 1$ where $\mathcal{B}_i \overset{\text{def}}{=} \{S \in \mathcal{B} \mid i \in S\}$.

In the development below, we will use the following notation: For any given vector $\mathbf{u} \in \mathcal{R}_+^N$ and any coalition $S \in \aleph$, we define $\mathbf{u}_S$ as

$$(\mathbf{u}_S)_i \overset{\text{def}}{=} \begin{cases} \mathbf{u}_i & i \in S \\ 0 & \text{otherwise} \end{cases} \qquad (9)$$

Definition 10 (Balanced NTU game) :
Let $(\aleph, \nu)$ be a NTU cooperative game such that $\nu(S)$ is closed in $\mathcal{R}_+^N$ for every $S \subset \aleph$. The NTU cooperative game $(\aleph, \nu)$ is said to be balanced, if for every balanced family $\mathcal{B}$ of coalitions of $\aleph$, $\{\mathbf{u} \in \mathcal{R}_+^N \text{ and } \mathbf{u}_S \in \nu(S) \text{ for every } S \in \mathcal{B}\} \Rightarrow \mathbf{u} \in \nu(\aleph)$.

The Bondareva-Scarf Theorem [14] states that a balanced NTU game has a non-empty core. In the following theorem, we use this fact to show that the core capacity region of an energy-limited network is non-empty.

Theorem 1 (Non-emptiness) :
The core capacity region is non-empty.

Proof: We will show that the network game is balanced. Then, by the Bondareva-Scarf Theorem, it will follow that the core of this game is non-empty. In order to show that the game is balanced, we first note that $\nu(S)$ is closed in $\mathcal{R}_+^N$ $\forall S \subset \aleph$. Next, we let $\mathcal{B}$ be a balanced family of coalitions of $\aleph$, and let $\mathbf{u} \in \mathcal{R}_+^N$ be such that $\mathbf{u}_S \in \nu(S) \; \forall S \in \mathcal{B}$. We must show that $\mathbf{u} \in \nu(\aleph)$. The main idea of the proof is as follows: For any $S \in \mathcal{B}$, $\mathbf{u}_S \in \nu(S)$ implies that there exists a $(\mathbf{x}_S, \mathbf{d}_S, \mathbf{u}_S)$ feasible in M_G^S. If we can construct a $(\mathbf{x}, \mathbf{d}, \mathbf{u})$ feasible in M_G for some $(\mathbf{x}, \mathbf{d})$, then we will have shown that $\mathbf{u} \in \nu(\aleph)$. We will construct such an $\mathbf{x}$ by superposing the $\{\mathbf{x}_S\}_{S \in \mathcal{B}}$ using the balancing scalars of $\mathcal{B}$ for the weights in the superposition. We will show that the $\mathbf{x}$ constructed this way (A) is a feasible flow in M_G, and

(B) achieves the utility vector $\mathbf{u}$. This will imply that $\mathbf{u}$ is feasible in M_G.

To this end, let $\mathbf{x} \stackrel{\text{def}}{=} \sum_{S\in\mathcal{B}} \delta^{(S)}\mathbf{x}_S$. To show (A), we first show that $\mathbf{x}$ satisfies the node energy constraint given by (6):

$$\sum_{m\in\aleph}\sum_{n\in\aleph}\sum_{k\in\aleph} c_{ik}\mathbf{x}_{ik}^{(m,n)} \tag{10}$$

$$= \sum_{m\in\aleph}\sum_{n\in\aleph}\sum_{k\in\aleph} c_{ik}\sum_{S\in\mathcal{B}}\delta^{(S)}\cdot(\mathbf{x}_S)_{ik}^{(m,n)} \tag{11}$$

$$= \sum_{S\in\mathcal{B}_i}\delta^{(S)}\sum_{m\in S}\sum_{n\in S}\sum_{k\in S} c_{ik}\cdot(\mathbf{x}_S)_{ik}^{(m,n)} \tag{12}$$

$$\leq E_i\sum_{S\in\mathcal{B}_i}\delta^{(S)} \tag{13}$$

$$= E_i \tag{14}$$

Above, (11) follows from the definition of $\mathbf{x}$, (12) from the fact that $(\mathbf{x}_S)_{ik}^{(m,n)} = 0$ if $(i,k,m,n) \notin S\times S\times S\times S$, (13) from the fact that $(\mathbf{x}_S, \mathbf{d}_S, \mathbf{u}_S)$ is feasible in M_G^S, and (14) from the fact that $\mathcal{B}$ is a balanced family of coalitions. Since the constraints (4) and (5) follow trivially, this shows that $\mathbf{x}$ is a feasible flow in M_G. Now, to show (B): Let $\tilde{\mathbf{d}}$ be the demand matrix and $\tilde{\mathbf{u}}$ be the utility vector achieved by the flow $\mathbf{x}$. Then,

$$\tilde{\mathbf{u}}_i = \sum_{m\in\aleph}\alpha_i^{(m)}\tilde{\mathbf{d}}^{(m,i)} + \sum_{n\in\aleph}\beta_i^{(n)}\tilde{\mathbf{d}}^{(i,n)} \tag{15}$$

$$= \sum_{m\in\aleph}\alpha_i^{(m)}\sum_{j\in\aleph}\mathbf{x}_{ji}^{(m,i)} + \sum_{n\in\aleph}\beta_i^{(n)}\sum_{k\in\aleph}\mathbf{x}_{ik}^{(i,n)} \tag{16}$$

$$= \sum_{m\in\aleph}\alpha_i^{(m)}\sum_{j\in\aleph}\sum_{S\in\mathcal{B}}\delta^{(S)}(\mathbf{x}_S)_{ji}^{(m,i)} \tag{17}$$

$$+\sum_{n\in\aleph}\beta_i^{(n)}\sum_{k\in\aleph}\sum_{S\in\mathcal{B}}\delta^{(S)}(\mathbf{x}_S)_{ik}^{(i,n)} \tag{18}$$

$$= \sum_{S\in\mathcal{B}_i}\delta^{(S)}\Big(\sum_{m\in S}\alpha_i^{(m)}\sum_{j\in S}(\mathbf{x}_S)_{ji}^{(m,i)} \tag{19}$$

$$+\sum_{n\in S}\beta_i^{(n)}\sum_{k\in S}(\mathbf{x}_S)_{ik}^{(i,n)}\Big) \tag{20}$$

$$= \sum_{S\in\mathcal{B}_i}\delta^{(S)}(\mathbf{u}_S)_i \tag{21}$$

$$= \mathbf{u}_i\sum_{S\in\mathcal{B}_i}\delta^{(S)} \tag{22}$$

$$= \mathbf{u}_i \tag{23}$$

Above, (15) follows from (8), (16) from (1) and (2), (17-18) from the definition of $\mathbf{x}$, (19-20) from the fact that $(\mathbf{x}_S)_{ij}^{(m,n)} = 0$ if $(i,j,m,n) \notin S\times S\times S\times S$, (21) from (8), (22) from (9), and (23) from the fact that the game is balanced. This shows that the utility vector achieved by $\mathbf{x}$ is precisely $\mathbf{u}$. Hence, $\mathbf{u} \in \nu(\aleph)$, which completes the proof.

CORE CAPACITY UNDER TRAFFIC MODELS

Our aim in this section is to examine the core capacity region under two representative traffic models. We define the "core capacity" of an energy-limited wireless ad hoc network to be the optimal value of an optimization program that takes $\nu(\aleph)$ as its feasible set. We examine the core capacity under two representative traffic models for wireless ad hoc networks.

Many-to-one Traffic Model

In the many-to-one model, each node generates traffic for only a single destination node for the whole network. This models the case where nodes want to send traffic to a base station or an access point into the wired Internet.

Definition 11 (Many-to-One Traffic Model) :
For an energy-limited network with more than 1 node, the many-to-one traffic model is defined as follows: Node 1 is called the "base station"; $\alpha_1^{(m)} = 0\ \forall m \in \aleph$, and the traffic matrix is given by

$$\Gamma_{mn} = \begin{cases} 1 & n = 1, m \neq 1 \\ 0 & \text{otherwise} \end{cases}$$

Definition 12 (Cellular Uplink Topology) :
Under the many-to-one traffic model, the cellular uplink topology is defined as the network model with the additional constraint $x_{ij}^{(m,n)} = 0\ \forall (i,j,m,n) \in \aleph\times(\aleph\setminus\{1\})\times\aleph\times\aleph$.

It can be shown that there exists a unique feasible $(\mathbf{x},\mathbf{d},\mathbf{u})$ achieved by the cellular uplink topology.

Theorem 2 (Core under Many-to-One Model) :
The only utility vector in the core capacity region under the many-to-one traffic model is the one achieved by the cellular uplink topology.

This result implies that in any energy-limited wireless network in which stationary, autonomous, rational agents want to send traffic to a single destination, the nodes will not relay each other's traffic and will instead transmit directly to the destination node.

One-to-one Traffic Model

In the one-to-one model, each node generates traffic for a randomly picked destination node in the network. This models the case where wireless ad hoc networks operate stand-alone without any access points into the wired Internet.

Definition 13 (One-to-one traffic model) :
For an energy-limited network with more than 1 node, the one-to-one traffic model is defined as follows: Each node $i \in \aleph$ has a demand for exactly one other randomly picked node $r(i)$, where r is a permutation on $\aleph$. Hence, the traffic matrix is given by

$$\Gamma_{mn} = \begin{cases} 1 & n = r(m), m \in \aleph \\ 0 & \text{otherwise} \end{cases}$$

We define the "sum capacity" of the network as the total utility that can be achieved in the feasible set. We define the "core sum capacity" to be the core capacity when the objective function of the optimization program is $\sum_{i\in\aleph} u_i$. For our simulations, we examine the core sum capacity under the one-to-one traffic model, with the parameters of the node utility model given by $\alpha_i^{(m)} = \beta_i^{(n)} = 1/2\ \forall (i,m,n) \in \aleph\times\aleph\times\aleph$. Under this particularization of the node utility

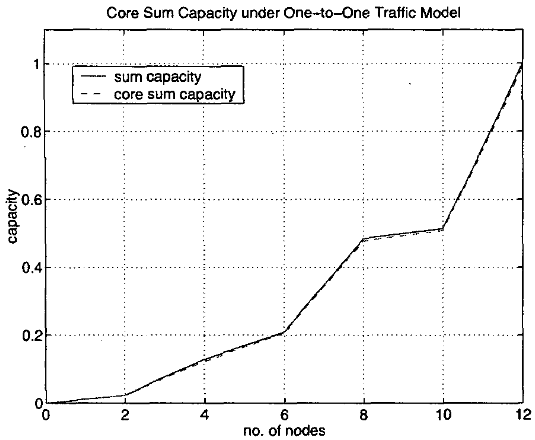

Figure 1. Sum capacity and core sum capacity under the one-to-one traffic model

model, $\sum_{i\in\aleph} u_i = \sum_{m\in\aleph}\sum_{n\in\aleph} d^{(m,n)}$. The latter quantity, namely the sum of the traffic demands that the network delivers, can be conceived of as the traditional sum capacity of such a network. Hence, this choice of the node utility parameters allows us to conceptually connect the total utility of the nodes and the total traffic demand delivered.

It is possible to construct networks in which the core sum capacity is much lower than the sum capacity of the network. However, we simulated a small randomly deployed ad hoc network and found that typically, the core sum capacity is close to the sum capacity. The simulation study was conducted for a network of nodes that are randomly deployed on a square area of 120 meters on each side under an urban outdoor propagation model with low antennas. We use the empirical results and model of [2] for path loss, and Gudmunson's model for correlated lognormal shadowing with its parameter obtained by a fit to urban data [3]. The positions of the nodes are generated independently from a uniform distribution. The node energies are generated independently of each other from a uniform distribution on the interval [0.2,1]. We display the results in Figure 1 where we have grown the network from 2 nodes to 12 nodes.

CONCLUSION

We have presented a framework in this paper which wireless ad hoc network protocols can utilize in order to allow these networks to grow over time based purely on the incentives of users to cooperate. This framework is centered on the notion of the core capacity of a wireless ad hoc network. The protocol requirements of these networks are very different from the ones encountered on the wired Internet and call for entirely new approaches to protocol design, based on the core capacity of these networks.

REFERENCES

[1] M. Cotton and L. G. Mason, "Adaptive isarithmic flow control in fast packet switching networks", IEEE Trans. Commun., vol. 43, no. 2/3/4, pp. 1580-1590, Feb/Mar/Apr. 1995.

[2] M. J. Feuerstein, K. L. Blackard, T. S. Rappaport, S. Y. Seidel, H. H. Xia, "Path loss, delay spread, and outage models as functions of antenna height for microcellular system design", IEEE Trans. on Veh. Technol., vol. 43, no. 3, pp. 487-498, Aug. 1994.

[3] M. Gudmunson, "Correlation Model for Shadow Fading in Mobile Radio Systems", Electr. Lttrs., vol. 27, pp. 2145-2146, Sep. 1991.

[4] T. Inohara, "On analysis of coalition formation with cooperative game theory and conflict analysis", in Proc. IEEE Int. Conf. Systems, Man and Cybernetics, vol. 2, pp. 1218-1223, 2001.

[5] E. Kalai and E. Zemel, "Totally Balanced Games and Games of Flow", Math. of Operations Research, Vol. 7, No. 3, pp. 476-478, Aug. 1982.

[6] E. Kalai and E. Zemel, "Generalized Network Problems Yielding Totally Balanced Games", Operations Research, Vol. 30, No. 5, pp. 998-f1008, Sep-Oct. 1982.

[7] A. A. Lazar, A. Orda, D. E. Pendarakis, "Virtual path bandwidth allocation in multiuser networks", IEEE/ACM Trans. Networking, vol. 5, no. 6, pp. 861-871, Dec. 1997.

[8] L. Libman and A. Orda, "The designer's perspective to atomic noncooperative networks", IEEE/ACM Trans. Networking, vol. 7, no. 6, pp. 875-884, Dec. 1999.

[9] R. Mazumdar, L. G. Mason and C. Douligeris, "Fairness in network optimal flow control: optimality of product forms", IEEE Trans. Commun., vol. 39, no. 5, pp. 775-782, May 1991.

[10] R. B. Myerson, "Game Theory: Analysis of Conflict", Harvard University Press, Cambridge, MA, London, England, 1991.

[11] G. Owen, "On the Core of Linear Production Games", Math. Programming 9, pp. 358-370, 1975.

[12] V. Rodoplu and T. H. Meng, "Minimum energy mobile wireless networks", IEEE J. Sel. Areas Commun., vol. 17, pp. 1333-1344, Aug. 1999.

[13] A. Orda, R. Rom, N. Shimkin, "Competitive routing in multiuser communication networks", IEEE/ACM Trans. Networking, vol. 1, no. 5, pp. 510-521, Oct. 1993.

[14] H. E. Scarf, "The core of an n-person game", Econometrica, vol. 35, issue 1, pp. 50-69, Jan. 1967.

[15] H. Yaiche, R. R. Mazumdar, C. Rosenberg, "A game theoretic framework for bandwidth allocation and pricing in broadband networks", IEEE/ACM Trans. Networking, vol. 8, no. 5, pp. 667-678, Oct. 2000.