Communication-Aware Position Control for Mobile Nodes in Vehicular Networks

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Abstract—**In this paper, we study a communication-aware position control problem for mobile nodes in vehicular networks, in which the positions of some nodes can be controlled considering the network performance. We model the average achievable data rate of a link between two nodes as a function of the distance between the two nodes, i.e., as a function of positions of the two nodes. We then try to find the positions of some nodes whose positions can be controlled so as to maximize the minimum weighted average data rate among those of all links in the network. To tackle this problem, we take two approaches: optimization and game theoretic approaches. In the optimization theoretic approach, even though the optimization problem is formulated as non-convex optimization, we can develop algorithms for the optimal solution. However, since those algorithms are centralized algorithms, which may not be applicable to some cases such as vehicular ad-hoc networks (VANETs), we also use the game theoretic approach to develop distributed algorithms. In addition to developing algorithms, we also analyze and compare the performances of our algorithms, showing that the game theoretic approach could provide not only distributed algorithms but also efficient algorithms in our problem.**

Index Terms—**Vehicular networks; Position control; Network performance; Non-convex optimization; Game theory.**

I. Introduction

THE MOBILITY of nodes is one of the unique characteristics of wireless networks such as vehicular networks, cellular networks, and ad-hoc networks. Since there is no physically connected link between nodes, a node can change its position, while still maintaining links with its neighbor nodes. However, since the path gain of a link depends on the distance between its transmitter and receiver, the mobility of nodes greatly affects the performance of the wireless network. Especially, in vehicular networks, communication modules are installed in moving vehicles or mobile robots. Hence, in general, nodes in vehicular networks have high mobility and efficient mobility control and management are important in vehicular networks.

Traditionally, in conventional wireless networks such as cellular and ad-hoc networks, mobility is treated as a (random) perturbation to the network that cannot be controlled. Hence, many network protocols that deal with mobility focus on preventing the mobility of nodes from deteriorating the network performance.

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As in conventional wireless networks, many researches in vehicular networks assumed that the mobility of nodes is not controllable from the point of the network protocol, and thus most network protocols are developed to resolve problems due to the mobility of nodes [1], [2], [3]. However, unlike conventional wireless networks, the mobility of nodes can be controlled relatively easily at the operator's own will in vehicular networks. For this reason, mobility control has also been one of the important issues in vehicular networks. However, thus far, most researches on mobility control in vehicular networks have focused on the mobility control according to vehicular applications [3], such as collision avoidance between vehicles and platooning maneuvers, without considering its impact on the network performance. However, as mentioned before, the mobility of nodes greatly affects the network performance, such as throughput, fairness, and so on, in wireless networks. Hence, when we control the mobility of nodes in vehicular networks, it is important to consider not only its impact on the applications but also its impact on the network performance, which is the main motivation of this paper.

There are many different applications of vehicular networks [3]. Among them, we consider an application where nodes (i.e, vehicles) are organized into a group and have a common mission (e.g., moving toward the same destination with sensing events or military operation). During performing the mission, all nodes have to exchange some information with each other, such as traffic condition, accident situation, and sensing data, and thus improving the network performance is important to exchange data with low delay and low loss rate. From the point of the network performance, we classify nodes into two classes. Some nodes in the network play an important role in performing their mission, and thus their mobility should be controlled only based on its mission. Hence, their mobility cannot be controlled to improve the network performance and we call them "nodes with uncontrollable mobility" from the point of the network performance. On the other hand, some nodes in the network play relatively less important role to perform their mission and their positions can be freely changed to some degree. In this case, we can control the positions of those nodes to improve the network performance, while satisfying constraints on their positions, and we call them "nodes with controllable mobility" from the point of the network performance.

In this paper, we study the problem that determines the optimal positions of nodes with controllable mobility adapting to the movement of nodes with uncontrollable mobility in order to maximize the network performance. We may define

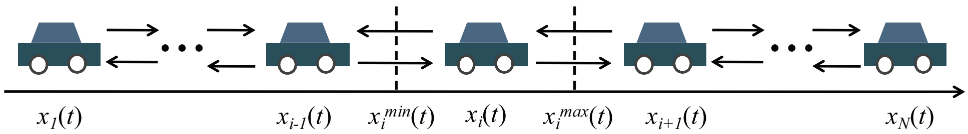


Fig. 1. Vehicular network.

the network performance by using various criteria such as efficiency and fairness. In this paper, we define it as the average data rate of the bottleneck link in the network. Hence, we try to maximize the average data rate of a link whose average data rate is the minimum among those of all links. As we will show later, our criterion for the network performance also provides an effect to improve the degree of fairness among links. In addition, to accommodate differences among links, such as traffic level and importance, we introduce the weight factor to each link, which has a similar role to the weight factor in the weighted max-min fairness. In conclusion, in this paper, we define the network performance as the weighted average data rate of the bottleneck link and we try to maximize it by controlling the position of each node with controllable mobility.

We study our problem considering a linear vehicular network, as in Fig. 1, in which mobile nodes are aligned on a straight line and move toward the same direction. This linear network corresponds to the situation that vehicles move along a highway. Some of nodes have uncontrollable mobility, while some of them have controllable mobility. We model the average data rate of each link as a function of the distance between its two end nodes. To solve the problem, we take two approaches: optimization and game theoretic approaches.

In the optimization theoretic approach, we find the global optimal position of each node with controllable mobility according to the positions of nodes with uncontrollable mobility. Since the capacity of a link is formulated as a non-concave function of positions of its two end nodes, the problem is in fact formulated as a non-convex optimization problem, which is in general difficult to solve. However, even though the problem is non-convex, we can develop algorithms that provide the optimal solution of the problem based on the duality theories.

With the optimization theoretic approach, we develop centralized algorithms that require information on the entire network. However, in practice, it may not be feasible to perform a centralized algorithm in some cases such as the vehicular ad-hoc network (VANET). Hence, we also develop distributed algorithms based on the game theory. We formulate the positioning game in which each node finds its position in a distributed manner with only local information on its neighbor nodes. We show that there exists a Nash equilibrium in the positioning game. We also provide an algorithm that converges to the Nash equilibrium.

Our contributions are summarized as follows:

- Unlike previous researches on position control of nodes in vehicular networks, which try to obtain the positions of nodes considering applications, in this paper, we study the position control problem in which the position of each node is determined considering the network performance.
- We provide analytic models and algorithms for the position control problem with optimization and game theoretic frameworks.
- We analyze and compare the performances of various algorithms.

This paper is organized as follows. We present some related work in Section II. In Section III, we introduce our system model and problem. We provide algorithms from the optimization theoretic approach and the game theoretic approach in Sections IV and V, respectively. We provide the numerical results in Section VI and conclude in Section VII.

II. Related Work

As mentioned in the previous section, most researches on position control problems in vehicular networks deal with position control considering only vehicular applications and communication between vehicles is used as a tool to exchange information for position control [3]. In [4], the inter-vehicle communication is used to prevent the rear-end collision accident between vehicles, which are moving in a string. In [5], the authors address the problem of the platoon formation of vehicles by using the hierarchical architecture for collaborative driving in an autonomous collaborative driving system. In [6], the authors propose a scheduling algorithm in order to prevent the congestion and accident at blind crossings by using the inter-vehicle communication. For more details, we refer readers to [3] and references therein. Recently, gateway placement problems according to the positions of access points are studied in [7]. In this work, the impacts of the placement of gateways on the network performance such as the number of hops to the gateway and total power consumption are considered.

In fact, recently, position control problems are extensively studied in sensor and ad-hoc networks. In [8], [9], [10], [11], problems for relay node placement considering power consumption or network lifetime are studied. In those works, the transmission power consumption is modeled as a function of the distance between transmitter and receiver nodes. Based on the model for the transmission power, the relay node placement that aims at minimizing power consumption or maximizing network lifetime is obtained.

In [12], [13], [14], problems that place the minimum number of relay nodes are studied. In [12], the algorithm that places the minimum number of relay nodes to ensure the connectivity of sensor nodes and base stations is proposed. In [13], joint relay node placement and channel assignment problem in wireless mesh networks is studied. Assuming that link capacities and traffic demands are fixed, joint relay node placement and channel assignment, which minimizes the number of required relay nodes while satisfying traffic demands, is obtained. In [14], considering cooperative transmissions, a heuristic algorithm for the relay node placement that minimizes the number of required relay nodes is proposed.

In [15], [16], [17], problems that place relay nodes considering the throughput of the network are studied. In [15], considering the probabilistic model for the positions of nodes, an algorithm for the relay node placement that maximizes the throughput of the network is proposed. In [16], [17], similar models to our model are considered. In [16], joint relay node placement and assignment problem is studied. Once the relay node assignment is determined, the problem in [16] is reduced to the problem to find the optimal position of a single relay node while other nodes that communicate with the relay node are assumed to be fixed. In [17], it is also assumed that only one relay node is mobile and all other nodes that communicate with the relay node are fixed. Hence, the problems in [16], [17] aim at finding the optimal position of a single relay node given a set of fixed nodes that communicate only with the relay node, which is different from ours.

III. System Model and Problem

We consider a vehicle network that consists of set $\mathbf{N}$ of N mobile nodes that are aligned on a line and move toward the same direction, as in Fig. 1. Nodes are divided into two subsets, $\mathbf{N}_F$ and $\mathbf{N}_M$. Nodes in set $\mathbf{N}_F$ have uncontrollable mobility and their positions cannot be controlled to improve the network performance. Nodes in set $\mathbf{N}_M$ have controllable mobility and their positions can be controlled to improve the network performance based on positions of nodes in set $\mathbf{N}_F$.

Since nodes are moving, the position of each node keeps varying over time and it can be represented as a function of time. However, in our problem below, since the link capacity between two nodes is modeled as a function of the distance between them, the absolute position of each node is not important but its relative position to its neighbor nodes is important. This implies that if all nodes are moving at the same and constant speed, i.e., *the reference speed*, nodes are relatively static with respect to other nodes, and thus we can model the vehicle network as a virtually static network, in which nodes are not moving. Hence, we model a node as a static node, if it moves at the reference speed. A node can change its position by increasing or decreasing its speed from the reference speed. However, we assume that once it is located at the desired position, it returns to the reference speed to keep its position. In this paper, we consider a situation when nodes with uncontrollable mobility return to the reference speed after they changed their positions according to their mission by changing their speed temporarily. This implies that we can consider each node with uncontrollable mobility as a static node with a fixed position.

With the above model, we can consider an equivalent linear wireless network that consists of set $\mathbf{N}$ of N nodes (i.e., nodes $1, 2, \cdots, N$) that are aligned on a line, as in Fig. 2. The set of nodes are divided into two subsets, $\mathbf{N}_F$ and $\mathbf{N}_M$. Nodes in subset $\mathbf{N}_F$ are static nodes whose positions are fixed. Nodes in subset $\mathbf{N}_M$ have controllable mobility, and thus we can control their positions according to the positions of nodes in subset $\mathbf{N}_F$.

Since we consider a one-dimensional linear network, the position of node i is denoted by x_i. Even though a node can be mobile, we assume that its movement is constrained within an interval, which is defined as

$$x_i^{min} \le x_i \le x_i^{max}, \quad \forall i \in \mathbf{N}.$$

Without loss of generality, we can assume that

$$x_i^{min} = x_i^{max}, \quad \forall i \in \mathbf{N}_F.$$

In addition, we assume that

$$x_{i-1}^{max} < x_i^{min}, \quad \forall i \in \mathbf{N},$$

i.e., nodes are ordered in an increasing order of their positions. The values of x_i^{min} and x_i^{max} can be determined considering various constraints such as the required safe distance between adjacent nodes and the area within which each node should remain to accomplish its mission. The values of x_i^{min} and x_i^{max} could affect the network performance. However, determining them is out of scope in this paper. Hence, we assume that the values of x_i^{min} and x_i^{max} are predetermined and do not discuss its effect on the network performance in this paper.

Each node can communicate with only its neighbor nodes and the neighbor node set of node i is defined as

$$\mathbf{N}_N(i) = \begin{cases} \{2\}, & \text{if } i = 1 \\ \{N-1\}, & \text{if } i = N \\ \{i-1, i+1\}, & \text{otherwise} \end{cases}.$$

We assume that the path gain of a link from node i to its neighbor node j depends on the distance between nodes i and j as

$$K_{i,j} d_{i,j}^{-\alpha}, \quad \forall i \in \mathbf{N},\ \forall j \in \mathbf{N}_N(i),$$

where $K_{i,j}$ is some constant that depends on transmitter and receiver nodes i and j, $d_{i,j}$ is the distance between nodes i and j, and α is the pass gain exponent. By using the position of each node, the distance between node i and its neighbor node j is obtained as

$$\begin{aligned} d_{i,j}(x_i, x_j) &= |x_i - x_j| \\ &= \begin{cases} x_i - x_j, & \text{if } j = i-1 \\ x_j - x_i, & \text{if } j = i+1 \end{cases}, \\ &\quad \forall i \in \mathbf{N},\ \forall j \in \mathbf{N}_N(i). \end{aligned} \tag{1}$$

Each node i can transmit its data to its neighbor node j at a rate given by the Shannon capacity according to the signal to noise ratio (SNR) at node j. The SNR of a link from node i to node j, $\gamma_{i,j}$, is obtained as

$$\gamma_{i,j}(x_i, x_j) = \frac{K_{i,j} d_{i,j}(x_i, x_j)^{-\alpha} P_i}{N_0}, \quad \forall i \in \mathbf{N},\ \forall j \in \mathbf{N}_N(i), \tag{2}$$

where P_i is the transmission power of node i and N_0 is the noise power level. Hence, the capacity of a link from node i to its neighbor node j, $C_{i,j}(x_i, x_j)$, is given as

$$C_{i,j}(x_i, x_j) = W \log_2 \left(1 + \gamma_{i,j}(x_i, x_j)\right), \quad \forall i \in \mathbf{N},\ \forall j \in \mathbf{N}_N(i), \tag{3}$$

where W is the bandwidth of the system. In addition, we assume that the underlying medium access control (MAC) protocol is given (either by scheduling or random access)

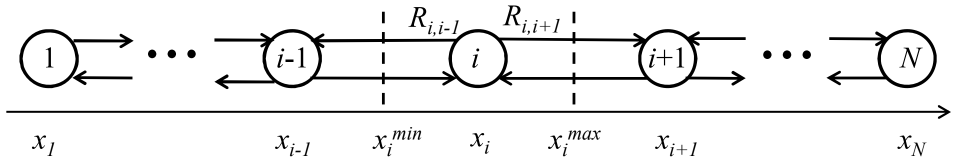


Fig. 2. Linear network.

such that the time fraction that node i can transmit its data to its neighbor node j successfully is given by $T_{i,j}$. Hence, the average data rate of a link from node i to its neighbor node j, $R_{i,j}(x_i, x_j)$, is obtained as

$$\begin{aligned} R_{i,j}(x_i, x_j) &= T_{i,j}C_{i,j}(x_i, x_j) \\ &= T_{i,j}W\log_2(1+\gamma_{i,j}(x_i, x_j)), \\ &\quad \forall i \in \mathbf{N},\ \forall j \in \mathbf{N}_N(i). \end{aligned} \tag{4}$$

For each link from node i to its neighbor node j, we define its weighted average data rate as

$$\begin{aligned} R^W_{i,j}(x_i, x_j) = \frac{1}{w_{i,j}} R_{i,j}(x_i, x_j), \\ \forall i \in \mathbf{N},\ \forall j \in \mathbf{N}_N(i), \end{aligned} \tag{5}$$

where $w_{i,j}$ is the weight factor for a link from node i to node j that determines its relative performance requirement against other links. In other words, the higher the weight factor of a link is, the higher its relative performance requirement is.

The objective of our problem is to find the position of each node with controllable mobility in $\mathbf{N}_M$ that maximizes the minimum of weighted average data rates of all links in the network. Hence, the problem is formulated as follows:

$$\begin{aligned} \max_{x_i, \forall i \in \mathbf{N}} \quad & \min_{i\in\mathbf{N},\, j\in\mathbf{N}_N(i),\, i\notin\mathbf{N}_F \vee j\notin\mathbf{N}_F} R^W_{i,j}(x_i, x_j) \\ \text{subject to} \quad & x_i^{min} \le x_i \le x_i^{max},\ \forall i \in \mathbf{N}_M \\ & x_i = x_i^*,\ \forall i \in \mathbf{N}_F, \end{aligned} \tag{6}$$

where x_i^* is the fixed position of node i in $\mathbf{N}_F$ and $A \vee B$ means A or B. In the above problem, $(i \notin \mathbf{N}_F \vee j \notin \mathbf{N}_F)$ implies that we do not consider links whose two end nodes are static when we solve the problem, since in such a case, its capacity is fixed and uncontrollable.

IV. Optimization theoretic approach

In this section, we develop an algorithm for determining the optimal position of each node in the network, i.e., an algorithm that solves problem (6). Note that problem (6) is a non-convex optimization problem, which is in general difficult to solve, since its objective function is not a concave function. In fact, a similar problem was considered in our previous work [17]. In [17], a system in which the position of only a single relay node can be controlled was considered assuming that the positions of other nodes are fixed. Hence, the system considered in this paper is more general than that in [17] in the sense that in the system in this paper, multiple nodes have controllable mobility. Hence, the mobility of one node affects the position of the other node. However, we can solve problem (6) with a similar approach in [17].

We first reformulate problem (6) by introducing a new variable Z as

$$\begin{aligned} \max_{Z, x_i, \forall i \in \mathbf{N}} \quad & Z \\ \text{subject to} \quad & R^W_{i,j}(x_i, x_j) \ge Z, \\ & \forall i \in \mathbf{N}, \forall j \in \mathbf{N}_N(i),\ i \notin \mathbf{N}_F \vee j \notin \mathbf{N}_F \\ & x_i^{min} \le x_i \le x_i^{max}, \quad \forall i \in \mathbf{N}_M \\ & x_i = x_i^*, \quad \forall i \in \mathbf{N}_F. \end{aligned} \tag{7}$$

In addition, by using (1)-(5), problem (7) can be rewritten as

$$\begin{aligned} \max_{Z, x_i, \forall i \in \mathbf{N}} \quad & Z \\ \text{subject to} \quad & (x_i - x_j)^2 \le M_{ij}(Z), \\ & \forall i \in \mathbf{N},\ \forall j \in \mathbf{N}_N(i),\ i \notin \mathbf{N}_F \vee j \notin \mathbf{N}_F \\ & x_i^{min} \le x_i \le x_i^{max}, \quad \forall i \in \mathbf{N}_M \\ & x_i = x_i^*, \quad \forall i \in \mathbf{N}_F, \end{aligned} \tag{8}$$

where $M_{ij}(Z) = \left(\frac{N_0}{K_{i,j}P_i}\left(2^{\frac{w_{i,j}Z}{WT_{i,j}}} - 1\right)\right)^{-2/\alpha}$.

We now define the feasible set of the above problem as a function of Z as

$$\begin{aligned} \Omega(Z) = \{(x_i), \forall i \in \mathbf{N} \mid (x_i - x_j)^2 \le M_{ij}(Z), \\ \forall i \in \mathbf{N}, \forall j \in \mathbf{N}_N(i), i \notin \mathbf{N}_F \vee j \notin \mathbf{N}_F, \\ x_i^{min} \le x_i \le x_i^{max}, \\ \forall i \in \mathbf{N}_M,\ \ x_i = x_i^*, \forall i \in \mathbf{N}_F\}. \end{aligned}$$

In addition, we also define the following set

$$\begin{aligned} \Lambda = \{(x_i), \forall i \in \mathbf{N} \mid x_i^{min} \le x_i \le x_i^{max},\ \forall i \in \mathbf{N}_M, \\ x_i = x_i^*, \forall i \in \mathbf{N}_F\}. \end{aligned}$$

Since $M_{i,j}(Z)$ is strictly decreasing in Z, we can easily show the following proposition.

Proposition 1: Feasible set $\Omega(Z)$ is decreasing in Z in the sense that for fixed Z_1 and Z_2, $Z_1 \ge Z_2$ if and only if $\Omega(Z_1) \subset \Omega(Z_2)$. In addition, for Z such that $\Omega(Z) \ne \emptyset$ and $\Omega(Z) \ne \Lambda$, $\Omega(Z)$ is strictly decreasing in Z in the sense that for fixed Z_1 and Z_2 such that $\Omega(Z_1) \ne \emptyset$, $\Omega(Z_1) \ne \Lambda$, $\Omega(Z_2) \ne \emptyset$, and $\Omega(Z_2) \ne \Lambda$, $Z_1 > Z_2$, if and only if $\Omega(Z_1) \subset \Omega(Z_2)$ and $\Omega(Z_1) \ne \Omega(Z_2)$.

Let us denote Z^* and x_i^*, $\forall i \in \mathbf{N}$ be the optimal solutions of problem (8). Without loss of generality, we can assume that $\Omega(Z^*) \ne \Lambda$. By using the decreasing property of $\Omega(Z)$ in Proposition 1, we can obtain the optimal Z^* by solving the following optimization problem:

$$\begin{aligned} \max_{Z} \quad & Z \\ \text{subject to} \quad & \Omega(Z) \ne \emptyset, \end{aligned} \tag{9}$$

and the problem can be solved by using a simple line search algorithm such as the bisection algorithm, as in Table I.

TABLE I
BISECTION ALGORITHM FOR OBTAINING OPTIMAL Z^*.

Let $Z^* = 0$.
Find two points a and b such that $\Omega(a) \neq \emptyset$ and $\Omega(b) = \emptyset$. Then, obviously $a < b$.
Let ϵ be a small constant.
while $(b - a > \epsilon)$ {
Let $Z = (a + b)/2$.
Check whether $\Omega(Z)$ is an empty set or not.
If $\Omega(Z) \neq \emptyset$, then $a = Z$ and $Z^* = Z$.
Else if $\Omega(Z) = \emptyset$, then $b = Z$.
} **end**

To obtain optimal Z^*, we should be able to check if feasible set $\Omega(Z)$ is empty or not for given Z and it can be checked by solving the following optimization problem with given Z by using its dual problem.

$$\begin{aligned} \max_{x_i, \forall i \in \mathbf{N}} \quad & 0 \\ \text{subject to} \quad & (x_i - x_j)^2 \leq M_{ij}(Z), \\ & \forall i \in \mathbf{N},\ \forall j \in \mathbf{N}_N(i),\ i \notin \mathbf{N}_F \vee j \notin \mathbf{N}_F \\ & x_i^{min} \leq x_i \leq x_i^{max}, \quad \forall i \in \mathbf{N}_M \\ & x_i = x_i^*, \quad \forall i \in \mathbf{N}_F. \end{aligned} \tag{10}$$

Note that with fixed Z, the above problem is a convex optimization problem. Hence, there is no duality gap between the above problem and its dual problem.

We first define the Lagrangian function of problem (10) as

$$L_Z(\boldsymbol{x}, \boldsymbol{\lambda}) = 0 + \sum_{i \in \mathbf{N}} \sum_{j \in \mathbf{N}_N(i),\ i \notin \mathbf{N}_F \vee j \notin \mathbf{N}_F} \lambda_{ij} \left(M_{ij}(Z) - (x_i - x_j)^2 \right),$$

where $\boldsymbol{x} = (x_i)_{i \in \mathbf{N}}$ and $\boldsymbol{\lambda} = (\lambda_{ij})_{i \in \mathbf{N},\ j \in \mathbf{N}_N(i),\ i \notin \mathbf{N}_F \vee j \notin \mathbf{N}_F}$. Then, the dual objective function and is defined as

$$D_Z(\boldsymbol{\lambda}) = \max_{\substack{x_i^{min} \leq x_i \leq x_i^{max},\ \forall i \in \mathbf{N}_M \\ x_i = x_i^*,\ \forall i \in \mathbf{N}_F}} L_Z(\boldsymbol{x}, \boldsymbol{\lambda}) \tag{11}$$

and the dual problem is defined as

$$\min_{\boldsymbol{\lambda} \geq \mathbf{0}} D_Z(\boldsymbol{\lambda}). \tag{12}$$

We first consider the optimization problem in (11) with fixed $\boldsymbol{\lambda}$ and obtain its solutions $x_i(\boldsymbol{\lambda})$, $\forall i \in \mathbf{N}$. Since this optimization problem is also a convex optimization, it can be easily solved by using one of standard methods such as dual approach, penalty approach, and KKT conditions.

We then solve dual problem (12) by using the gradient projection algorithm as:

$$\lambda_{ij}^{(n+1)} = \left[\lambda_{ij}^{(n)} - a^{(n)} \frac{\partial D_Z(\boldsymbol{\lambda}^{(n)})}{\partial \lambda_{ij}} \right]^+, \quad \forall i \in \mathbf{N},\ \forall j \in \mathbf{N}_N(i), \tag{13}$$

where

$$\frac{\partial D_Z(\boldsymbol{\lambda}^{(n)})}{\partial \lambda_{ij}} = M_{ij}(Z) - \left(x_i(\boldsymbol{\lambda}^{(n)}) - x_j(\boldsymbol{\lambda}^{(n)}) \right)^2.$$

As $n \to \infty$, $\boldsymbol{\lambda}^{(n)}$ converges to the optimal solution $\boldsymbol{\lambda}^*$ and also dual objective value $D_Z(\boldsymbol{\lambda}^{(n)})$ converges to the optimal dual objective value D_Z^*.

TABLE II
ALGORITHM FOR CHECKING WHETHER $\Omega(Z)$ IS AN EMPTY SET OR NOT FOR GIVEN Z.

Let m be a large constant.
Let $n = 0$.
Initialize $\boldsymbol{\lambda}^{(0)}$ and $\boldsymbol{x}(\boldsymbol{\lambda}^{(0)})$.
while $(n < m)$ {
$n = n + 1$
Calculate $\boldsymbol{\lambda}^{(n)}$ in (13).
Calculate $\boldsymbol{x}(\boldsymbol{\lambda}^{(n)})$ by solving the optimization problem in (11) with fixed $\boldsymbol{\lambda}^{(n)}$.
If $D(\boldsymbol{\lambda}^{(n)}) < 0$ in (11), then $\Omega(Z) = \emptyset$ and **break**.
} **end**
If $D(\boldsymbol{\lambda}^{(n)}) \geq 0$ in (11), then $\Omega(Z) \neq \emptyset$.

Then, by the duality theorem to the convex optimization [18], we have the following properties:

- If $D_Z^* = 0$, which is the optimal primal objective value, then problem (10) has a non-empty feasible set;
- If $D_Z^* = -\infty$, then problem (10) has an empty feasible set.

In other words, by checking which value $D_Z(\boldsymbol{\lambda}^{(n)})$ converges to, we can check if problem (10) has an empty feasible set or not. In addition, by the weak duality theorem, if problem (10) has a non-empty feasible set,

$$0 \leq D_Z^* \leq D_Z(\boldsymbol{\lambda}^{(n)}), \quad \forall \boldsymbol{\lambda}^{(n)} \geq \mathbf{0}.$$

Hence, $D_Z(\boldsymbol{\lambda}^{(n)}) < 0$ for some iteration n, then problem (10) has an empty feasible set. The pseudo-code of the algorithm that checks whether $\Omega(Z)$ is a empty set or not for given Z is provided in Table II.

After obtaining the optimal Z^*, the optimal positions of nodes $\boldsymbol{x}^*$ can be found by solving problem (10) with fixed Z^*, which is a convex optimization problem. Thus far, we have derived the algorithm that provides the global optimal solution of (6). In the following, we will show that the global optimal solution of (6) may not be unique. In addition, we will develop an improved algorithm that selects a better solution among solutions of (6).

Proposition 2: The optimal position of each node in (6) may not be unique.

Proof: We prove this proposition by taking a simple example that allows multiple optimal positions. Consider a linear network that consists of four nodes that are homogeneous, i.e., all nodes have the same parameters except for their positions. Nodes 1 and 4 have their fixed positions at 1 and 10, respectively. Nodes 2 and 3 have their positioning range $1.5 \leq x_2 \leq 2.5$ and $2.5 \leq x_3 \leq 3.5$, respectively. Since node 4 is too far away from node 3, regardless of positions of nodes 2 and 3, a link between nodes 3 and 4 has always the minimum weighted average data rate, and thus the optimal position of node 3 is 3.5 in (6). Note that in this case, any position between 1.5 and 2.5 is the optimal position of node 2 in (6), since it does not affect the objective function, which proves the proposition. ■

As shown in the above proposition, there may exist multiple optimal solutions all of which maximize the minimum of weighted average data rates of all links. Even though all of them provide the same weighted average data rate for the link with the minimum weighted average data rate in the network,

TABLE III
ALGORITHM FOR THE IMPROVEMENT OF THE DEGREE OF FAIRNESS.

1:	Initialize sets $\mathbf{N}_F$ and $\mathbf{N}_M$ from the system.
2:	**while** ($\mathbf{N}_M \neq \emptyset$){
3:	Solve problem (6) by using algorithms in Tables I and II.
4:	Obtain two nodes i^* and j^* that are end-nodes of the link that achieves the minimum weighted average rate and their positions $x^*_{i^*}$ and $x^*_{j^*}$.
5:	Let $\mathbf{N}_M = \mathbf{N}_M - \{i^*, j^*\}$ and $\mathbf{N}_F = \mathbf{N}_F \cup \{i^*, j^*\}$ with $x_{i^*} = x^*_{i^*}$ and $x_{j^*} = x^*_{j^*}$.
6:	} **end**

each of which may provide different weighted average data rates for the other links. Some of them may provide a better degree of fairness, while others provide a worse degree of fairness. In the example in the proof of the above proposition, even though all positions between 1.5 and 2.5 are the optimal position of node 2, we can easily see that if node 2 is located at 2.25, a link between nodes 1 and 2 and a link between nodes 2 and 3 have the same weighted average data rate, and thus the degree of fairness could be improved. Hence, to improve the degree of fairness of the solution maintaining its optimality, we propose an improved algorithm in Table III.

V. NON-COOPERATIVE GAME THEORETIC APPROACH

In the previous section, we developed algorithms that provide the global optimal position of each node that solves problem (6). However, algorithms developed in the previous section are centralized algorithms that require a central controller. In many cases such as VANET, such a centralized approach may not be feasible. Hence, in this section, instead of obtaining the global optimal position of each node with a centralized optimization theoretic approach, we consider a distributed game theoretic approach, in which each node with controllable mobility tries to find its own position considering the positions of only its neighbor nodes. We consider two different cases: a case in which each node considers only its transmitting links and a case in which each node considers not only its transmitting links but also its receiving links.

A. Non-cooperative game approach considering transmitting links

We first consider the case in which each node considers only its transmitting links. The Positioning Game is defined as $PG^T(P, A, U)$, where P is the set of players of the game, which are nodes with controllable mobility in set $\mathbf{N}_M$, A is the strategy set, in which the strategy of each player i, $A_i = \{x_i \mid x_i^{min} \le x_i \le x_i^{max}\}$, and $U = (U_i)_{i \in P}$, in which U_i is the utility function of each player i. The utility function of player i is defined as the lowest weighted average data rate among those of its all transmitting links, i.e.,

$$U_i(\boldsymbol{x}) = U_i(x_i, \boldsymbol{x}_{-i}) = \min_{j \in \mathbf{N}_N(i)} R^W_{i,j}(x_i, x_j), \qquad (14)$$

where $\boldsymbol{x} = (x_j)_{j \in \mathbf{N}}$ and $\boldsymbol{x}_{-i} = (x_j)_{j \in \mathbf{N},\ j \neq i}$.

We first show that the Positioning Game $PG^T(P, A, U)$ admits the Nash equilibrium.

Lemma 1: The utility function of each player i, $U_i(x_i, \boldsymbol{x}_{-i})$ in (14) is a quasiconcave function of x_i.

Proof: We first show that with fixed x_j, $j \neq i$, the weighted data rate of node i to its neighbor node j is a quasiconcave function of x_i. To this end, we only have to show that for every c, its superlevel set $S^c_{i,j}$,

$$S^c_{i,j} = \{x_i \mid R^W_{i,j}(x_i, x_j) \ge c\}$$

is a convex set. From (1)-(5), the weighted data rate of node i to its neighbor node j can be written as a function of its location x_i as

$$R^W_{i,j}(x_i, x_j) = \frac{T_{i,j} W}{w_{i,j}} \log_2 \left(1 + \frac{K_{i,j} |x_i - x_j|^{-\alpha} P_i}{N_0}\right).$$

For given c, from the following

$$\frac{T_{i,j} W}{w_{i,j}} \log_2 \left(1 + \frac{K_{i,j} |x_i - x_j|^{-\alpha} P_i}{N_0}\right) \ge c,$$

we have

$$|x_i - x_j| \le \left(\frac{N_0}{K_{i,j} P_i}\left(2^{\frac{c w_{i,j}}{W T_{i,j}}} - 1\right)\right)^{-1/\alpha}.$$

Note that the righthand side of the above inequality is a constant. Hence,

$$S^c_{i,j} = \left\{x_i \mid |x_i - x_j| \le \left(\frac{N_0}{K_{i,j} P_i}\left(2^{\frac{c w_{i,j}}{W T_{i,j}}} - 1\right)\right)^{-1/\alpha}\right\}$$

and $S^c_{i,j}$ is a convex set for any c. Since every superlevel set of $R^W_{i,j}(x_i, x_j)$ is a convex set with fixed x_j, $R^W_{i,j}(x_i, x_j)$ is a quasiconcave function of x_i [18]. In addition, since $U_i(x_i, \boldsymbol{x}_{-i})$ is the minimum of $R^W_{i,j}(x_i, x_j)$'s over $j \in N_N(i)$, which are all quasiconcave functions of x_i, $U_i(x_i, \boldsymbol{x}_{-i})$ is also a quasiconcave function of x_i [18]. ■

Theorem 1: The Positioning Game $PG^T(P, A, U)$ has a Nash equilibrium.

Proof: From the above lemma, the utility function of each node i, which is the minimum of the weighed data rates of its transmitting links to its neighbor nodes, is quasiconcave. Since the utility function of each node i, $U_i(x_i, \boldsymbol{x}_{-i})$, is a continuous and quasiconcave function of x_i and its strategy set A_i is nonempty compact convex set, Positioning Game $PG^T(P, A, U)$ has a Nash equilibrium [19]. ■

We now develop the algorithm for each node that converges to the Nash equilibrium. To this end, we first study the properties of the Nash equilibrium position of each node. Since nodes 1 and N have only one neighbor, i.e., node 2 and node $N-1$, respectively, their Nash equilibrium position is easily obtained as the following theorem.

Theorem 2: The Nash equilibrium positions of x_1 and x_N, x_1^{Nash} and x_N^{Nash}, are obtained as

$$x_1^{Nash} = x_1^{max} \quad \text{and} \quad x_N^{Nash} = x_N^{min}.$$

Hence, we can assume that nodes 1 and N have fixed positions at x_1^{max} and x_N^{min}, respectively and in the following, we will develop the algorithm for node i, $i = 2, 3, \cdots, N-1$, that converges to the Nash equilibrium. Since each node i has two neighbor nodes, nodes $(i-1)$ and $(i+1)$, which are in the opposite directions from node i, we can easily show the following theorem.

Theorem 3: For node i, $i = 2, 3, \cdots, N-1$, the Nash equilibrium position, x_i^{Nash}, is obtained as

$$x_i^{Nash} = \begin{cases} x_i^{min}, & \text{if } x_i^* \le x_i^{min} \\ x_i^{max}, & \text{if } x_i^* \ge x_i^{max} \\ x_i^*, & \text{otherwise} \end{cases}, \quad i = 2, 3, \cdots, N-1, \tag{15}$$

where x_i^* is a position that satisfies

$$R_{i,i-1}^W(x_i^*, x_{i-1}^{Nash}) = R_{i,i+1}^W(x_i^*, x_{i+1}^{Nash}),$$

when other nodes are at their Nash equilibrium, i.e., $x_j = x_j^{Nash}$, $j \ne i$.

Proof: Since nodes are ordered in an increasing order of their positions, given x_{i-1}^{Nash} and x_{i+1}^{Nash}, $R_{i,i-1}^W(x_i, x_{i-1}^{Nash})$ is strictly decreasing in x_i and $R_{i,i+1}^W(x_i, x_{i+1}^{Nash})$ is strictly increasing in x_i. Hence, $U_i(x_i) = \min[R_{i,i-1}^W(x_i, x_{i-1}^{Nash}), R_{i,i+1}^W(x_i, x_{i+1}^{Nash})]$ is maximized at a unique position x_i^* such that

$$R_{i,i-1}^W(x_i^*, x_{i-1}^{Nash}) = R_{i,i+1}^W(x_i^*, x_{i+1}^{Nash}). \tag{16}$$

In addition, since we have a constraint, $x_i^{min} \le x_i \le x_i^{max}$, the Nash equilibrium position of node i is obtained as in (15). ■

Corollary 1: We have the following properties:

1) If $x_i^{min} < x_i^{Nash} < x_i^{max}$, then $R_{i,i-1}^W(x_i^{Nash}, x_{i-1}^{Nash}) = R_{i,i+1}^W(x_i^{Nash}, x_{i+1}^{Nash})$.
2) If $x_i^{Nash} = x_i^{min}$, then $R_{i,i-1}^W(x_i^{Nash}, x_{i-1}^{Nash}) \le R_{i,i+1}^W(x_i^{Nash}, x_{i+1}^{Nash})$ and $R_{i,i-1}^W(x_i, x_{i-1}^{Nash}) < R_{i,i+1}^W(x_i, x_{i+1}^{Nash})$, $\forall x_i > x_i^{min}$.
3) If $x_i^{Nash} = x_i^{max}$, then $R_{i,i-1}^W(x_i^{Nash}, x_{i-1}^{Nash}) \ge R_{i,i+1}^W(x_i^{Nash}, x_{i+1}^{Nash})$ and $R_{i,i-1}^W(x_i, x_{i-1}^{Nash}) > R_{i,i+1}^W(x_i, x_{i+1}^{Nash})$, $\forall x_i < x_i^{max}$.

From Corollary 1, we now consider the following iterative algorithm:

$$x_i(n+1) = x_i(n) - \gamma f_i(\boldsymbol{x}(n)), \quad i = 2, 3, \cdots, N-1, \tag{17}$$

where

$$f_i(\boldsymbol{x}) = R_{i,i+1}^W(x_i, x_{i+1}) - R_{i,i-1}^W(x_i, x_{i-1}) + \beta x_i,$$

and γ and β are small positive constants.

We first show that it is a contraction mapping.

Proposition 3: The algorithm in (17) is a contraction mapping.

Proof: From (1) - (5),

$$R_{i,i+1}^W(x_i, x_{i+1}) = \frac{T_{i,i+1}W}{w_{i,i+1}} \times \log_2\left(1 + \frac{K_{i,i+1}(x_{i+1} - x_i)^{-\alpha} P_i}{N_0}\right)$$

and

$$R_{i,i-1}^W(x_i, x_{i-1}) = \frac{T_{i,i-1}W}{w_{i,i-1}} \times \log_2\left(1 + \frac{K_{i,i-1}(x_i - x_{i-1})^{-\alpha} P_i}{N_0}\right).$$

Hence,

$$\frac{\partial R_{i,i+1}^W(x_i, x_{i+1})}{\partial x_i} = \frac{T_{i,i+1}W}{w_{i,i+1}} \times \frac{1}{\log 2 \times \left(1 + \frac{K_{i,i+1}(x_{i+1}-x_i)^{-\alpha}P_i}{N_0}\right)} \times \frac{\alpha K_{i,i+1}(x_{i+1} - x_i)^{-\alpha-1} P_i}{N_0} \tag{18}$$

and

$$\frac{\partial R_{i,i-1}^W(x_i, x_{i-1})}{\partial x_i} = -\frac{T_{i,i-1}W}{w_{i,i-1}} \times \frac{1}{\log 2 \times \left(1 + \frac{K_{i,i-1}(x_i-x_{i-1})^{-\alpha}P_i}{N_0}\right)} \times \frac{\alpha K_{i,i-1}(x_i - x_{i-1})^{-\alpha-1} P_i}{N_0}. \tag{19}$$

This implies that

$$\begin{aligned}\frac{\partial f_i(\boldsymbol{x})}{\partial x_i} &= \frac{\partial R_{i,i+1}^W(x_i, x_{i+1})}{\partial x_i} - \frac{\partial R_{i,i-1}^W(x_i, x_{i-1})}{\partial x_i} + \beta \\ &= \frac{T_{i,i+1}W}{w_{i,i+1}} \times \frac{1}{\log 2 \times \left(1 + \frac{K_{i,i+1}(x_{i+1}-x_i)^{-\alpha}P_i}{N_0}\right)} \times \frac{\alpha K_{i,i+1}(x_{i+1} - x_i)^{-\alpha-1} P_i}{N_0} \\ &\quad + \frac{T_{i,i-1}W}{w_{i,i-1}} \frac{1}{\log 2 \times \left(1 + \frac{K_{i,i-1}(x_i-x_{i-1})^{-\alpha}P_i}{N_0}\right)} \times \frac{\alpha K_{i,i-1}(x_i - x_{i-1})^{-\alpha-1} P_i}{N_0} + \beta,\end{aligned}$$

$$\begin{aligned}\frac{\partial f_i(\boldsymbol{x})}{\partial x_{i+1}} &= -\frac{T_{i,i+1}W}{w_{i,i+1}} \times \frac{1}{\log 2 \times \left(1 + \frac{K_{i,i+1}(x_{i+1}-x_i)^{-\alpha}P_i}{N_0}\right)} \times \frac{\alpha K_{i,i+1}(x_{i+1} - x_i)^{-\alpha-1} P_i}{N_0} \\ &= -\frac{\partial R_{i,i+1}^W(x_i, x_{i+1})}{\partial x_i},\end{aligned}$$

$$\begin{aligned}\frac{\partial f_i(\boldsymbol{x})}{\partial x_{i-1}} &= \frac{T_{i,i-1}W}{w_{i,i-1}} \times \frac{1}{\log 2 \times \left(1 + \frac{K_{i,i-1}(x_i-x_{i-1})^{-\alpha}P_i}{N_0}\right)} \times \frac{\alpha K_{i,i-1}(x_i - x_{i-1})^{-\alpha-1} P_i}{N_0} \\ &= -\frac{\partial R_{i,i-1}^W(x_i, x_{i-1})}{\partial x_i},\end{aligned}$$

and

$$\frac{\partial f_i(\boldsymbol{x})}{\partial x_j} = 0, \quad \text{if } j \ne i-1, i, i+1.$$

Hence, there exists a positive constant M such that

$$\frac{\partial f_i(\boldsymbol{x})}{\partial x_i} \le M, \quad x_i^{min} \le x_i \le x_i^{max}.$$

$$\sum_{j \ne i} \left| \frac{\partial f_i(\boldsymbol{x})}{\partial x_j} \right| = \frac{\partial f_i(\boldsymbol{x})}{\partial x_i} - \beta.$$

In addition, $f_i(\boldsymbol{x})$ is continuously differentiable and $\{x_i \mid x_i^{min} \leq x_i^{max}\}$ is a convex set. Therefore, f_i satisfies conditions at Proposition 1.1 in Chapter 3 of [20], and thus the algorithm in (17) is a contraction mapping. ∎

Since the algorithm in (17) is a contraction mapping, if γ is sufficiently small, it converges to the unique fixed point x_i^* such that

$$\begin{aligned} f_i(\boldsymbol{x}^*) &= R_{i,i+1}^W(x_i^*, x_{i+1}^*) - R_{i,i-1}^W(x_i^*, x_{i-1}^*) + \beta x_i^* \\ &= 0, \quad i = 2, 3, \cdots, N-1. \end{aligned} \tag{20}$$

We now consider the following iterative algorithm:

$$\begin{aligned} x_i(n+1) = [x_i(n) - \gamma f_i(\boldsymbol{x}(n))]_{x_i^{min}}^{x_i^{max}}, \\ i = 2, 3, \cdots, N-1, \end{aligned} \tag{21}$$

where $[a]_b^c$ is the projection of a onto $\{x : b \leq x \leq c\}$, i.e., $[a]_b^c = \min[\max[a, b], c]$.

Proposition 4: The algorithm in (21) is a contraction mapping.

Proof: The algorithm in (21) is the projected version of the algorithm in (17). Since the projection is nonexpansive by the Projection Theorem (Proposition 3.2 in Chapter 3 of [20]) and the algorithm in (17) is a contraction mapping by Proposition 3, the algorithm in (21) is also a contraction mapping. ∎

Since the algorithm in (21) is a contraction mapping, it converges to the following fixed point $\boldsymbol{x}_\beta^*$:

$$x_{i,\beta}^* = \begin{cases} x_i^{min}, & \text{if } x_i^* \leq x_i^{min} \\ x_i^{max}, & \text{if } x_i^* \geq x_i^{max} \\ x_i^*, & \text{otherwise} \end{cases}, \quad i = 2, 3, \cdots, N-1, \tag{22}$$

where x_i^* is a position that satisfies the condition in (20). Hence, the position $\boldsymbol{x}_\beta^*$ is different from the Nash equilibrium position in Theorem 3. In fact, if $\beta = 0$, then the condition in (20) is equivalent to that in Theorem 3. We need the positivity of β to guarantee that the algorithm in (17) is a contraction mapping. Even though we cannot obtain the exact Nash equilibrium position with a positive β, if it is sufficiently small, we can obtain a closed approximation to the Nash equilibrium position. We call the position that satisfies the condition in (22) the *β-approximate Nash equilibrium position* and we have the following theorem:

Theorem 4: With the algorithm in (21), the position $x_i(n)$ of each node i converges to the β-approximate Nash equilibrium position.

The pseudo-code of the non-cooperative game algorithm considering transmitting links is provided in Table IV.

We now study the efficiency of the Nash equilibrium in (15). We first define "Pareto-efficiency" of a solution as follows:

Definition 1: A solution $\boldsymbol{x}$ is said to be Pareto-efficient, if we cannot find another feasible solution $\boldsymbol{x}'$ such that

$$U_i(\boldsymbol{x}') \geq U_i(\boldsymbol{x}), \quad \forall i$$

and there exists an i^* such that

$$U_{i^*}(\boldsymbol{x}') > U_{i^*}(\boldsymbol{x}).$$

In general, the Nash equilibrium is not Pareto-efficient. However, in our case, we can show that in fact the Nash equilibrium

TABLE IV
NON-COOPERATIVE GAME ALGORITHM CONSIDERING TRANSMITTING LINKS.

Let γ and β are small positive constants.
Let $n = 0$.
while {
  $n = n + 1$
  Obtain $x_j(n)$ for each neighbor node j by notification from neighbor node j or measuring it.
  Calculate $R_{i,j}^W(x_i(n), x_j(n))$ for each neighbor node j.
  Calculate $x_i(n+1)$ in (21).
  Move to position $x_i(n+1)$ and broadcast its position $x_i(n+1)$ to all neighbor nodes.
} **end**

is Pareto-efficient, if some conditions are satisfied. Without loss of generality, we assume that only two boundary nodes are fixed, i.e., $\{1, N\} \in \mathbf{N}_F$ and $\{2, 3, \cdots, N-1\} \in \mathbf{N}_M$. If some inner nodes are fixed, we can divide the network into several subnetworks such that each subnetwork consists of inner nodes with controllable mobility and two fixed boundary nodes and apply the following results for each subnetwork.

Theorem 5: If the Nash equilibrium position of each node i, x_i^{Nash}, satisfies (16), then the Nash equilibrium is Pareto-efficient.

Proof: Let us define the distance between node i and its neighbor node j when their positions are x_i and x_j, respectively, $d_{i,j}(x_i, x_j) = |x_i - x_j|$. Then, for any $\boldsymbol{x}$, we have the following relationship:

$$\sum_{i=1}^{N-1} d_{i,i+1}(x_i, x_{i+1}) = x_N^* - x_1^*, \tag{23}$$

where $x_N^* - x_1^*$ is a constant, since nodes 1 and N are fixed. Hence, for the Nash equilibrium, we also have

$$\sum_{i=1}^{N-1} d_{i,i+1}(x_i^{Nash}, x_{i+1}^{Nash}) = x_N^* - x_1^*$$

We now assume that $\boldsymbol{x}^{Nash}$ is not Pareto-efficient. In this case, we can find another position $\boldsymbol{x}'$ such that the conditions in Definition 1 are satisfied. Note that

$$U_i(\boldsymbol{x}) = \min\{R_{i,i-1}^W(x_i, x_{i-1}), R_{i,i+1}^W(x_i, x_{i+1})\}.$$

Hence, from the conditions in Definition 1, we have

$$R_{i,i+1}^W(x_i', x_{i+1}') \geq R_{i,i+1}^W(x_i^{Nash}, x_{i+1}^{Nash})$$

and

$$R_{i,i-1}^W(x_i', x_{i-1}') \geq R_{i,i-1}^W(x_i^{Nash}, x_{i-1}^{Nash}), \ \forall i$$

and there exists an i^* such that

$$R_{i^*,i^*+1}^W(x_{i^*}', x_{i^*+1}') > R_{i^*,i^*+1}^W(x_{i^*}^{Nash}, x_{i^*+1}^{Nash})$$

and

$$R_{i^*,i^*-1}^W(x_{i^*}', x_{i^*-1}') > R_{i^*,i^*-1}^W(x_{i^*}^{Nash}, x_{i^*-1}^{Nash}).$$

Since $R_{i,j}^W(x_i, x_j)$ is a strictly decreasing function of $d_{i,j}(x_i, x_j)$, this implies that

$$d_{i,i+1}(x_i', x_{i+1}') \leq d_{i,i+1}(x_i^{Nash}, x_{i+1}^{Nash})$$

and

$$d_{i,i-1}(x_i', x_{i-1}') \leq d_{i,i-1}(x_i^{Nash}, x_{i-1}^{Nash}), \ \forall i$$

and there exists an i^* such that

$$d_{i^*,i^*+1}(x_{i^*}', x_{i^*+1}') < d_{i^*,i^*+1}(x_{i^*}^{Nash}, x_{i^*+1}^{Nash})$$

and

$$d_{i^*,i^*-1}(x_{i^*}', x_{i^*-1}') < d_{i^*,i^*-1}(x_{i^*}^{Nash}, x_{i^*-1}^{Nash}),$$

and thus

$$\sum_{i=1}^{N-1} d_{i,i+1}(x_i', x_{i+1}') < x_N^* - x_1^*,$$

which contradicts to (23). Hence, there is no other solution that satisfies the conditions in Definition 1, and thus the Nash equilibrium is Pareto-efficient. ■

In addition, if all nodes are homogeneous, then we have the following theorem.

Theorem 6: If all nodes are homogeneous (i.e., all nodes have the same parameters except for their positions) and the Nash equilibrium position of each node i, x_i^{Nash}, satisfies (16), then the Nash equilibrium is the global optimal solution of problem (6).

Proof: Since all nodes are homogeneous, we have

$$R_{i,j}^W(x_i^{Nash}, x_j^{Nash}) = R_{j,i}^W(x_j^{Nash}, x_i^{Nash}), \quad \forall i \in \mathbf{N}, j \in \mathbf{N}_N(i).$$

In addition, since we consider a linear network, this implies that all links achieve the same weighted average rate, i.e.,

$$R_{i,j}^W(x_i^{Nash}, x_j^{Nash}) = c, \quad \forall i \in \mathbf{N}, j \in \mathbf{N}_N(i),$$

where c is a positive constant. Since all links achieve the same weighted average rate at the Nash equilibrium, all nodes achieve the same utility, i.e., $U_i(\boldsymbol{x}^{Nash}) = U_j(\boldsymbol{x}^{Nash})$, $\forall i, j$. In addition, since $\boldsymbol{x}^{Nash}$ is Pareto-efficient, by the definition of the Pareto-efficiency, it is a global optimal solution of problem (6). ■

Hence, if all nodes are homogeneous and the condition in (16) is satisfied, we can find the global optimal solution of problem (6) in a distributed way by using the game theoretic approach in this subsection.

B. Non-cooperative game approach considering transmitting and receiving links

In the previous subsection, we developed the game theoretic algorithm in which each node controls its position considering only weighted average data rates of its transmitting links. In this subsection, we also develop a game theoretic algorithm, as in the previous subsection. However, we allow each node to consider not only its transmitting links but also its receiving links. The Positioning Game $PG^{TR}(P, A, U)$ is defined as the same as in the previous subsection except for the utility function of each node. Since each node i now considers both its transmitting and receiving links, its utility function is defined as the lowest weighted average data rate among those of its all transmitting and receiving links, i.e.,

$$U_i(\boldsymbol{x}) = U_i(x_i, \boldsymbol{x}_{-i}) = \min_{j \in \mathbf{N}_N(i)} F_{i,j}(x_i, x_j), \quad (24)$$

where $\boldsymbol{x} = (x_j)_{j \in \mathbf{N}}$, $\boldsymbol{x}_{-i} = (x_j)_{j \in \mathbf{N}, j \neq i}$, and

$$F_{i,j}(x_i, x_j) = \min\{R_{i,j}^W(x_i, x_j), R_{j,i}^W(x_j, x_i)\}. \quad (25)$$

In a similar way to the previous subsection, we can show that the utility function of each node i is a quasiconcave function of its position x_i. Hence, we have the following theorem.

Theorem 7: The Positioning Game $PG^{TR}(P, A, U)$ has a Nash equilibrium.

In addition, in a similar way to the previous subsection, we can show the following properties for the Nash equilibrium.

Theorem 8:

$$x_i^{Nash} = \begin{cases} x_i^{min}, & \text{if } x_i^* \leq x_i^{min} \text{ or if } i = N \\ x_i^{max}, & \text{if } x_i^* \geq x_i^{max} \text{ or if } i = 1 \\ x_i^*, & \text{otherwise} \end{cases}, \quad \forall i \in \mathbf{N}, \quad (26)$$

where x_i^* is a position that satisfies

$$F_{i,i-1}(x_i^*, x_{i-1}^{Nash}) = F_{i,i+1}(x_i^*, x_{i+1}^{Nash}), \quad (27)$$

when other nodes are at their Nash equilibrium, i.e., $x_j = x_j^{Nash}$, $j \neq i$.

Corollary 2: We have the following properties:

1) If $x_i^{min} < x_i^{Nash} < x_i^{max}$, then $F_{i,i-1}^W(x_i^{Nash}, x_{i-1}^{Nash}) = F_{i,i+1}^W(x_i^{Nash}, x_{i+1}^{Nash})$.
2) If $x_i^{Nash} = x_i^{min}$, then $F_{i,i-1}^W(x_i^{Nash}, x_{i-1}^{Nash}) \leq F_{i,i+1}^W(x_i^{Nash}, x_{i+1}^{Nash})$ and $F_{i,i-1}^W(x_i, x_{i-1}^{Nash}) < F_{i,i+1}^W(x_i, x_{i+1}^{Nash})$, $\forall x_i > x_i^{min}$.
3) If $x_i^{Nash} = x_i^{max}$, then $F_{i,i-1}^W(x_i^{Nash}, x_{i-1}^{Nash}) \geq F_{i,i+1}^W(x_i^{Nash}, x_{i+1}^{Nash})$ and $F_{i,i-1}^W(x_i, x_{i-1}^{Nash}) > F_{i,i+1}^W(x_i, x_{i+1}^{Nash})$, $\forall x_i < x_i^{max}$.

Hence, from the above properties, we can consider an algorithm for the β-approximated Nash equilibrium as:

$$x_i(n+1) = [x_i(n) - \gamma f_i(\boldsymbol{x}(n))]_{x_i^{min}}^{x_i^{max}}, \quad i = 2, 3, \cdots, N-1, \quad (28)$$

where

$$f_i(\boldsymbol{x}) = F_{i,i+1}(x_i, x_{i+1}) - F_{i,i-1}(x_i, x_{i-1}) + \beta x_i.$$

To show the convergence of the algorithm in (28), we first consider the case when the transmission time fraction $T_{i,j}$ of a link from node i to node j is set to be proportional to its weight $w_{i,j}$, i.e.,

$$\frac{T_{i,j}}{w_{i,j}} = c, \quad \forall i \in \mathbf{N}, j \in \mathbf{N}_N(i), \quad (29)$$

where c is a positive constant.

Lemma 2: With the assumption in (29), $\frac{K_{i,j}P_i}{N_0} \leq \frac{K_{j,i}P_j}{N_0}$ in (2) if and only if $R_{i,j}^W(x_i, x_j) \leq R_{j,i}^W(x_j, x_i)$ for all x_i and x_j.

Proof: Since $d_{i,j}(x_i, x_j) = d_{j,i}(x_j, x_i)$ for all x_i and x_j, $\frac{K_{i,j}P_i}{N_0} \leq \frac{K_{j,i}P_j}{N_0}$ if and only if $\gamma_{i,j}(x_i, x_j) \leq \gamma_{j,i}(x_j, x_i)$ in (2) for all x_i and x_j. Since the capacity of a link in (3) is an increasing function of its SNR, $\gamma_{i,j}(x_i, x_j) \leq \gamma_{j,i}(x_j, x_i)$ if and only if $C_{i,j}(x_i, x_j) \leq C_{j,i}(x_j, x_i)$ for all x_i and x_j. Since we assume that $\frac{T_{i,j}}{w_{i,j}} = c$ for all i and j, from (4) and (5), $C_{i,j}(x_i, x_j) \leq C_{j,i}(x_j, x_i)$ if and only if $R_{i,j}^W(x_i, x_j) \leq R_{j,i}^W(x_j, x_i)$ for all x_i and x_j, which completes the proof. ■

TABLE V
NON-COOPERATIVE GAME ALGORITHM CONSIDERING TRANSMITTING AND RECEIVING LINKS.

Let γ and β are small positive constants.
Let $n = 0$.
while {
  $n = n + 1$
  Obtain $x_j(n)$ for each neighbor node j by notification from neighbor node j or measuring it.
  Calculate $R^W_{i,j}(x_i(n), x_j(n))$ and broadcast it to neighbor node j.
  Obtain $R^W_{j,i}(x_j(n), x_i(n))$ for each neighbor node j by notification from neighbor node j.
  Calculate $x_i(n+1)$ in (28).
  Move to position $x_i(n+1)$ and broadcast its position $x_i(n+1)$ to all neighbor nodes.
} **end**

With the condition in (29), we can rewrite the utility function of each node i as

$$U_i(\boldsymbol{x}) = U_i(x_i, \boldsymbol{x}_{-i}) = \min_{j \in \mathbf{N}_N(i)} F_{i,j}(x_i, x_j), \tag{30}$$

where

$$F_{i,j}(x_i, x_j) = \begin{cases} R^W_{i,j}(x_i, x_j), & \text{if } \frac{K_{i,j}P_i}{N_0} \le \frac{K_{j,i}P_j}{N_0} \\ R^W_{j,i}(x_j, x_i), & \text{otherwise} \end{cases}.$$

We can easily show that all the results in the previous subsection are still valid, if we substitute $R^W_{i,j}(x_i, x_j)$ in the previous subsection with $F_{i,j}(x_i, x_j)$ defined above. Hence, we have the following theorem.

Theorem 9: With the algorithm in (28), if the condition in (29) is satisfied, the position $x_i(n)$ of each node i converges to the β-approximate Nash equilibrium position.

We proved the algorithm in (28) converges to the β-approximated Nash equilibrium only when the condition in (29) is satisfied. However, through extensive studies of numerical results, we could conclude that it also converges to the β-approximated Nash equilibrium, even though such a condition is not satisfied. The pseudo-code of the non-cooperative game algorithm considering transmitting and receiving links is provided in Table V.

We now study the efficiency of the Nash equilibrium in (26). First, in a similar way to the previous subsection, we can easily show the following theorem.

Theorem 10: If the Nash equilibrium position of each node i, x_i^{Nash}, satisfies (27), then the Nash equilibrium is Pareto-efficient.

In addition, since $U_i(\boldsymbol{x}^{Nash}) = U_j(\boldsymbol{x}^{Nash}), \forall i, j$ and $\boldsymbol{x}^{Nash}$ is Pareto efficient if it satisfies (16), we can easily show the following theorem.

Theorem 11: If the Nash equilibrium position of each node i, x_i^{Nash}, satisfies (27), then the Nash equilibrium is the global optimal solution of problem (6).

Hence, if the condition in (27) is satisfied, we can find the global optimal solution of problem (6) in a distributed way by using the game theoretic approach in this subsection.

VI. NUMERICAL RESULTS

In this section, we provide numerical results [1] of our algorithms, i.e., Global Optimal Solution Algorithm (GOSA), Improved Global Optimal Solution Algorithm (I-GOSA), Non-Cooperative Game Algorithm considering only Transmitting links (NCGA-T), and Non-Cooperative Game Algorithm considering Transmitting and Receiving links (NCGA-TR). In addition, we also compare the performances of our algorithms with that achieved by the Fixed Mobile Node Placement (FMNP). We focus on comparing node positions, achieved minimum weighted average data rates, and degrees of fairness in these algorithms. We consider the fairness in terms of the minimum weighted average data rate of links between two end nodes. In order to compare the degree of fairness, we use the following fairness index, which is defined in [21]:

$$f(\mathbf{F}(\mathbf{x})) = \frac{\left[\sum_{i\in\mathbf{N}}\sum_{j\in\mathbf{N}_N(i)} F_{i,j}(x_i, x_j)\right]^2}{|L| \sum_{i\in\mathbf{N}}\sum_{j\in\mathbf{N}_N(i)} F_{i,j}(x_i, x_j)^2}, \tag{31}$$

where $|L|$ is the number of links in the network and $F_{i,j}(\mathbf{x})$, which is defined in (25), is the minimum weighed average data rate of links between nodes i and j. In general, the larger the value of fairness index is, the higher degree of fairness we have.

We consider a network topology in Fig. 3, which consists of nodes 1, 2, 3, and 4. Nodes 1 and 4 have uncontrollable mobility, while nodes 2 and 3 have controllable mobility. We set $x_1 = 100$, $400 \le x_2 \le 600$, $700 \le x_3 \le 900$, $x_4 = 1500$, and initially set $x_2 = 400$, $x_3 = 800$, which are used as the positions of nodes 2 and 3, respectively, at the FMNP. In addition, we set $\alpha = 2, N_0 = 10^{-6}$, $W = 10^6$, and $K_{i,j} = 1, \forall i, j$. For our algorithms, i.e., GOSA, I-GOSA, NCGA-T and NCGA-TR, we set $\epsilon = 10^{-1}$ in Table I, $m = 10^{-3}, \boldsymbol{\lambda}^{(0)} = \mathbf{0}$ in Table II, and $\gamma = 10^{-7}, \beta = 10^{-5}$ in Tables IV and V.

We first consider the case that all nodes are homogeneous and set $T_{i,j} = 0.25$, $w_{i,j} = 1$, $P_i = 1$, $\forall i, j$. In Fig. 4, we show the convergence of our algorithms. In Fig. 4(a), we compare the convergence of the node positions in GOSA and I-GOSA. Since the distance between node 3 and 4 (i.e., 700) is much longer than the distance between other nodes, the minimum of weighted average data rate is always achieved at links between nodes 3 and 4, i.e., $R^W_{3,4}$ and $R^W_{4,3}$, regardless of the positions of nodes. Therefore, in GOSA and I-GOSA, node 3 approaches node 4 to maximize $R^W_{3,4}$ and $R^W_{4,3}$. However, once $R^W_{3,4}$ and $R^W_{4,3}$ are maximized, GOSA is terminated, while I-GOSA keeps trying to maximize the second minimum of weighted average data rates (i.e., $R^W_{2,3}$ and $R^W_{3,2}$). Hence, in I-GOSA, node 2 approaches node 3 to maximize $R^W_{2,3}$ and $R^W_{3,2}$. In Fig. 4(b), we provide the convergence of the node positions in NCGA-T and NCGA-TR. Since all nodes are homogeneous, NCGA-T and NCGA-TR provide the same results. In addition, from Figs. 4(a) and 4(b), we can see that if all nodes are homogeneous, I-GOSA, NCGA-T, and NCGA-TR provide the same results. In Table VI, we summarize the results of all

[1] To obtain numerical results, we used a customized program written with C++ programming language.

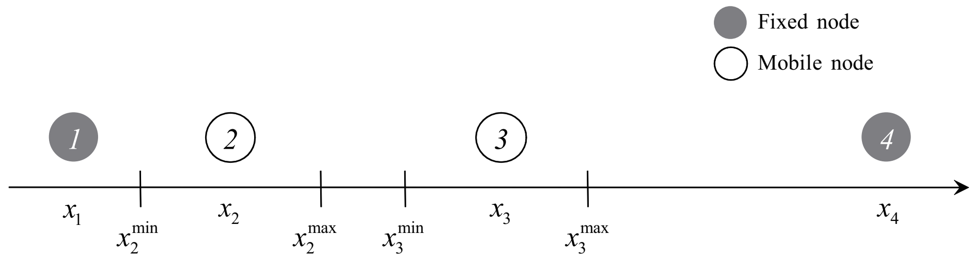


Fig. 3. Network topology.

TABLE VI
PERFORMANCE COMPARISON OF ALL ALGORITHMS.

(a) Node positions.

Node: i		1	2	3	4
x_i	FMNP	100	400	800	1500
x_i^*	GOSA	100	400	900	1500
	I-GOSA	100	500	900	1500
x_i^{Nash}	NCGA-T	100	500	900	1500
	NCGA-TR	100	500	900	1500

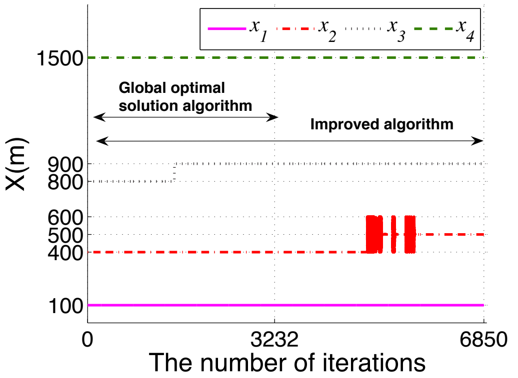
(b) Weighted average data rates and fairness indices.

Node: i		1	2		3		4	Minimum	Fairness index: $f(\mathbf{F}(\mathbf{x}))$
Neighbor node: $j \in \mathbf{N}_N(i)$		2	1	3	2	4	3		
$R_{i,j}^W$ (Unit: 10^3)	FMNP	900	900	714	714	401	401	401	0.914
	GOSA	900	900	580	580	479	479	479	0.930
	I-GOSA	714	714	714	714	479	479	479	0.971
	NCGA-T	714	714	714	714	479	479	479	0.971
	NCGA-TR	714	714	714	714	479	479	479	0.971

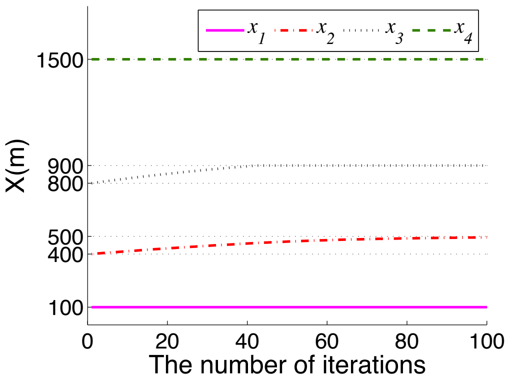
algorithms. In Table VI(b), we can see that I-GOSA provides the improved degree of fairness compared with GOSA.

We now consider the case that nodes are not homogeneous. We first examine how the link weight factor affects the performance of our algorithms with varying the weight factor of the link from node 2 to node 3, $w_{2,3}$. We set the weight factors of the other links to be 1 and $T_{i,j} = 0.25, P_i = 1, \forall i, j$. As mentioned above, since the distance between nodes 3 and 4 is much longer than the distance between other nodes, the minimum of weighted average data rates is still achieved at links between nodes 3 and 4, i.e., $R_{3,4}^W$ and $R_{4,3}^W$, regardless of the positions of nodes, if the value of $w_{2,3}$ is lower than a certain certain threshold (i.e., approximately 1.8 in this topology). Hence, as in the previous case, when the value of $w_{2,3}$ is less than 1.8, node 3 is located at position 900 to maximize $R_{3,4}^W$ and $R_{4,3}^W$. In addition, as shown in Fig. 5(a), in I-GOSA, NCGA-T, and NCGA-TR, as the value of $w_{2,3}$ increases, node 2 more approaches node 3 to improve $R_{2,3}^W$. However, in GOSA, node 2 can be located at any position between 400 and 600, since GOSA tries to maximize only the minimum weighted average data rate (i.e., $R_{3,4}^W$ and $R_{4,3}^W$). When the the value of $w_{2,3}$ exceeds the threshold (i.e., 1.8), $R_{2,3}^W$ becomes the minimum weighted average data rate. Therefore, in GOSA, I-GOSA, and NCGA-TR, nodes 2 and 3 get closer to each other to maximize $R_{2,3}^W$. However, in NCGA-T, node 3 does not approach node 2, since NCGA-T considers only the transmitting rates of each node. In other words, in NCGA-T, the value of $w_{2,3}$, which is the weight factor of the receiving link of node 3, does not affect the position of node 3 directly. Due to this reason, NCGA-T ineffectively maximizes the maximum weighted average data rate (i.e., $R_{2,3}^W$) compared with the other algorithms, as shown in Fig. 5(b). In addition, as shown in Fig. 5(c), the degrees of fairness in I-GOSA and NCGA-TR are always higher than those in GOSA and NCGA-T.

We now examine how the power of a node affects the performance of our algorithms with varying the power of node 2, i.e., P_2. We set $P_i = 1$ for each node i except node 2 and $T_{i,j} = 0.25, w_{i,j} = 1, \forall i, j$. If P_2 is higher than a certain threshold (i.e., approximately 0.5 in this topology), the minimum of weighted average data rate is also still achieved at the link between nodes 3 and 4. In this case, in I-GOSA, NCGA-T, and NCGA-TR, node 3 is located at position 900 that maximizes $R_{3,4}^W$ and $R_{4,3}^W$ and node 2 is located at position 500, where $R_{2,1}^W = R_{2,3}^W$. However, in GOSA, node 2 can be located at any position between 400 and 600, since GOSA tries to maximize only the minimum weighted average data rate (i.e., $R_{3,4}^W$ and $R_{4,3}^W$). In this figure, when $P_2 = 1$, node 2 is located at position 600, which provides a lower degree of fairness of GOSA than the other algorithms, as shown in Fig. 6(c). However, as the value of P_2 decreases, $R_{2,1}^W$ and $R_{2,3}^W$ also decease and if P_2 is lower than the threshold (i.e., 0.5), $R_{2,1}^W$ and $R_{2,3}^W$ become the minimum weighted average data rates. Therefore, as shown in Fig. 6(a), in GOSA, I-GOSA, and NCGA-TR, as the value of P_2 decreases, nodes 2 and 3 approach nodes 1 and 2, respectively. However, in NCGA-T, since node 3 does not consider its receiving rate (i.e., $R_{2,3}^W$), node 3 maintains its position at 900, and thus node 2 also maintains its position at 500, where $R_{2,1}^W = R_{2,3}^W$. Similar to the previous case, due to this reason, NCGA-T ineffectively maximizes the minimum weighted average data rate (i.e., $R_{2,3}^W$). Therefore, as shown in Fig. 6(b), the minimum

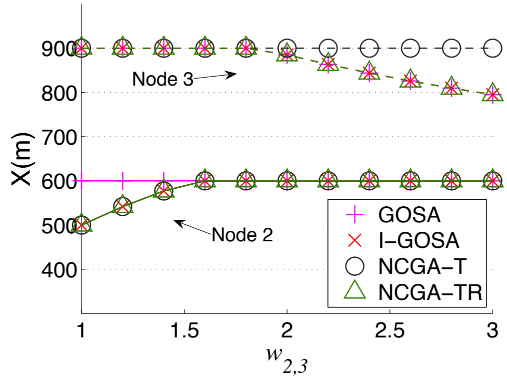


(a) Optimization theoretic approaches (GOSA vs. I-GOSA).

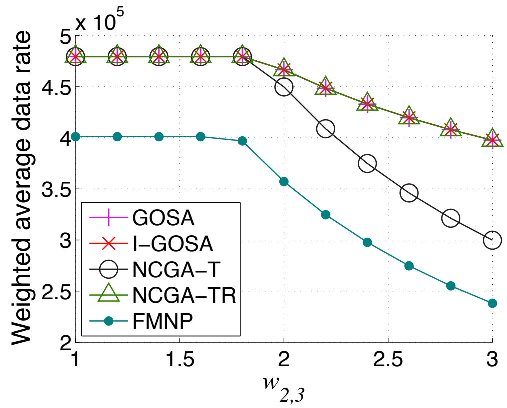


(b) Game theoretic approaches (NCGA-T vs. NCGA-TR).

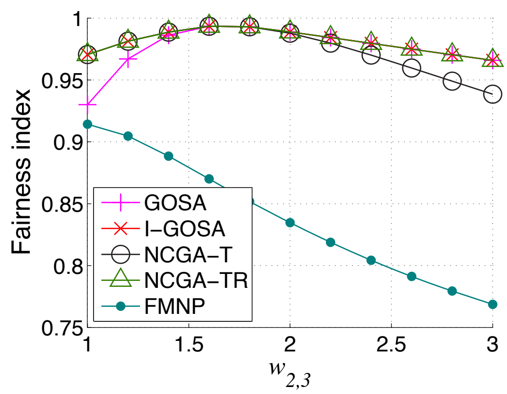
Fig. 4. Convergence of our algorithms (i.e., GOSA, I-GOSA, NCGA-T, and NCGA-TR).



(a) Positions of nodes 2 and 3.



(b) Minimum weighted average data rates.



(c) Fairness indices.

Fig. 5. Performance comparison of algorithms with varying $w_{2,3}$.

weighted average data rates of GOSA, I-GOSA, and NCGA-TR are always higher than that of NCGA-T regardless of the value of node power. In addition, as shown in Fig. 6(c), the degrees of fairness in I-GOSA and NCGA-TR are always higher than those in GOSA and NCGA-T.

From Figs. 4, 5, 6, and Table VI, we can see that the minimum weight average data rates and degrees of fairness in I-GOSA and NCGA-TR are equal to each other, while they are always higher than those in GOSA and NCGA-T. In addition, due to the ineffectiveness, the minimum average data rate of NCGA-T is lower than or equal to those of GOSA, I-GOSA, and NCGA-TR. This also results in the decrease of the degree of fairness in NCGA-T. Since GOSA tries to maximize only the minimum average data rate, its degree of fairness is also lower than or equal to those of I-GOSA and NCGA-TR. In other words, in order to achieve the maximum performance, we have to consider the information of both transmitting and receiving links of each node. However, although I-GOSA and NCGA-TR provide the same maximum performance for all cases (i.e., homogeneous and non-homogeneous cases), it is better to implement NCGA-TR since it can be implemented in a distributed manner. since its rate of convergence is higher than that of I-GOSA and it can be implemented in a distributed manner.

We now examine whether I-GOSA and NCGA-TR provide the same results or not in a more complex case. We randomly locate 5 nodes in a linear network, and assume that all nodes are mobile. We set $\alpha = 2, N_0 = 10^{-6}, W = 10^6$, and $T_{i,j} = 0.25$. In addition, we set $w_{i,j}$, $K_{i,j}$, and P_i as a value between 0.5 and 2 randomly (i.e., $w_{i,j}, K_{i,j}, P_i \in [0.5, 2], \forall i, j$). In Table VII, we summarize the parameters of each node and the results of I-GOSA and NCGA-TR for a specific setting and they provide the same results. Even though we do not provide more results due to the page limitation, we examined more different situations and in all other situations, both algorithms also provide almost the same results.

With the above all results, even though we cannot prove it theoretically, we can infer that I-GOSA and NCGA-TR provide higher performance than GOSA and NCGA-T, when we consider both the achievable minimum weighted average

TABLE VII
PERFORMANCE COMPARISON OF I-GOSA AND NCGA-TR.

(a) Node positions.

Node: i		1	2	3	4	5
Power: P_i		0.50	1.64	1.61	1.69	1.79
x_i^{min}		89	443	1001	1302	1584
x_i^{max}		300	861	1205	1461	1869
Initial position: x_i		224	737	1005	1448	1774
x_i^*	I-GOSA	300	536	1020	1302	1584
x_i^{Nash}	NCGA-TR	300	536	1020	1302	1584

(b) Weighted average data rates and fairness indices.

Node: i		1	2		3		4		5	Minimum	Fairness $f(\mathbf{F}(\mathbf{x}))$
Neighbor node: $j \in \mathbf{N}_N(i)$		2	1	3	2	4	3	5	4		
$w_{i,j}$		1.39	1.18	1.08	1.09	0.92	1.79	1.17	1.15	-	-
$K_{i,j}$		1.91	1.59	1.20	1.29	1.96	1.90	0.92	1.58	-	-
$R_{i,j}^W$	I-GOSA	750	1177	750	753	1443	750	929	1133	750	0.991
(Unit: 10^3)	NCGA-TR	750	1177	750	753	1443	750	929	1133	750	0.991

data rate and degree of fairness. In addition, we can also infer that I-GOSA and NCGA-TR provide similar results.

VII. CONCLUSION

We studied the position control problem for mobile nodes in one dimensional linear vehicular networks that aims at finding the position of each node with controllable mobility considering the network performance. We developed four algorithms, two based on optimization theoretic approach to find the global optimal position and two based on the game theoretic approach to find the Nash equilibrium position in a distributed way. From numerical results, we can clearly see that appropriately controlling mobility of nodes considering the network performance is critical to improve the network performance. One of the interesting findings in this paper is that in our problem, the game theoretic approach, i.e., NCGA-TR, can provide an efficient solution that is comparable to the global optimal solution, which is not true in many other cases.

In this paper, we formulated the average data rate of a link by using a simple wireless channel model. In our future work, we will consider the various vehicular channel models, e.g., those in [22], for the more realistic system model of vehicular networks. In addition, we studied the position control problem in a linear topology, which is applicable only a limited situation. The generalization of our work to more general topologies such as two-dimensional networks will be our future work. We also plan to generalize our work to the position control considering network performance and application specific missions jointly. We believe that the results in this paper will be useful bases for our future work.

APPENDIX: UNFAIRNESS OF THE SOCIAL OPTIMAL SOLUTION

An alternative problem that we can consider is to obtain the positions of nodes that provide the social optimum in the sense that they maximize the sum of weighted average data rates of all links in the network, which can be obtained by solving the following optimization problem:

$$\begin{array}{ll} \max\limits_{x_i, \forall i \in \mathbf{N}} & \sum\limits_{i \in \mathbf{N}} \sum\limits_{j \in \mathbf{N}_N(i), i \notin \mathbf{N}_F \vee j \notin \mathbf{N}_F} R_{i,j}^W(x_i, x_j) \\ \text{subject to} & x_i^{min} \le x_i \le x_i^{max}, \ \forall i \in \mathbf{N}_M \\ & x_i = x_i^*, \ \forall i \in \mathbf{N}_F, \end{array} \tag{32}$$

The above problem is one of the most extensively studied problems when we develop a network control algorithm considering the network performance. In many other cases, the above approach provides a good solution that improves the network performance. However, in our case, it may provide highly unfair positions for nodes in the sense that the difference between weighted average data rates of links could be very large. We will show it with a simple example.

Consider a network that consists of three nodes. Nodes 1 and 3 have fixed position and node 2 has controllable mobility. The positions of nodes 1 and 3 are x_1^* and x_3^* ($x_1^* < x_3^*$), respectively. The maximum and minimum positions of node 2 are $x_2^{min} = x_1^* + \epsilon$ and $x_2^{max} = x_3^* - \epsilon$, respectively, where ϵ is a very small positive constant, i.e., $0 < \epsilon \approx 0$. In addition, we assume that only node 2 has transmitting links to nodes 1 and 3. Hence, problem in (32) is reduced as

$$\begin{array}{ll} \max\limits_{x_2} & R_{2,1}^W(x_2, x_1^*) + R_{2,3}^W(x_2, x_3^*) \\ \text{subject to} & x_1^* + \epsilon \le x_2 \le x_3^* - \epsilon. \end{array} \tag{33}$$

We can easily show that $R_{2,1}^W(x_2, x_1^*)$ is a strictly decreasing function of x_2 and $R_{2,3}^W(x_2, x_3^*)$ is a strictly increasing function of x_2. In addition, we can also show that both $R_{2,1}^W(x_2, x_1^*)$ and $R_{2,3}^W(x_2, x_3^*)$ are strictly convex functions of x_2. This implies that the optimal position of node 2 will be either $x_1^*+\epsilon$ or $x_3^*-\epsilon$ depending on parameters. If $x_2^* = x_1^*+\epsilon$, then $\infty \approx R_{2,1}^W(x_2, x_1^*) \gg R_{2,3}^W(x_2, x_3^*)$ and if $x_2^* = x_3^* - \epsilon$, then $R_{2,1}^W(x_2, x_1^*) \ll R_{2,3}^W(x_2, x_3^*) \approx \infty$, which confirms our claim.

REFERENCES

[1] Y. Toor, P. Muhlethaler, A. Laouiti, and A. de La Fortelle, "Vehicle ad hoc netowrks: applications and related technical issues," *IEEE Commun. Survey & Tutorials*, vol. 10, no. 3, pp. 74–86, 3rd Quarter 2008.

[2] M. L. Sichitiu and M. Kihl, "Inter-vehicle communication systems: a survey," *IEEE Commun. Survey & Tutorials*, vol. 10, no. 2, pp. 88–105, 3nd Quarter 2008.

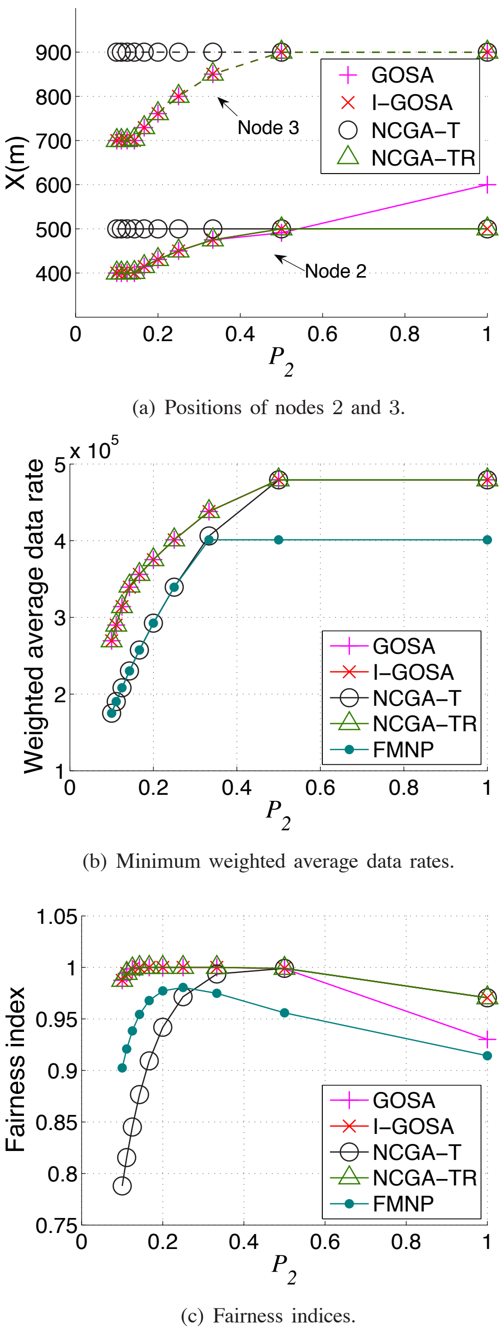


Fig. 6. Performance comparison of algorithms with varying P_2.

[3] T. L. Willke, P. tientrakool, and N. F. Maxemchuk, "A survey of inter-vehicle communication protocols and their applications," *IEEE Commun. Survey & Tutorials*, vol. 11, no. 2, pp. 3–20, 2nd Quarter 2009.

[4] Y. Liu and U. Ozguner, "Effect of inter-vehicle communication on rear-end collision avoidance," in *IEEE IVS*, 2003.

[5] S. Halle, J. Laumonier, and B. Chaib-draa, "A decentralized approach to collaborative driving coordination," in *IEEE ITSC*, 2004.

[6] L. Li and F.-Y. Wang, "Cooperative driving at blind crossings using intervehicle communication," *IEEE Trans. Veh. Technol.*, vol. 55, no. 6, pp. 1712–1724, Nov. 2006.

[7] P. Li, X. Huang, Y. Fang, and P. Lin, "Optimal placement of gateways in vehicular networks," *IEEE Trans. Veh. Technol.*, vol. 56, no. 6, pp. 3421–3430, Nov. 2007.

[8] J. Pan, Y. T. Hou, L. Cai, and Y. S. nd S. X. Shen, "Topology control for wireless sensor networks," in *ACM Mobicom*, 2003, pp. 286–299.

[9] D. K. Goldenberg, J. Lin, A. S. Morse, B. E. Rosen, and Y. R. Yang, "Towards mobility as a network control primitive," in *ACM Mobihoc*, 2004, pp. 163–174.

[10] Q. Wang, G. Takahara, H. hassanein, and K. Xu, "On relay node placement and locally optimal traffic allocation in heterogeneous wireless sensor networks," in *IEEE LCN*, 2005, pp. 656–664.

[11] E. R. Chittimalla, A. Venkateswaran, V. Sarangan, and R. Acharya, "On the use of nodes with controllable mobility for conserving power in manets," in *IEEE ICDCSW*, 2006, pp. 88–93.

[12] S. Misra, S. D. Hong, G. Xue, and J. Tang, "Constrained relay node placement in wireless sensor networks to meet connectivity and survivability requirements," in *IEEE INFOCOM*, 2008, pp. 281–285.

[13] A. So and B. Liang, "Optimal placement and channel assignment of relay stations in heterogeneous wireless mesh networks by modified Bender's decomposition," *Ad Hoc Networks*, vol. 1, no. 1, pp. 118–135, Jan. 2009.

[14] B. Lin, P.-H. Ho, L. l. Xie, and X. Shen, "Relay station placement in ieee 802.16j dual-relay mmr networks," in *IEEE ICC*, 2008, pp. 3437–3441.

[15] A. So and B. Liang, "Enhancing WLAN capacity by strategic placement of tetherless relay points," *IEEE Trans. Mobile Comput.*, vol. 6, no. 5, pp. 522–535, May 2007.

[16] A. Srinivas and E. Modiano, "Joint node placment and assignment for throughput optimization in mobile backbone networks," in *IEEE INFOCOM*, 2008, pp. 1804–1812.

[17] H.-T. Roh and J.-W. Lee, "Optimal placement of a relay node with controllable mobility in wireless networks considering fairness," in *IEEE CCNC*, 2010.

[18] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, 2004.

[19] M. J. Osborne and A. Rubinstein, *A Course in Game Theory*. The MIT Press, 1994.

[20] D. P. Bertsekas and J. N. Tsitsiklis, *Parallel and Distributed Computation: Numerical Methods*. Athena Scientific, 1999.

[21] R. Jain, D. Chiu, and W. Hawe, "A quantitative measure of fairness and discrimination for resource allocation in shared computer systems," *Tech. Rep.*, Sept. 1984.

[22] C.-X. Wang, X. Cheng, and D. Laurenson, "Vehicle-to-vehicle channel modeling and measurements: recent advances and future challenges," *IEEE Commun. Mag.*, vol. 47, no. 11, pp. 96–103, Nov. 2009.

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