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COOPERATION DIVERSITY IN MULTIHOP WIRELESS NETWORKS USING OPPORTUNISTIC DRIVEN MULTIPLE ACCESS

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ABSTRACT

Cooperation among users in a multihop wireless network adds diversity to the system and thus it allows us to reduce the overall transmit power. However, cooperation requires signaling among users and this reduces the overall rate gain. In this work, we provide the optimal coding strategy for meshed wireless networks, where more links are active simultaneously, assuming as optimality criterion the rates of all the links. This is a multi-objective optimization problem that has interesting applications in multihop and ad-hoc networks. We evaluate the optimal codes in closed form for the case where there are two pairs of users acting simultaneously and we provide an iterative algorithm for the general case. Then we evaluate the loss, in terms of information rate, resulting from simultaneous cooperation.

1. INTRODUCTION

Cooperation among users in a wireless network creates the possibility for a capacity and/or diversity gains [8] [4], [7], [6]. Furthermore, as suggested in [1], if two users or a user and a relay terminal share data without errors, they create consequently a *virtual* transmit multiple antenna, whose elements are the antennas of the cooperating users. Hence, if the base station is equipped with a real multi-element antenna and the users are perfectly synchronous, the system has the possibility to induce a virtual multiple-input/multiple-output (MIMO) system, capable of increasing the capacity by a factor potentially equal to the minimum of the transmit and receive antennas. This makes possible a considerable potential gain which can be exploited, for example, to reduce the average power necessary to achieve the desired bit error rate. However, cooperation inevitably requires the allocation of resources dedicated to this purpose. For example, in Opportunistic Driven Multiple Access (ODMA), which was considered during the standardization of 3G systems, in Europe, within each transmitted frame there are time slots dedicated to peer-to-peer communications. Clearly, the introduction of these time slots entails a corresponding rate loss. Therefore, in considering the possible benefits of cooperation in terms of rate, it is necessary to consider the balance between the waste due to sharing and the gain due to the increase of diversity and capacity.

In this paper, we consider a scenario with Q Mobile Terminals (MT) and as many Relay Terminals (RT). We focus on the uplink channel, with two hops (from MT to RT and from the pair MT/RT to the BS). We assign one time slot of duration T_{coop} (per frame) for Q simultaneous MT-to-RT links. Then, the Q successive time slots, of duration T_s are dedicated to the transmission from each MT/RT pair to the base station (BS). If the BS has at least two receive antennas, in the Q slots assigned to the MT/RT-to-BS links, we have a potential capacity increase by a factor of two, due to virtual MIMO. At the same time, there is rate reduction for data exchange equal to $QT_s/(QT_s + T_{coop})$ and a rate loss due to the interference, quantified by a factor $\alpha(Q) < 1$. Combining all these factors, there is a potential rate gain due to cooperation if $2\alpha(Q)QT_s/(QT_s + T_{coop}) > 1$. In this paper, we derive the optimal coding strategy maximizing the information rate between MT and RT, in the presence of interference from the other MT-to-RT links. In particular, we provide a closed form expression for the case when there are two pairs of MT/RT's and we propose an iterative algorithm valid for the general case. The bounds on information rate allow us to establish under which condition there is a real benefit from cooperation, as shown in the last section.

Throughout the paper, we use the following setup. Each user transmits blocks of M symbols using linear (redundant) precoding. We denote with $\boldsymbol{s}(n)$ the n-th block of information symbols and with $\boldsymbol{x}(n) = \boldsymbol{F}\boldsymbol{s}(n)$ the corresponding transmitted block, where $\boldsymbol{F}$ is an $N \times M$ full-rank matrix. All channels are FIR, time-invariant, with maximum order L. We denote with $h_{kl}(n)$ the impulse response between the k-th MT and the l-th RT. Two MT and RT are considered *paired*, when $k = l$. We append a cyclic prefix CP of order L to each block to facilitate elimination of inter-block interference (IBI). We assume, without any loss of generality, that the information symbols are uncorrelated with variance σ_s^2, and that the receiver noise vector $\boldsymbol{\eta}(n)$ is white Gaussian, with covariance matrix $\boldsymbol{C}_v = \sigma_n^2 \boldsymbol{I}$.

2. OPTIMAL SHARING STRATEGY

During the time slot dedicated to cooperation, each MT is allowed to communicate with one RT, but more MT/RT links are active simultaneously. We assume first that each MT has already chosen its RT and then we will remove this assumption by providing a method to establishing the best pairing between MT and RT. Assuming perfect synchronization between MT/RT pairs and no cooperation between MT's, the N-size vector $\boldsymbol{y}_k(n)$, received by the k-th relay, after discarding the guard interval, is

$$\boldsymbol{y}_k(n) = \boldsymbol{H}_{kk}\boldsymbol{F}_k\boldsymbol{s}_k(n) + \sum_{j=1,\, j\neq k}^{Q} \boldsymbol{H}_{jk}\boldsymbol{F}_j\boldsymbol{s}_j(n) + \boldsymbol{\eta}_k(n), \quad (1)$$

with $k = 1, \ldots, Q$, where $\boldsymbol{H}_{kk}$, thanks to the insertion of the CP, is an $N \times N$ circulant Toeplitz matrix with entries $H_{kk}(i,j) = h_{kk}((i-j) \mod N)/\sqrt{r_{kk}^{\alpha}}$, where $h_{kk}(n)$ and r_{kk} are the channel impulse response and the distance between the MT and the corresponding RT in the k-th pair, respectively. We have made explicit the dependance of the channel impulse response on the transmitter-to-receiver distance r, as this will give a physical justification of the optimal coding strategy that we will show in the next section. We assume that the transmitter power decreases as $1/r^{2\alpha}$, with $\alpha \geq 1$. Since no cooperation between MT's is allowed, the second term on the right-hand side of (1) represents the

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multi-user interference (MUI) received by the k-th relay caused by the other active MT-RT links. Therefore, the Q input-output relationships (1) can be modelled as a Gaussian interference channel with Q transmitters and Q receivers.

Our goal is to find out the coding matrices $\{\boldsymbol{F}_k\}_{k=1}^{Q}$ that maximize the information rates of all cooperating pairs MT-RT *jointly*, subject to the constraint that each transmitter has a maximum power P_T. Stated in mathematical terms, denoting by R_k the rate of the k-th MT/RT link, which is a function of the power budget P_T and of the coding matrices $\{\boldsymbol{F}_k\}_{k=1}^{Q}$ assigned to each cooperating pair, we look for the coding matrices solution of the following optimization problem

$$\begin{aligned} &\{\boldsymbol{F}_1^{\star}, \cdots, \boldsymbol{F}_Q^{\star}\} = argmax\{R_1, R_2, \ldots, R_Q\}, \\ &\text{subject to} \quad tr\{\sigma_s^2 \boldsymbol{F}_k \boldsymbol{F}_k^H\} \le P_T, \;\; k = 1, \ldots, Q, \end{aligned} \tag{2}$$

where the stars indicate the optimal solution. This is a multi-objective problem, whose optimal solution has to be intended in the Pareto's sense.[1] This multi-objective formulation is made necessary by the interfering nature of the problem (1), so that increasing the rate of a link would increase the interference on other links and then decrease their rates. The full characterization of the problem would require the determination of the capacity region (CR) of the interference channel (1), but this is still an open problem. Partial results have been achieved in the simple case of two input/output white Gaussian interference channels. Specifically, in [3] and [5] it was shown that the boundary of the CR of such a system is achieved, under the assumption of strong interference, by interference cancellation. However, the assumption made in [3] and [5] that the interference level must be stronger than the useful signal is not applicable to our problem. Furthermore, to achieve the boundary of the CR, some cooperation among different pairs could be required and this would be to difficult to implement in a real network.

Thus, we approach the problem searching for an optimum solution of (2), under the following assumption: a1) no interference cancellation is performed at the receiver; a2) multi-user interference is treated as additive noise at each receiver; a3) no cooperation among different pairs is allowed; a4) all channels $\{\boldsymbol{H}_{kj}\}_{k,j=1}^{Q}$ are perfectly known to all transmitters and receivers. Only for the sake of simplicity, we also assume that a5) multi-carrier modulation is performed from each pair, but no constraint on the bandwidth that each pair may use, is imposed. Thus all pairs, in principle, could share all the sub-carriers. Because of a1), a2) and a3), the information rate of the k-th MT/RT link can be computed as the maximum mutual information $I(\boldsymbol{x}_k; \boldsymbol{y}_k)$ between the transmitted block $\boldsymbol{x}_k(n)$ and the received block $\boldsymbol{y}_k(n)$, *assuming the other received signals as additive noise.* Under the hypothesis of additive Gaussian noise and the power constraint $\text{tr}\{\sigma_s^2 \boldsymbol{R}_{x_k}\} \le P_T$, mutual information $I(\boldsymbol{x}_k; \boldsymbol{y}_k)$ is maximum when the symbol vectors $\{\boldsymbol{s}_k(n)\}_{k=1}^{Q}$ are Gaussian. Using a4) and a5), mutual information exchanged by the k-th MT/RT pair is [2]

$$R_k = \frac{1}{N} \sum_{i=0}^{N-1} \log\left(1 + \frac{|H_{kk}(i)|^2 p_k(i)/r_{kk}^{\alpha}}{\sigma_n^2/\sigma_s^2 + \sum_{j\neq k}^{Q} \frac{1}{r_{jk}^{\alpha}} |H_{jk}(i)|^2 p_j(i)}\right)$$

where $H_{jk}(i)$ are the samples of the channel transfer function, i.e. $H_{jk}(i) = \sum_{q=0}^{L} h_{jk}(q)\, e^{-j2\pi iq/N}$, and $p_k(i)$ is the power allocated over the i-th sub-carrier from the k-th MT. For each MT/RT link, such power allocation can be found solving the following maximization problem

$$\begin{aligned} &\{\boldsymbol{p}_1^{\star}, \cdots, \boldsymbol{p}_Q^{\star}\} = argmax\{R_1, R_2, \ldots, R_Q\}, \\ &\text{with} \;\; R_k = \tfrac{1}{N}\textstyle\sum_{i=0}^{N-1} \log\left(1 + \frac{|H_{kk}(i)|^2 p_k(i)/r_{kk}^{\alpha}}{\sigma_n^2/\sigma_s^2 + \sum_{j\neq k}^{Q} \frac{1}{r_{jk}^{\alpha}} |H_{jk}(i)|^2 p_j(i)}\right) \\ &subject\ to \quad \sigma_s^2 \textstyle\sum_{i=0}^{N-1} p_k(i) = P_T, \;\; k = 1, \ldots, Q, \end{aligned} \tag{3}$$

where $\boldsymbol{p}_k := \{p_k(0), \ldots, p_k(N-1)\}, \forall k$. The equality in the power constraint of (3) follows from the fact that, in a non-cooperative scenario, all MT's transmit with the maximal average power. Since the multi-objective maximization (3) is not convex (because the rates are not convex with respect to the power vectors), the corresponding weighted scalar optimization problem also is not convex. Thus, numerical algorithms are able to find only local optima and there is no guarantee that *all* the local optima be checked. Furthermore, the high number of unknowns does not allow an exhaustive search, whose computational complexity would be prohibitive. Since only some local optima of (3) can be found, we introduce a new optimality criterion, having an interesting physical interpretation. We start reformulating the above optimization problem as a competitive non-cooperative game, where multiple players with conflicting interests compete through self-optimization. Under this perspective, the optimal solutions we are looking for are the stable equilibrium points, called Nash Equilibrium (NE), in the sense that each player (pair), given the power allocation of the other players (pairs), does not get any rate increase by changing its own power. In this paper, we provide: i) The sufficient conditions for the existence and *uniqueness* of a stable NE; ii) The optimal solution in closed form, in the simple case of two active cooperating pairs ($Q = 2$). When $Q > 2$, it is not easy to find a closed form solution. In such a case, we provide a numerical solution based on a simple iterative *distributed* algorithm, which is *always* convergent to the unique NE.

It is important to remark that the solutions corresponding to NE points are, in general, sub-optimal solutions of the problem (3) and thus they are Pareto inefficient[2]. On the other hand, since (3) is non-convex, its numerical (generally suboptimal) solutions are not guaranteed to be Pareto-dominant of NE points. We will show in the last section that, if the sufficient conditions for the existence and uniqueness of NE hold true, the rate loss of each pair with respect to its maximum (achievable without interference) is practically negligible.

3. GAME THEORETIC FORMULATION

We reformulate the problem (3) as a non-cooperative strategic game [12] with the following structure $\mathscr{G} = (\Omega, \{\mathscr{P}_k\}_{k\in\Omega}, \{R_k\}_{k\in\Omega}, \boldsymbol{H}, \sigma_n^2, \sigma_s^2)$, where $\Omega := \{1, 2, \ldots, Q\}$ is the set of pairs indices; $\mathscr{P}_k$ is the set of the admissible strategies (power distribution) for the k-th player; the information rates R_k are the payoff functions; the power distribution $\boldsymbol{p}_k := \{p_k(i)\}_{i=1}^{N} \in \mathscr{P}_k$ over the N available sub-carriers, subject to the power constraint $P_T = \sigma_s^2 \sum_{i=1}^{N} p_k(i)$, represents the game strategy for the k-th player. The game structure, i.e. the channels $\{H_{ij}\}_{i,j=1}^{Q}$ and the variances σ_n^2 and σ_s^2 are assumed to be known to all players. Moreover, only pure strategies are allowed. In such a game, each player competes with the others in order to maximize its own information rate R_k (given by (3)), regardless of all other players. In this competition, if there exists a NE, it means that there is an optimum strategy profile $\overline{\boldsymbol{p}} := (\overline{\boldsymbol{p}}_1, \overline{\boldsymbol{p}}_2, \ldots, \overline{\boldsymbol{p}}_Q) \in \mathscr{P}_1 \times \ldots \times \mathscr{P}_Q$

[1]Indicating with $\mathscr{F}$ the set of admissible points $(\boldsymbol{F}) \equiv \{\boldsymbol{F}_1, \cdots, \boldsymbol{F}_Q\}$ satisfying the constraint in (2), the solution $(\boldsymbol{F}^{\star})$ is an optimum according to Pareto criterion, iff it is Pareto dominant, i.e. there does not exist any other point $(\overline{\boldsymbol{F}}) \in \mathscr{F}$ and $(\overline{\boldsymbol{F}}) \neq (\boldsymbol{F}^{\star})$, such that $R_k(\overline{\boldsymbol{F}}) \ge R_k(\boldsymbol{F}^{\star})$ for $k = 1, 2, \ldots, Q$, with at least one of the above inequalities satisfied in strict-sense.

[2]Generally, Pareto optimum points are not stable solutions in a non-cooperative game, and the Pareto boundary can often be reached only by a cooperative approach, that is not applicable in our context.

where "... each player's strategy is an optimal response to the other players' strategies" [12]. Note that, directly from the definition of NE, it follows that the solution at NE is robust with respect to the worst-case. In fact, at a NE, for each pair, setting $\boldsymbol{p}_{-k} := [\boldsymbol{p}_1, .., \boldsymbol{p}_{k-1}, \boldsymbol{p}_{k+1}, .., \boldsymbol{p}_Q]$, it has to be $R_k(\overline{\boldsymbol{p}}_k, \overline{\boldsymbol{p}}_{-k}) = \max_{\boldsymbol{p}_k} R_k(\boldsymbol{p}_k, \overline{\boldsymbol{p}}_{-k}) \geq \max_{\boldsymbol{p}_k} \min_{\boldsymbol{p}_{-k}} R_k(\boldsymbol{p}_k, \boldsymbol{p}_{-k})$. Thus, at a NE, each player maximizes, at least, its worst rate.

In the following, **1)** we prove that the game $\mathscr{G}$ always admits al least one NE and we provide the sufficient conditions for the uniqueness of the NE. Then, **2)** we propose a simple iterative algorithm able to achieve such a unique NE.

1) The existence of a NE was already discussed in [9] for the case of two users transmitting over frequency selective channels, where a sufficient condition for the existence and uniqueness of a NE was provided. We extend the solution to an arbitrary number of interfering users through the following

Theorem: Given the game $\mathscr{G}$, there exists at least one stable NE. If, for all $i \in [1, Q]$, $k \in [0, N-1]$ and $c < 1$, the following conditions hold true

$$\frac{1}{2}\sum_{j\neq i=1}^{Q}\left[\frac{r_{ii}^{\alpha}}{r_{ij}^{\alpha}}\frac{|H_{ij}(k)|^2}{|H_{ii}(k)|^2}+\frac{r_{ii}^{2\alpha}}{r_{jj}^{\alpha}r_{ji}^{\alpha}}\frac{|H_{jj}(k)|^2|H_{ji}(k)|^2}{|H_{ii}(k)|^2|H_{ii}(k)|^2}\right]\leq c, \tag{4}$$

then the NE of $\mathscr{G}$ is unique.

Proof. See Appendix.

In [2], we proved that the solution set of the above NQ inequalities is always non-empty, and one possible set of solutions is given, $\forall i, j = 1, \dots, Q$ and $i \neq j$, by

$$r_{ij}^{\alpha} > f_{ij}\alpha_{ij}^{-1}(\boldsymbol{r})\, r_{ii}^{\alpha}, \tag{5}$$

where $f_{ij} = \max_k |H_{ij}(k)|^2/|H_{ii}(k)|^2$, $\boldsymbol{r} := [r_{11}, \dots r_{QQ}]$ and the sets of admissible coefficients $\{\alpha_{ij}(\boldsymbol{r})\}_{i\neq j=1}^{Q}$ represent the maximal elements ([13], p. 28) of a polyhedron in $\mathscr{R}_{+}^{Q(Q-1)}$ with respect to componentwise inequality. Using derivations in [10], it can be seen that, e.g. $\alpha_{ij}^{-1}(\boldsymbol{r}) = Q-1$ for all $\boldsymbol{r}$, $i \neq j$ is a set of admissible coefficients. Interestingly, expression (5) has a physical interpretation: In order to assure the uniqueness of the competitive equilibrium, a minimum distance between the cooperating pairs has to be guaranteed. Such a distance corresponds to the maximum level of interference that may be tolerated by each pair and, as we expected, it depends on i) the number Q of pairs; ii) the distance r_{ii} between the MT and RT in each pair; and iii) the worst ratio (f_{ij}) between the channel transfer function of direct link and the channel transfer function of all interference links. It is important to remark that conditions (4) are only sufficient, i.e. a unique NE could exist also if they are not met.

2) In the simple case of two cooperating pairs, the power allocation at NE can be found in closed form and it is given by [2]

$$p_1(k) = \frac{1}{\sigma_s^2}\left(\frac{A_2(k)}{\mu_2}-\frac{A_1(k)}{\mu_1}-\sigma_n^2 B(k)\right)^+, \quad k\in I_1,$$

$$p_2(k) = \frac{1}{\sigma_s^2}\left(\frac{C_1(k)}{\mu_1}-\frac{A_1(k)}{\mu_2}-\sigma_n^2 D(k)\right)^+, \quad k\in I_2, \tag{6}$$

where $(x)^+ := max(0, x)$ and

$$\begin{aligned}
A_1(k) &= (|\widetilde{H}_{22}(k)|^2|\widetilde{H}_{11}(k)|^2)/E(k);\\
A_2(k) &= (|\widetilde{H}_{21}(k)|^2|\widetilde{H}_{22}(k)|^2)/E(k);\\
B(k) &= (|\widetilde{H}_{21}(k)|^2-|\widetilde{H}_{22}(k)|^2)/E(k);\\
C_1(k) &= |\widetilde{H}_{12}(k)|^2|\widetilde{H}_{11}(k)|^2/E(k);\\
D(k) &= (|\widetilde{H}_{12}(k)|^2-|\widetilde{H}_{11}(k)|^2)/E(k);
\end{aligned}$$

where $E(k) = |\widetilde{H}_{12}(k)|^2\,|\widetilde{H}_{21}(k)|^2 - |\widetilde{H}_{22}(k)|^2\,|\widetilde{H}_{11}(k)|^2$ and $|\widetilde{H}_{ij}(k)|^2 := |H_{ij}(k)|^2/r_{ij}^{\alpha}$, for $i, j = 1, \dots, Q$. The symbols I_1 and I_2 denote the sets of sub-carriers allocated to the two MT/RT links, with $I_1, I_2 \subseteq \{0, 1, \dots, N\}$, and, in general, $I_1 \cap I_2 \neq \emptyset$. The constants μ_1 and μ_2 are chosen in order to satisfy the power constraint in (3). More specifically, we get

$$\frac{1}{\mu_1} = \frac{P_T\left[1+\frac{\sum_{k\in I_2}A_1(k)}{\sum_{k\in I_1}A_2(k)}\right]+\sigma_\eta^2\left[\frac{\sum_{k\in I_1}B(k)\sum_{k\in I_2}A_1(k)}{\sum_{k\in I_1}A_2(k)}+\sum_{k\in I_2}D(k)\right]}{\sum_{k\in I_2}C_1(k)-\frac{\sum_{k\in I_1}A_1(k)\sum_{k\in I_2}A_1(k)}{\sum_{k\in I_1}A_2(k)}}$$

$$\frac{1}{\mu_2} = \frac{P_T\left[1+\frac{\sum_{k\in I_1}A_1(k)}{\sum_{k\in I_2}C_1(k)}\right]+\sigma_\eta^2\left[\frac{\sum_{k\in I_2}D(k)\sum_{k\in I_1}A_1(k)}{\sum_{k\in I_2}C_1(k)}+\sum_{k\in I_1}B(k)\right]}{\sum_{k\in I_1}A_2(k)-\frac{\sum_{k\in I_2}A_1(k)\sum_{k\in I_1}A_1(k)}{\sum_{k\in I_2}C_1(k)}}.$$

In [2], we provide a simple iterative algorithm to compute the sets I_1 and I_2. The above solution has been derived assuming that no channel has zeros on the unit circle and that[3] $|\widetilde{H}_{11}(k)|^2|\widetilde{H}_{22}(k)|^2 - |\widetilde{H}_{12}(k)|^2|\widetilde{H}_{21}(k)|^2 \neq 0$, for $k \in I_2$, and $\mu_1|\widetilde{H}_{22}(k)|^2 - \mu_2 |\widetilde{H}_{12}(k)|^2 \neq 0$, for $k \in I_1$.

If $Q > 2$, the closed form solution is not available. In such a case, to achieve the unique NE, an iterative algorithm based on the gradient descent method is proposed in [2]. However, this algorithm requires a *centralized* control. A simpler *distributed* algorithm that achieves the NE can be obtained as follows. From the definition of NE, we deduce that, for each NE, the optimal power allocation strategy for every player of the game $\mathscr{G}$, must be the water-filling power distribution over the available sub-carriers subject to the power constraint P_T and regarding the interference due to the other players as additive (colored) noise. Hence, denoting by $\overline{\boldsymbol{p}} := (\overline{\boldsymbol{p}}_1, \dots, \overline{\boldsymbol{p}}_Q)$ the optimal power distribution adopted by the players, the power allocation reaching one NE must be solution of the following system of implicit equations

$$\overline{p}_k(i) = \left(\frac{1}{\mu_k}-\frac{\sigma_n^2+\sigma_s^2\sum_{j\neq k}^{Q}|\widetilde{H}_{jk}(i)|^2\overline{p}_j(i)}{\sigma_s^2|\widetilde{H}_{kk}(i)|^2}\right)^+,$$

$$\frac{1}{\mu_k} = \frac{P_T+\sum i\in I_k\frac{\sigma_n^2+\sigma_s^2\sum_{j\neq k}^{Q}|\widetilde{H}_{jk}(i)|^2\overline{p}_j(i)}{\sigma_s^2|\widetilde{H}_{kk}(i)|^2}}{N_k}, \quad i\in I_k,\ k\in\Omega, \tag{7}$$

where I_k is the set of sub-carriers allocated for the k-th pair and N_k the cardinality of I_k. Since our game $\mathscr{G}$ admits at least one stable NE, the existence of a simultaneous water-filling solution (7) is guaranteed. It follows that an iterative procedure among the players (cooperating pairs), where at every step, each player (pair) performs the single-user water-filling power distribution (7), regarding the interference from the other players as noise, if it converges, it has to converge to one of the stable NE's, from any starting point. In [10], it has proved that if conditions (5), with $\alpha_{ij}^{-1}(\boldsymbol{r}) = Q-1$, hold true, the iterative water-filling algorithm always converges to the unique NE.

Pairing among MT and RT can be performed by introducing a cost function $f(R_1(\overline{\boldsymbol{p}}), \dots, R_Q(\overline{\boldsymbol{p}})) : \mathscr{R}^Q \mapsto \mathscr{R}$ of the information rates $R_1(\overline{\boldsymbol{p}}), \dots, R_Q(\overline{\boldsymbol{p}})$, like e.g. the sum-rate $f = \sum_{k=1}^{Q} R_k(\overline{\boldsymbol{p}})$, achieved by the proposed IWFA. We may in fact rank each pairing according to $f(R_1(\overline{\boldsymbol{p}}), \dots, R_Q(\overline{\boldsymbol{p}}))$. Although we are not able to insure the convergence of the algorithm to a global solution of (3), we will show via simulation, that, if the best pairing is performed, the rate loss of each pair with respect to its maximum rate achievable without interference, is negligible when $r_{ij}/r_{ii} \gg 1$.

4. SIMULATION RESULTS

We checked our theoretical derivations via numerical results. We have simulated our algorithm using the following setup. The number of active MT's and RT's is $Q = 2$ for Fig.1 and 2, whereas

[3]For fading channels, $\{h_{kj}(n)\}_{k,j=1}^{Q}$ are continuous random variables, so that these events have zero measure.

$Q = 3$ for Fig.3. The size of each transmitted block is $N = 64$ and the number of information symbols in each block is $M = N$; the channels are simulated as FIR filters of order $L = 6$, whose taps are iid complex Gaussian random variables with zero mean and unit variance; the additive noise $\eta_r(n)$, for all $r = 1, 2, \ldots, Q$ is assumed to be drawn from a complex white Gaussian random process with zero mean and variance $\sigma_n^2 = 1$, for each component; the signal to noise ratio $SNR := P_T/\sigma_n^2$ is 5 dB. Each transmitter and receiver is equipped with one antenna. For the sake of simplicity, we have assumed also $r_{ii} = r_{jj}$ and $r_{ij} = r_{ji}$ for all $i, j = 1, 2, \ldots, Q$.

In Fig.1 we compare the results of our closed form expression (6) with the numerical results provided by IWFA. Specifically, we report, for $Q = 2$ (each subplot refers to one of the pairs), the optimal power spectral density (PSD) distribution over the available sub-carriers provided by IWFA (red solid line) and by (6) (red markers). In the same sub-plots, we report also (blue curves) the PSD of the equivalent noise (thermal plus interference noise) normalized by the channel transfer function square modulus of the cooperating pair which the subplot refers to. We have set $r_{11}/r_{12} = r_{22}/r_{21} = 1/5$ (i.e., the distance between MT and RT in each pair is one fifth of the distance between any two different pairs). We have found that, in this case, IWFA converged in only three iterations. In Fig.2, subplots a) and b) depict the same quantities as in Fig.1, but for distance ratios $r_{11}/r_{12} = r_{22}/r_{21} = 1$ (high interference level). To quantify the convergence speed of IWFA, in subplot c) of Fig.2, we report the information rates (normalized by the corresponding rates provided by the closed form as a function of iterations, for the pairs of subplots a) and b). From Figs.1 and 2 we infer that: i) for each pair, the optimal power allocation performs the simultaneous water-filling solution (7); ii) The IWFA converges to the *unique*[4] optimal Nash equilibrium in a very few iterations, also in a high interference level environments (see Fig.2c); iii) depending on the interference level, some sub-carriers are shared, as in Fig.1 where the interference is low, or not, as in Fig.2, where the interference is high and different MT's opt for transmission over non-overlapping bands. In both cases, the simultaneous water-filling solution is reached.

We have pointed out that, if conditions (5) do not hold true, IWFA could not converge. Furthermore, even if IWFA converges, the final NE is not, in general, a global optimal solution of (3). In order to quantify the rate loss due to possible sub-optimal solutions provided by IWFA, we introduce, for each pair, the normalized rate loss $R_k^{loss} := 1 - R_k/R_k^*$, where R_k denotes the k-th pair's rate reached by IWFA for a given channels set, whereas R_k^* is the maximum k-th available rate, achievable in the absence of interference from any other pair. In Fig.3, we report the rate loss R_k^{loss}, for 500 independent channel realizations, in case of three cooperating MT/RT pairs, as a function (for each channels realization) of the ratio r_{kj}/r_{kk}. Experimentally, we have found that IWFA has always converged to the same value, regardless of conditions (5) and the channels and the power budgets for each pair. Interestingly, the rate loss becomes negligible as r_{kj}/r_{kk} increases (e.g., as $r_{kj}/r_{kk} > 10$ the loss falls below 5%).

In summary, in this paper we have shown how to find the optimal coding strategy for MT's in a wireless network, where each MT has an associated RT. The solution is optimal in the sense that it leads to a stable Nash equilibrium for a non-cooperative environment, i.e. when each MT send its data only to the terminal, but it does not share any information with the other MT's. In general, allowing for cooperations among the MT's, one could get an improvement and tend towards the optimal Pareto solutions. However, this would come at the cost of signalling among the MT's, so that the final balance in terms of rate is still an open problem. We have shown under which conditions, depending on the transmit power, relative distance and channel transfer functions, the solution is unique. As expected, the conditions require that the inter-pair (MT-to-MT) distances be sufficiently greater than the intra-pair (MT-to-RT) distance. Nevertheless, we have observed, by simulation, that the proposed iterative water-filling algorithm has always converged, even when the conditions for the uniqueness did not hold true. The suggested strategy can be used in meshed or ad-hoc networks, where there is no infrastructure and each MT can communicate, in principle, with another MT, without passing through a BS, or in multihop networks. In the latter case, specific time slots have to be allocated for MT-to-RT links. Simultaneous exchanges among Q pairs are necessary, to avoid excessive rate losses. In practice, the number Q of pairs will result as a trade-off between the information rate loss due to interference, arising as Q increases, and the waste of resources occurring for low values of Q.

[4]The conditions required in [9] to guarantee a unique Nash equilibrium hold true in this case.

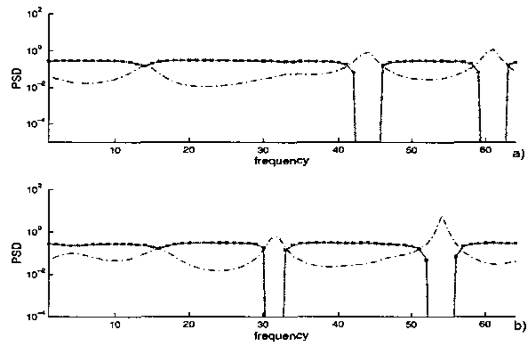

Fig. 1. Optimal PSD's for the two links: IWFA (solid line), theoretical values (stars) and interference from the other pair (dot and dashed line).

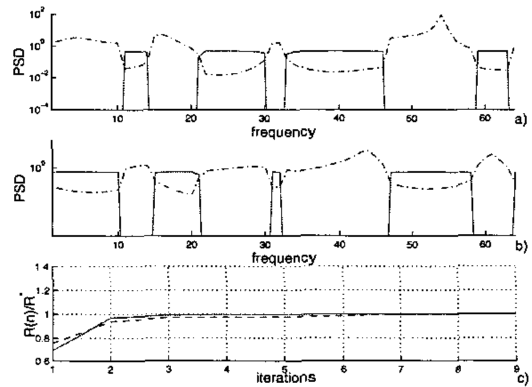

Fig. 2. a)-b) Optimal PSD's for the two links: IWFA (solid line) and interference from the other pair (dot and dashed line); c) rate vs. iteration index.

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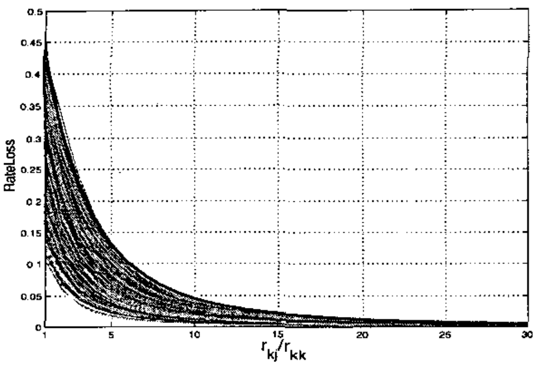

Fig. 3. Rate loss due to simultaneous interfering MT/RT links, as a function of the ratio between the distance between pairs and the distance between MT and RT of each pair.

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6. APPENDIX

We briefly outline the proof of the theorem, building on the general game theory formulation of [11]. As in [11], we denote with $\boldsymbol{x}_k \in E^{m_k}$ the m_k-size vector that represents the strategy (power allocation) of the k-th player, where E^{m_k} is the m_k-dimensional Euclidean space; $\widetilde{\Omega}$ is the set $\{1, 2, \ldots, n\}$ of the n players. We collect all the vectors $\boldsymbol{x}_k$ in $\boldsymbol{x} := [\boldsymbol{x}_1, \ldots, \boldsymbol{x}_n] \in E^m$, where $E^m = E^{m_1} \times \cdots E^{m_n}$ is the product space and $m = \sum_{k=1}^{n} m_k$. Because of the constraints, the overall admissible strategy $\boldsymbol{x}$ is assumed to belong to a subset R of E^m. Denoting by Q_k the orthogonal projection of the set R onto E^{m_k}, we introduce also the set $S := Q_1 \times Q_2 \cdots \times Q_n \supseteq R$. The payoff of the k-th player (its transmission rate) is given by the function $\phi_k(\boldsymbol{x}) : E^m \mapsto \mathscr{R}$, which depends on the strategies of *all* players. We define the weighted nonnegative sum of the payoff functions $\phi_i(\boldsymbol{x})$ as $\sigma(\boldsymbol{x}, \boldsymbol{r}) = \sum_{i=1}^{n} r_i \phi_i(\boldsymbol{x})$, for each nonnegative vector $\boldsymbol{r} := [r_1, \ldots, r_n] \in \mathscr{R}_+^n$, and the associated pseudo-gradient vector $\boldsymbol{g}(\boldsymbol{x}, \boldsymbol{r}) = \left[r_1 \nabla_1 \phi_1^T(\boldsymbol{x}), \ldots, r_n \nabla_n \phi_n^T(\boldsymbol{x})\right]^T$. We introduce the following

Definition: The function $\sigma(\boldsymbol{x}, \boldsymbol{r}) : R \mapsto \mathscr{R}$ is Diagonal Strict Concave (DSC) if, for every $\boldsymbol{x}_0, \boldsymbol{x}_1 \in R$ it holds that $(\boldsymbol{x}_1 - \boldsymbol{x}_0)^T(\boldsymbol{g}(\boldsymbol{x}_0, \boldsymbol{r}) - \boldsymbol{g}(\boldsymbol{x}_1, \boldsymbol{r})) > 0$. As shown in [11], a sufficient condition for $\sigma(\boldsymbol{x}, \boldsymbol{r})$ to be DSC is that the $m \times m$ matrix $\boldsymbol{G}(\boldsymbol{x}, \boldsymbol{r}) + \boldsymbol{G}(\boldsymbol{x}, \boldsymbol{r})^T$ be positive definite for $\boldsymbol{x} \in R$, where $\boldsymbol{G}(\boldsymbol{x}, \boldsymbol{r})$ denotes the Jacobian of $-\boldsymbol{g}(\boldsymbol{x}, \boldsymbol{r})$ with respect to $\boldsymbol{x}$, defined as $[G(\boldsymbol{x}, \boldsymbol{r})]_{ij} = -\partial g_i(\boldsymbol{x}, \boldsymbol{r}) / \partial x_j$, for $i, j = 1, \ldots m$. The game $\widetilde{\mathscr{G}} := \{\widetilde{\Omega}, R, \{\phi_k\}_{k=1}^{n}\}$ *admits at least one stable NE if the following conditions hold true* [12, Theorem 1]: i) The set R is a convex, closed and bounded set; ii) Every payoff function $\phi_k(\boldsymbol{x})$ is continuous in $\boldsymbol{x} \in S$ and concave in $\boldsymbol{x}_k$, for each fixed value of $(\boldsymbol{x}_1, .., \boldsymbol{x}_{k-1}, \boldsymbol{x}_{k+1}, .., \boldsymbol{x}_n)$. The NE is unique if the function $\sigma(\boldsymbol{x}, \boldsymbol{r})$ is DSC for some $\boldsymbol{r} = \overline{\boldsymbol{r}} \succ \boldsymbol{0}$, and such an equilibrium point is independent of $\overline{\boldsymbol{r}}$. To prove our theorem, we start showing that our game $\mathscr{G}$, as defined in Section 3, satisfies the conditions i) and ii). In fact, $\mathscr{G}$ is a special case of $\widetilde{\mathscr{G}}$, where the admissible strategy $\boldsymbol{x}_k$ of each player is restricted to a subset (call it $\overline{Q}_k$) of E^{m_k}, that is orthogonal to the other constraint sets $\overline{Q}_i$, with $i \neq k$. Using the following equivalences $\widetilde{\Omega} \to \Omega$, $E^{m_k} \to \mathscr{R}^N\ \forall k \in \Omega$, $\phi_k(\boldsymbol{x}) \to R_i(\boldsymbol{p}) = \frac{1}{N} \sum_{k=0}^{N-1} \log\left(1 + \frac{|\widetilde{H}_{ii}(k)|^2 p_i(k)}{\sigma_\eta^2/\sigma_s^2 + \sum_{j \neq i=1}^{Q} |\widetilde{H}_{ji}(k)|^2 p_j(k)}\right)$, $\boldsymbol{x}_k \to \boldsymbol{p}_k$, $[\boldsymbol{p}_1, \ldots, \boldsymbol{p}_Q] \to \boldsymbol{x}$, it can be verified that the game $\mathscr{G}$ admits at least one stable NE. Indeed i) $R = S = Q_1 \times, \ldots, \times Q_n$ where $Q_i = \overline{Q}_i = \{\boldsymbol{p}_i \in \mathscr{R}_+^N | \sum_{k=1}^{N} \boldsymbol{p}_i(k) \leq P_T\}$ is the simplex in $\mathscr{R}_+^N$, thus it is a convex, closed and bounded set. Since each Q_i is convex, closed and bounded, it follows that R is also convex, closed and bounded [13]; ii) each function $R_k(\boldsymbol{p})$ is continuous in $\boldsymbol{p}$ and concave with respect to $\boldsymbol{p}_k$ for any fixed $(\boldsymbol{p}_1, \ldots, \boldsymbol{p}_{k-1}, \boldsymbol{p}_{k+1}, \ldots, \boldsymbol{p}_Q)$.

We provide now the sufficient conditions for the NE of the game $\mathscr{G}$ to be unique. Let $\boldsymbol{G}_k(\boldsymbol{p}, \boldsymbol{r})$ be the $Q \times Q$ matrix, defined as

$$\begin{aligned}[G_k(\boldsymbol{p}, \boldsymbol{r})]_{ij} &= -r_i \frac{\partial^2 R_i}{\partial p_i(k) \partial p_j(k)} \\ &= r_i \frac{\log_2 e}{N} \frac{|\widetilde{H}_{ii}(k)|^2 |\widetilde{H}_{ij}(k)|^2}{\sigma_n^2/\sigma_s^2 + \sum_{j=1}^{Q} |\widetilde{H}_{ji}(k)|^2 p_j(k)} \end{aligned} \quad (8)$$

for $i, j = 1, \ldots, Q$ and $k = 0, \ldots, N-1$. Since there always exists a permutation matrix $\boldsymbol{T}$ such that $\boldsymbol{T}\boldsymbol{G}(\boldsymbol{p}, \boldsymbol{r})\boldsymbol{T}^T = \mathrm{diag}\{G_1(\boldsymbol{p}, \boldsymbol{r}), \ldots, G_{N-1}(\boldsymbol{p}, \boldsymbol{r})\}$ [2], the NE of the game $\mathscr{G}$ is unique if the matrix $\boldsymbol{G}_k(\boldsymbol{p}, \boldsymbol{r}) + \boldsymbol{G}_k(\boldsymbol{p}, \boldsymbol{r})^T$ is positive definite for every $\boldsymbol{p}, k \in [0, N-1]$ and some $\boldsymbol{r} \succ \boldsymbol{0}$. Using Gerschgorin's theorem [14], it may be proved that the matrix $\boldsymbol{G}_k(\boldsymbol{p}, \boldsymbol{r}) + \boldsymbol{G}_k(\boldsymbol{p}, \boldsymbol{r})^T$ is positive definite if [2]

$$2\, r_i \left| \frac{\partial^2 R_i}{\partial p_i^2(k)} \right| > \sum_{j \neq i}^{Q} \left[r_i \left| \frac{\partial^2 R_i}{\partial p_i(k) \partial p_j(k)} \right| + r_j \left| \frac{\partial^2 R_j}{\partial p_j(k) \partial p_i(k)} \right| \right] \quad (9)$$

for all $i \in [1, Q]$,$k \in [0, N-1]$ and some $\boldsymbol{r} \succ \boldsymbol{0}$. Introducing (8) in (9) and setting $r_i = r_j$, we obtain the conditions (4).