Adaptive Channel Allocation Spectrum Etiquette for Cognitive Radio Networks

Nie Nie · Cristina Comaniciu

Abstract In this work, we propose a game theoretic framework to analyze the behavior of cognitive radios for distributed adaptive channel allocation. We define two different objective functions for the spectrum sharing games, which capture the utility of selfish users and cooperative users, respectively. Based on the utility definition for cooperative users, we show that the channel allocation problem can be formulated as a potential game, and thus converges to a deterministic channel allocation Nash equilibrium point. Alternatively, a no-regret learning implementation is proposed for both scenarios and it is shown to have similar performance with the potential game when cooperation is enforced, but with a higher variability across users. The no-regret learning formulation is particularly useful to accommodate selfish users. Non-cooperative learning games have the advantage of a very low overhead for information exchange in the network. We show that cooperation based spectrum sharing etiquette improves the overall network performance at the expense of an increased overhead required for information exchange.

Keywords cognitive radio · channel allocation · potential game · no-regret learning

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1 Introduction

The explosive growth of wireless services and the increased users' population density call for intelligent ways of managing the scarce spectrum resources. In the same time, recent measurements of the spectrum utilization show very limited usage of certain licensed frequency bands, for certain geographical regions. With not much spectrum available to be licensed for new wireless services, the solution proposed marks a new paradigm shift in the FCC's spectrum management policy [1] that creates opportunities for new, more aggressive, spectrum reuse. The cognitive radio technology lays the foundation for the deployment of smart flexible networks that cooperatively adapt to increase the overall network performance.

The cognitive radio terminology was coined by Mitola [11], and refers to a smart radio which has the ability to sense the external environment, learn from the history, and make intelligent decisions to adjust its transmission parameters according to the current state of the environment. In a cognition cycle, a radio receives stimuli (information about its operating environment) from the external environment through direct observation or signaling information exchanges and then reacts to these stimuli in four steps: orientation, planning, decision and acting. The radio's action will influence the outside world and will in turn be received as stimuli by the other radios. Compared to a traditional radio node, a cognitive radio node has a more flexible physical platform based on Software Defined Radio (SDR) technology, which enables it to interact with the external environment by dynamically reconfiguring its physical transmission parameters. Some examples

of the application of cognitive radios can be found in [2, 7, 15].

The potential contributions of cognitive radios to spectrum sharing and an initial framework for formal radio etiquette have been discussed in [10]. According to the proposed etiquette, the users should listen to the environment, determine the radio temperature of the channels and estimate their interference contributions on their neighbors. Based on these measurements, the users should react by changing their transmission parameters if some other users may need to use the channel.

While it is clear that this etiquette promotes cooperation between cognitive radios, the behavior of networks of cognitive radios that implement distributed resource allocation algorithms is less well understood.

As the cognitive radios are essentially autonomous agents that are learning their environment and are optimizing their performance by modifying their transmission parameters, their interactions can be modeled using a game theoretic framework. In this framework, the cognitive radios are the players and their actions are the selection of new transmission parameters and new transmission frequencies, etc., which influence their own performance, as well as the performance of the neighboring players.

Game theory has been extensively applied in microeconomics, and only more recently has received attention as a useful tool to design and analyze distributed resource allocation algorithms (e.g., [4, 9]). Some game theoretic models for cognitive radio networks were presented in [13], which has identified potential game formulations for power control, call admission control and interference avoidance in cognitive radio networks. The convergence conditions for various game models in cognitive radio networks are investigated in [14].

In this work, we propose a game theoretic formulation of the adaptive channel allocation problem for cognitive radios. Our current work assumes that the radios can measure the local interference temperature on different frequencies and can adjust by optimizing the information transmission rate for a given channel quality (using adaptive channel coding) and by possibly switching to a different frequency channel. The cognitive radios' decisions are based on their perceived utility associated with each possible action. We propose two different utility definitions, which reflect the amount of cooperation enforced by the spectrum sharing etiquette. We then design adaptation protocols based on both a potential game formulation, as well as no-regret learning algorithms. We study the convergence properties of the proposed adaptation algorithms, as well as the tradeoffs involved.

2 System model

The cognitive radio network we consider consists of a set of N transmitting-receiving pairs of nodes, uniformly distributed in a square region of dimension $D^* \times D^*$. We assume that the nodes are either fixed, or are moving slowly (slower than the convergence time for the proposed algorithms). Figure 1 shows an example of a network realization, where we used dashed lines to connect the transmitting node to its intended receiving node. The nodes measure the spectrum availability and decide on the transmission channel. We assume that there are K frequency channels available for transmission, with $K < N$. By distributively selecting a transmitting frequency, the radios effectively construct a channel reuse distribution map with reduced co-channel interference.

The transmission link quality can be characterized by a required Bit Error Rate target (BER), which is specific for the given application. An equivalent SIR target requirement can be determined, based on the modulation type and the amount of channel coding.

The Signal-to-Interference Ratio (SIR) measured at the receiver j associated with transmitter i can be expressed as:

$$SIR_{ij} = \frac{p_i G_{ij}}{\sum_{k=1,k\neq i}^{N} p_k G_{kj} I(k,j)}, \qquad (1)$$

where p_i is the transmission power at transmitter i, G_{ij} is the link gain between transmitter i and receiver j. $I(i,j)$ is the interference function characterizing the interference created by node i to node j and is defined as

$$I(i,j) = \begin{cases} 1 \text{ if transmitters } i \text{ and } j \text{ are transmitting} \\ \quad \text{over the same channel} \\ 0 \text{ otherwise} \end{cases} \qquad (2)$$

Analyzing Eq. 1 we see that in order to maintain a certain BER constraint the nodes can adjust at both the physical and the network layer level. At the network level, the nodes can minimize the interference by appropriately selecting the transmission channel frequency. At the physical layer, power control can reduce interference and, for a feasible system, results in all users meeting their SIR constraints. Alternatively, the target SIR requirements can be changed (reduced or increased) by using different modulation levels and various channel coding rates. As an example of adap-

Figure 1 A snapshot of the nodes' positions and network topology
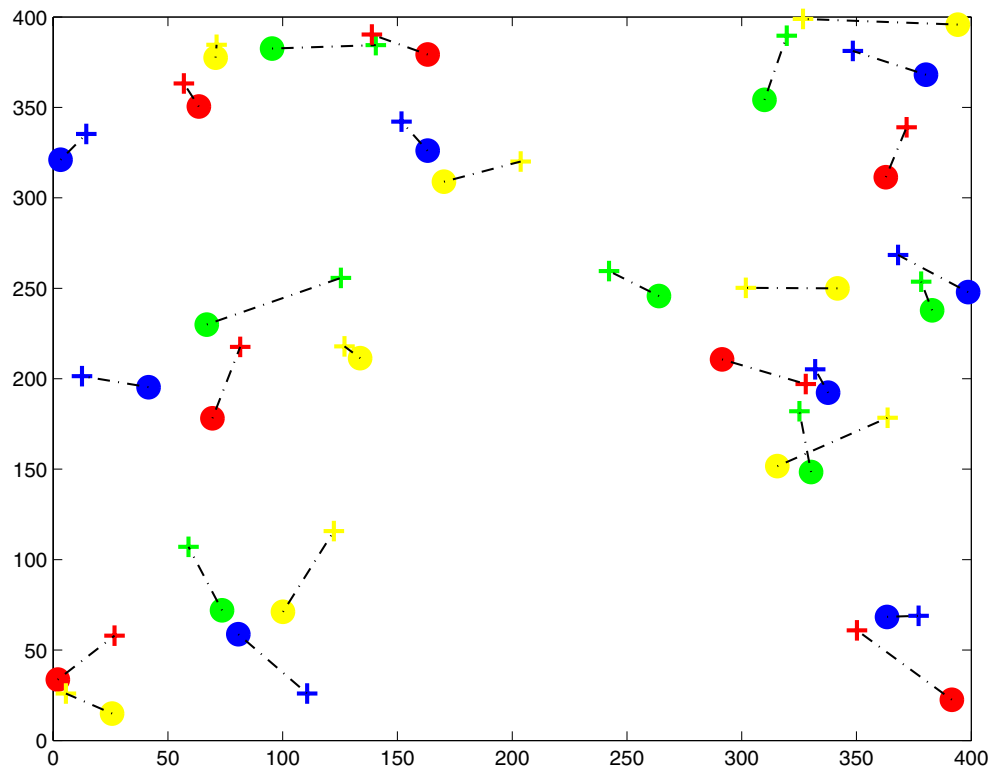
tation at the physical layer, we have assumed that for a fixed transmission power level, software defined radios enable the nodes to adjust their transmission rates and consequently the required SIR targets by varying the amount of channel coding for a data packet.

For our simulations we have assumed that all users have packets to transmit at all times (worst case scenario). Multiple users are allowed to transmit at the same time over a shared channel. We assume that users in the network are identical, which means they have an identical action set and identical utility functions associated with the possible actions.

Table 1 Code rates of Reed-Muller code RM $(1, m)$ and corresponding SIR requirement for target BER=10^{-3}

m	Code Rate	SIR (dB)
2	0.75	6
3	0.5	5.15
4	0.3125	4.6
5	0.1875	4.1
6	0.1094	3.75
7	0.0625	3.45
8	0.0352	3.2
9	0.0195	3.1
10	0.0107	2.8

The BER requirement selected for simulations is 10^{-3}, and we assume the use of a Reed-Muller channel code RM $(1, m)$. In Table 1 we show the coding rate combinations and the corresponding SIR target requirements used for our simulations [8].

3 A game theoretic framework

Game theory represents a set of mathematical tools developed for the purpose of analyzing the interactions in decision processes. Particularly, we can model our channel allocation problem as the outcome of a game, in which the players are the cognitive radios, their actions (strategies), are the choice of a transmitting channel and their preferences are associated with the quality of the channels. The quality of channels is determined by the cognitive radios by measurements on different radio frequencies.

We model our channel allocation problem as a normal form game, which can be mathematically defined as $\Gamma = \{N, \{S_i\}_{i \in N}, \{U_i\}_{i \in N}\}$, where N is the finite set of players (decision makers), and S_i is the set of strategies associated with player i. We define $\mathbb{S} = \times S_i, i \in N$ as the strategy space, and $U_i: \mathbb{S} \to \mathbb{R}$ as the set of utility functions that the players associate with their strategies.

For every player i in game Γ, the utility function, U_i, is a function of s_i, the strategy selected by player i, and of the current strategy profile of its opponents: s_{-i}.

In analyzing the outcome of the game, as the players make decisions independently and are influenced by the other players' decisions, we are interested to determine if there exist a convergence point for the adaptive channel selection algorithm, from which no player would deviate anymore, i.e., a Nash equilibrium (NE). A strategy profile for the players, $S = [s_1, s_2, ..., s_N]$, is a NE if and only if

$$U_i(S) \geq U_i(s_i', s_{-i}), \ \forall i \in N, s_i' \in S_i. \tag{3}$$

If the equilibrium strategy profile in Eq. 3 is deterministic, a pure strategy Nash equilibrium exists. For finite games, even if a pure strategy Nash equilibrium does not exist, a mixed strategy Nash equilibrium can be found (equilibrium is characterized by a set of probabilities assigned to the pure strategies).

As becomes apparent from the above discussion, the performance of the adaptation algorithm depends significantly on the choice of the utility function which characterizes the preference of a user for a particular channel. The choice of a utility function is not unique. It must be selected to have physical meaning for the particular application, and also to have appealing mathematical properties that will guarantee equilibrium convergence for the adaptation algorithm. We have studied and proposed two different utility functions, that capture the channel quality, as well as the level of cooperation and fairness in sharing the network resources.

3.1 Utility functions

The first utility function ($U1$) we propose accounts for the case of a "selfish" user, which values a channel based on the level of interference perceived on that particular channel:

$$U1_i(s_i, s_{-i}) = - \sum_{j \neq i, j=1}^{N} p_j G_{ij} f(s_j, s_i).$$
$$\forall i = 1, 2, ..., N \tag{4}$$

For the above definition, we denoted $P = [p_1, p_2, ..., p_N]$ as the transmission powers for the N radios, $S = [s_1, s_2, ..., s_N]$ as the strategy profile and $f(s_i, s_j)$ as an interference function:

$$f(s_i, s_j) = \begin{cases} 1 & \text{if } s_j = s_i, \text{ transmitter } j \text{ and } i \text{ choose} \\ & \text{the same strategy (same channel)} \\ 0 & \text{otherwise} \end{cases}$$

This choice of the utility function requires a minimal amount of information for the adaptation algorithm,

namely the interference measurement of a particular user on different channels.

The second utility function we propose accounts for the interference seen by a user on a particular channel, as well as for the interference this particular choice will create to neighboring nodes. Mathematically we can define $U2$ as:

$$U2_i(s_i, s_{-i}) = - \sum_{j \neq i, j=1}^{N} p_j G_{ij} f(s_j, s_i)$$
$$- \sum_{j \neq i, j=1}^{N} p_i G_{ji} f(s_i, s_j)$$
$$\forall i = 1, 2, ..., N \tag{5}$$

The complexity of the algorithm implementation will increase for this particular case, as the algorithm will require probing packets on a common access channel for measuring and estimating the interference a user will create to neighboring radios.

The above defined utility functions, characterize a user's level of cooperation and support a selfish and a cooperative spectrum sharing etiquette, respectively.

3.2 A potential game formulation

In the previous section we have discussed the choice of the utility functions based on the physical meaning criterion. However, in order to have good convergence properties for the adaptation algorithm we need to impose some mathematical properties on these functions. There are certain classes of games that have been shown to converge to a Nash equilibrium when a best response adaptive strategy is employed. In what follows, we show that for the $U2$ utility function, we can formulate an exact potential game, which converges to a pure strategy Nash equilibrium solution.

Characteristic for a potential game is the existence of a potential function that exactly reflects any unilateral change in the utility function of any player. The potential function models the information associated with the improvement paths of a game instead of the exact utility of the game [12].

An exact potential function is defined as a function

$$\textbf{Pot} : \mathbb{S} \to \mathbb{R}, \ \text{if for all } i, \text{ and } s_i, \ s_i' \in S_i,$$

with the property that

$$U_i(s_i, s_{-i}) - U_i(s_i', s_{-i}) = Pot(s_i, s_{-i}) - Pot(s_i', s_{-i}). \tag{6}$$

If a potential function can be defined for a game, the game is an exact potential game. In an exact potential game, for a change in actions of a single player the change in the potential function is equal to the value

of the improvement deviation. Any potential game in which players take actions sequentially converges to a pure strategy Nash equilibrium that maximizes the potential function.

For our previously formulated channel allocation game with utility function $U2$, we can define an exact potential function to be

$$Pot(S) = Pot(s_i, s_{-i})$$
$$= \sum_{i=1}^{N} \left(-\frac{1}{2} \sum_{j \neq i, j=1}^{N} p_j G_{ij} f(s_j, s_i) - \frac{1}{2} \sum_{j \neq i, j=1}^{N} p_i G_{ji} f(s_i, s_j) \right)$$
$$\forall i = 1, 2, ..., N. \quad (7)$$

The function in Eq. 7 essentially reflects the network utility. It can be seen thus that the potential game property Eq. 6 ensures that an increase in individual users' utilities contributes to the increase of the overall network utility. We note that this property holds only if users take actions sequentially, following a best response strategy.

The proof that Eq. 7 is an exact potential function is given in the Appendix.

Consequently, to ensure convergence for the spectrum allocation game, either a centralized or a distributed scheduler should be deployed. In an ad hoc network, the latter solution is preferable. To this end, we propose a random access for decision making in which each user is successful with probability $p_a = 1/N$. More specifically, at the beginning of each time slot, each user flips a coin with probability p_a, and, if successful, makes a new decision based on the current values for the utility functions for each channel; otherwise takes no new action. We note that the number of users that attempt to share each channel, can be determined from channel listening as we will detail shortly. The proposed random access ensures that on average exactly one user makes decisions at a time, but of course has a nonzero probability to have two or more users taking actions simultaneously. We have determined experimentally that the convergence of the game is robust to this phenomenon: when two or more users simultaneously choose channels, the potential function may temporarily decrease (decreasing the overall network performance) but then the upward monotonic trend is re-established.

The proposed potential game formulation requires that users should be able to evaluate the candidate channels' utility function $U2$. To provide all the information necessary to determine $U2$, we propose a signaling protocol based on a three way handshake protocol.

The signaling protocol is somewhat similar to the RTS-CTS packet exchange for the IEEE 802.11 protocol, but intended as a call admission reservation protocol, rather than packet access reservation protocol. When a user needs to make a decision on selecting the best transmission frequency (a new call is initiated or terminated, and user is successful in the Bernoulli trial), such a handshaking is initiated. In contrast to the RTS-CTS reservation mechanism, the signaling packets, START, START_CH, ACK_START_CH (END, ACK_END) in our protocol, are not used for deferring transmission for the colliding users, but rather to measure the interference components of the utility functions for different frequencies and to assist in computing the utility function. The signaling packets have a double role: to announce the action of the current user to select a particular channel for transmission, and to serve as probing packets for interference measurements on the selected channel. The signaling packets are transmitted with a fixed transmission power on a common control channel. To simplify the analysis, we assume that no collisions occur on the common control channel. As we mentioned before, the convergence of the adaptation algorithm was experimentally shown to be robust to collision situations. For a better frequency planning, it is desirable to use a higher transmission power for the signaling packets than for the transmitted packets. This will permit the users to learn the potential interferers over a larger area. For our simulations, we have selected the ratio of transmitted powers between signaling and data packets to be equal to two.

We note that the $U2$ utility function has two parts: a) a measure of the interference created by others on the desired user I_d; b) a measure of the interference created by the user on its neighbors' transmissions I_o. The first part of $U2$ can be estimated at the receiving node, while the second part can only be estimated at the transmitter node. Therefore, the protocol requires that both transmitter and receiver listen to the control channel, and each maintain an information table on all frequencies, similar to the NAV table in 802.11. In what follows, we outline the steps of the protocol.

Protocol steps:

1. Bernoulli trial with p_a
   if 0, listen to the common control channel; *break*.
   if 1, go to 2.
2. Transmitter sends START packet: includes current estimates for the interference created to neighboring users on all possible frequencies, $I_o(f)$ (this information is computed based on information saved in the Channel Status Table);

3. Receiver computes current interference estimate for the user $I_d(f)$, determines $U2(f) = -I_d(f) - I_o(f)$ for all channels, and decides on the channel with the highest $U2$ (in case of equality, the selection is randomized, with equal probability of selecting the channels);
4. Receiver includes the newly selected channel information on a signaling packet START_CH which is transmitted on the common channel;
5. Transmitter sends ACK_START_CH which acknowledges the decision of transmitting on the newly selected frequency, and starts transmitting on the newly selected channel;
6. All the other users (transmitters and receivers) that heard the START_CH and ACK_START_CH packets update their Channel Status Tables (CST) accordingly.

We note that when a call ends, only a two-way handshake is required: END, ACK_END to announce the release of the channel for that particular user. Upon hearing these end-of-call signaling packets, all transmitters and receivers, update their CSTs accordingly.

We can see that a different copy of the CST should be kept at both the transmitter and the receivers (CST_t and CST_r, respectively). The entries of each table will contain the neighboring users that have requested a channel, the channel frequency, and the estimated link gain to the transmitter/receiver of that particular user (for CST_r and CST_t, respectively).

The proposed potential game framework has the advantage that an equilibrium is reached very fast following a best response dynamic, but requires substantial information on the interference created to other users and additional coordination for sequential updates. We note however, that the sequential updates procedure also resolves the potential conflicts on accessing the common control channel.

The potential game formulation is suitable for designing a cooperative spectrum sharing etiquette, but cannot be used to analyze scenarios involving selfish users, or scenarios involving heterogeneous users (with various utility functions corresponding to different QoS requirements). In the following section, we present a more general design approach, based on no-regret learning techniques, which alleviates the above mentioned problems.

3.3 Φ-no-regret learning for dynamic channel allocation

While we showed in the previous section that the game with the $U2$ utility function fits the framework of an ex-

act potential game, the $U1$ function lacks the necessary symmetry properties that will ensure the existence of a potential function. In order to analyze the behavior of the selfish users game, we resort to the implementation of adaptation protocols using regret minimization learning algorithms. No regret learning algorithms are probabilistic learning strategies that specify that players explore the space of actions by playing all actions with some non-zero probability, and exploit successful strategies by increasing their selection probability. While traditionally, these types of learning algorithms have been characterized using a regret measure (e.g., external regret is defined as the difference between the payoffs achieved by the strategies prescribed by the given algorithm, and the payoffs obtained by playing any other fixed sequence of decisions in the worst case), more recently, their performance have been related to game theoretic equilibria.

A general class of no-regret learning algorithms called Φ-no-regret learning algorithm are shown in [5] to relate to a class of equilibria named Φ-equilibria. No-external-regret and no-internal regret learning algorithms are specific cases of Φ-no-regret learning algorithm. Φ describes the set of strategies to which the play of a learning algorithm is compared. A learning algorithm is said to be Φ-no-regret if and only if no regret is experienced for playing as the algorithm prescribes, instead of playing according to any of the transformations of the algorithm's play prescribed by elements of Φ. It is shown in [5] that the empirical distribution of play of Φ-no-regret algorithms converges to a set of Φ-equilibria. It is also shown that no-regret learning algorithms have the potential to learn mixed strategy (probabilistic) equilibria. We note that Nash equilibrium is not a necessary outcome of any Φ-no regret learning algorithm [5].

We propose an alternate solution for our spectrum sharing problem, based on a no-external-regret learning algorithm with exponential updates, proposed in [3].

Let $U_i^t(s_i)$ denote the cumulative utility obtained by user i through time t by choosing strategy s_i: $U_i^t(s_i) = \sum_{st=1}^{t} U_i(s_i, S_{-i}^{st})$. For $\beta > 0$, the weight (probability) assigned to strategy s_i at time $t+1$, is given by:

$$w_i^{t+1}(s_i) = \frac{(1+\beta)^{U_i^t(s_i)}}{\sum_{s_i' \in S_i}(1+\beta)^{U_i^t(s_i')}}. \tag{8}$$

In [6], based on simulation results, it is shown that the above learning algorithm converges to Nash equilibrium in games for which pure strategy Nash equilibrium exists. We also show by simulations that the

proposed channel allocation no-regret algorithm converges to a pure strategy Nash equilibrium for cooperative users (utility $U2$), and to a mixed strategy equilibrium for selfish users (utility $U1$).

By following our proposed learning adaptation process, the users learn how to choose the frequency channels to maximize their rewards through repeated play of the game.

For the case of selfish users, the amount of information required by this spectrum sharing algorithm is minimal: users need to measure the interference temperature at their intended receivers (function $U1$) and to update their weights for channel selection accordingly, to favor the channel with minimum interference temperature (equal transmitted powers are assumed). We note that the no-regret algorithm in Eq. 8 requires that the weights are updated for all possible strategies, including the ones that were not currently played. The reward obtained if other actions were played can be easily estimated by measuring the interference temperature for all channels.

For the case of cooperative users, the information needed to compute $U2$ is similar to the case of potential game formulation. We note that, while the learning algorithm does not require sequential updates to converge to an equilibrium, the amount of information exchange on the common control channel requires co-ordination to avoid collisions. One possible approach to reduce the amount of signaling, would be to maintain the access scheme proposed in the previous section, which would ensure that on average only one user at the time will signal changes in channel allocation.

4 Simulation results

In this section, we present some numerical results to illustrate the performance of the proposed channel allocation algorithms for both cooperative and selfish users' scenarios. For simulation purposes, we consider a fixed wireless ad hoc network (as described in the system model section) with $N = 30$ and $D = 200$ (30 transmitters and their receivers are randomly distributed over a $200\,m \times 200\,m$ square area). The adaptation algorithms are illustrated for a network of 30 transmitting radios, sharing $K = 4$ available channels. A random channel assignment is selected as the initial assignment and for a fair comparison, all the simulations start from the same initial channel allocation.

We first illustrate the convergence properties of the proposed spectrum sharing algorithms. We can see that for cooperative games, both the potential game formulation, as well as the learning solution converge to a pure strategy Nash equilibrium (Figs. 2, 4, 10 and 11).

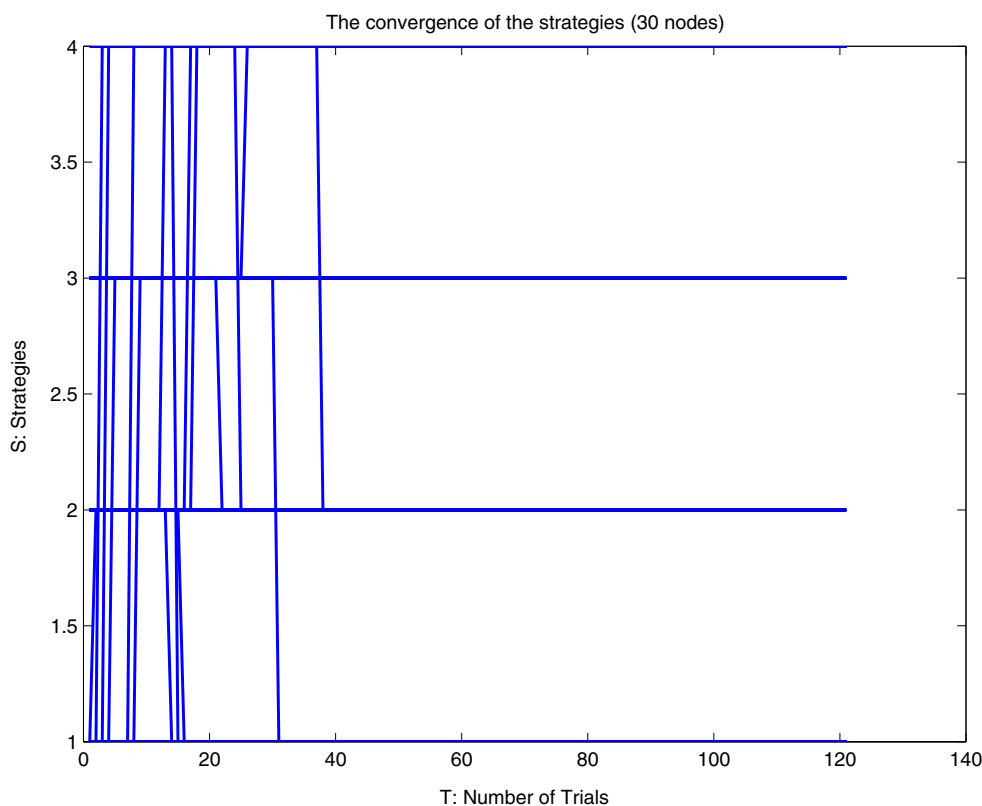**Figure 2** Potential game: convergence of users' strategies

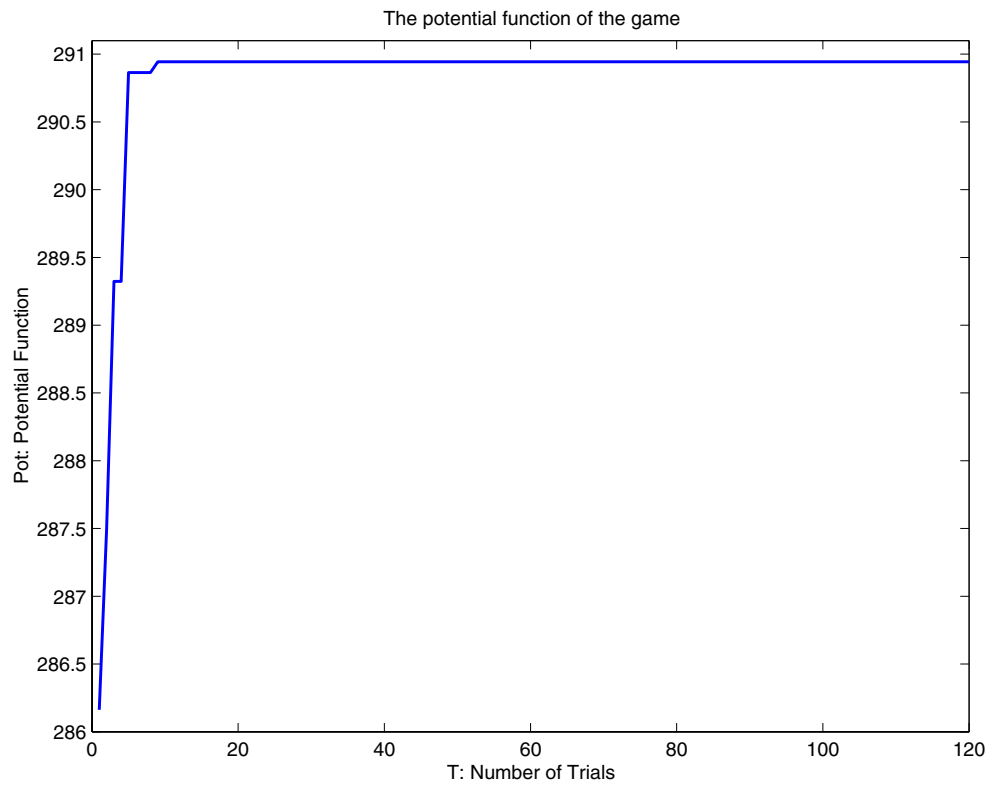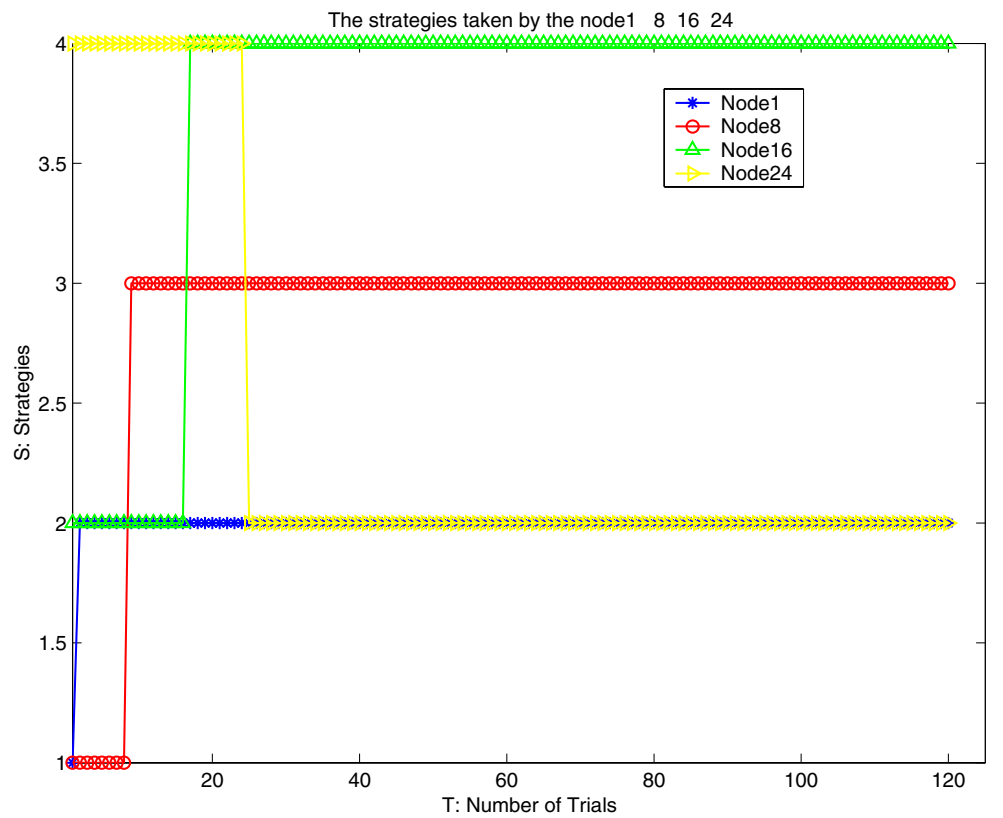**Figure 3** Evolution of
potential function

Figure 4 Potential game:
strategy evolution for
selected arbitrary users

In Fig. 3, we illustrate the changes in the potential function as the potential game evolves, and it can be seen that indeed by distributively improving their utility, the users positively affect the overall utility of the network, which is approximated by the potential function.

By contrast, the selfish users' learning strategy converges to a mixed strategy equilibrium, as it can be seen in Figs. 13 and 14.

As performance measures for the proposed algorithms we consider the achieved SIRs and throughputs (adaptive coding is used to ensure a certain BER target, as previously explained in Section 2). We consider the average performance per user as well as the variability in the achieved performance (fairness), measured in terms of variance and CDF.

We first give results for the potential game based algorithm.

The choice of the utility function for this game enforces a certain degree of fairness in distributing the network resources, as it can be seen in Figs. 5, 6, 7, and 8. Figures 5 and 6 illustrate the SIR achieved by the users on each of the four different channels for initial and final assignments, respectively. An SIR improvement for the users that initially had a low performance can be noticed, at the expense of a slight penalty in performance for users with initially high SIR. It can be seen in Fig. 7 that at the Nash equilibrium point, the number of users having an SIR below 0 dB has been reduced. Furthermore, Fig. 8 shows that the percentage of the users who have an SIR below 5 dB decreases from 60% to about 24%, at the expense of a slight SIR decrease for users with an SIR greater than 12.5 dB.

The advantage of the potential game is illustrated in Fig. 9, in terms of the normalized achievable throughput at each receiver. For the initial channel assignment, 62% of the users have a throughput less than 0.75. At the equilibrium, this fraction is reduced to 38%. Aggregate normalized throughput improvements for the potential game forzmulation are illustrated in Table 2.

Our simulation results show very similar performance for the learning algorithm in cooperative scenarios, with the potential game formulation. Figures 7 and 12, show the initial and final assignment for this algorithm, as well as the achieved SIRs after convergence for all users in the network. In terms of fairness, the learning algorithm performs slighly worse than the potential game formulation (Fig. 9). However, even though the equilibrium point for learning is different than that of the potential game, the two algorithms achieve very close throughput performance (Table 2).

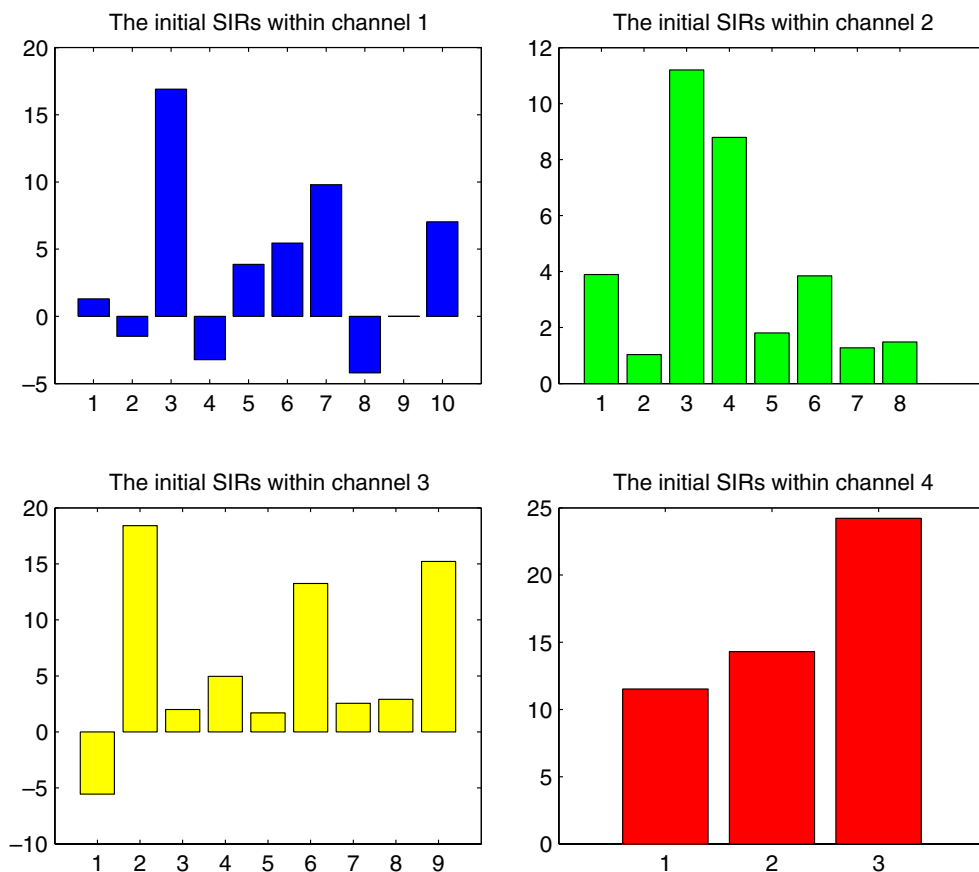**Figure 5** SIRs for initial channel assignment channels

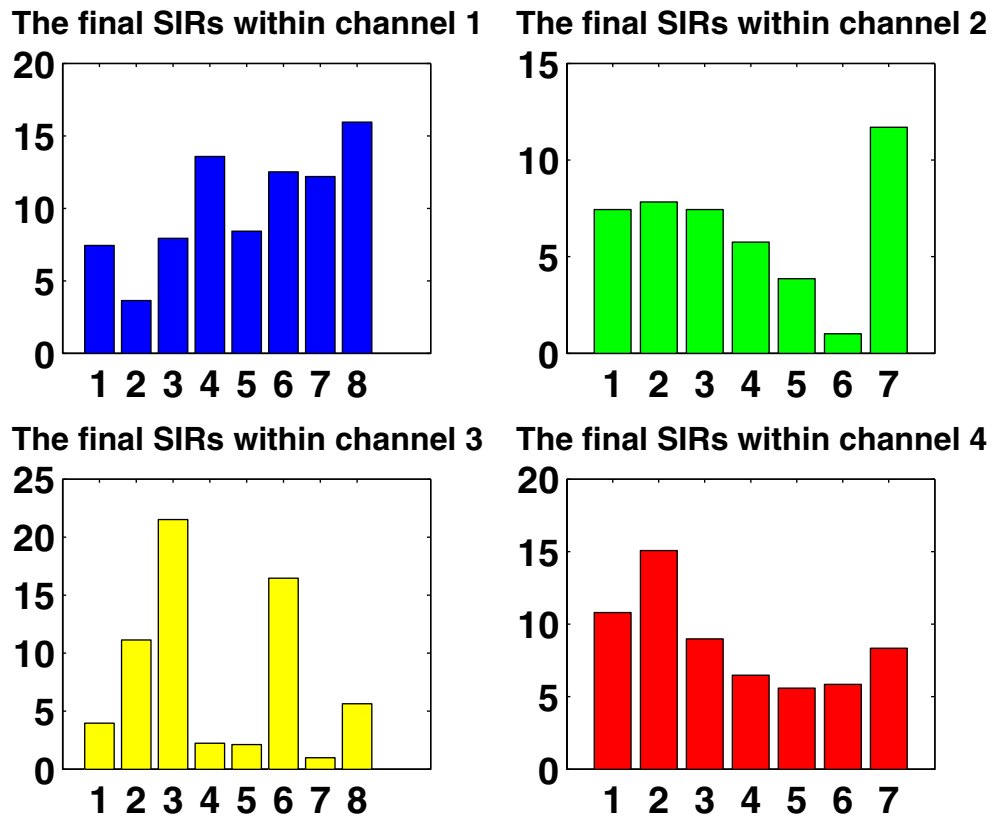**Figure 6** Potential game:
SIRs at final channel
assignment

Figure 7 SIRs histogram.
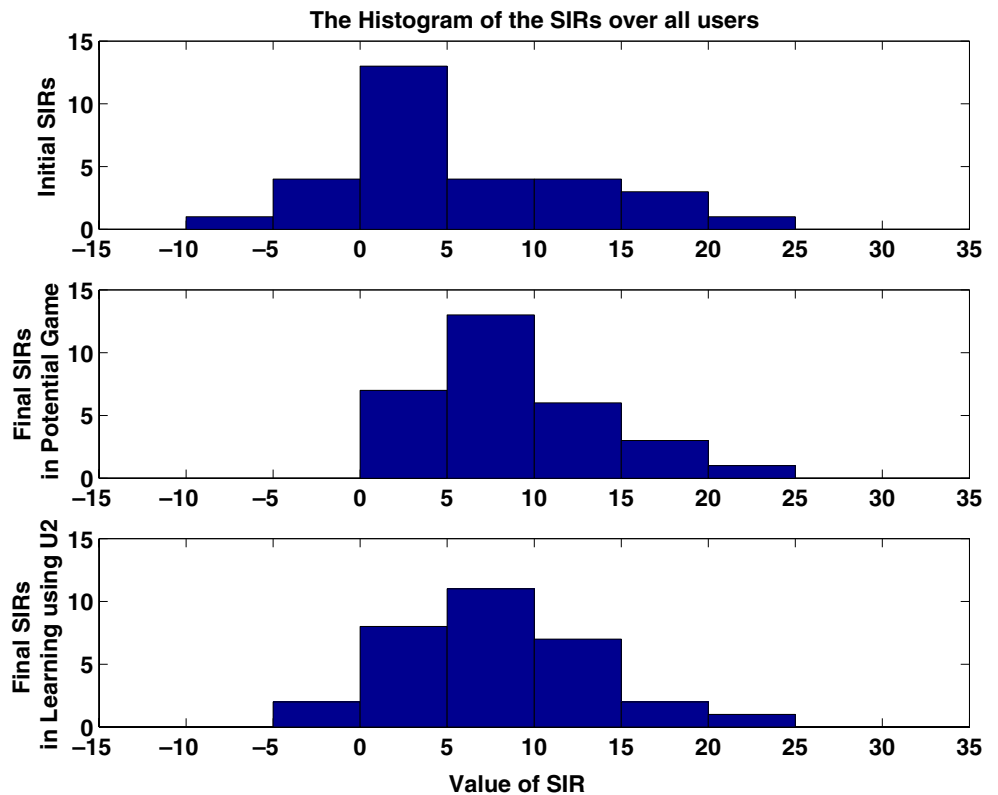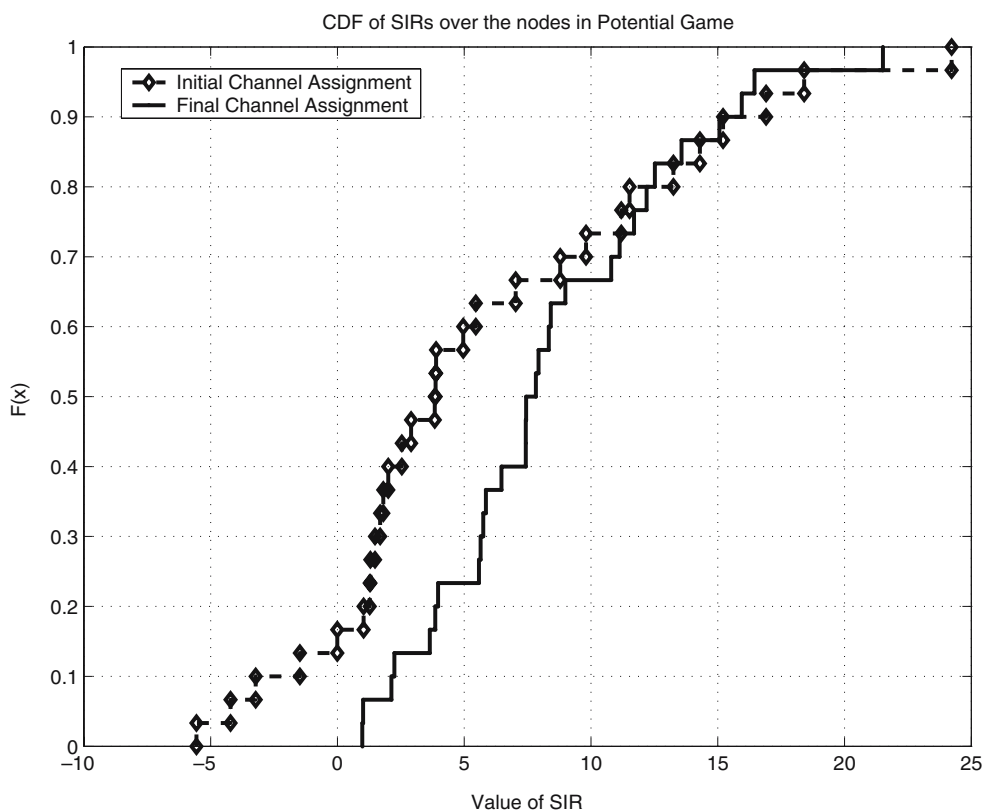Initial channel assignment vs.
final channel assignment

As we previously mentioned, the learning algorithm for selfish users does not lead to a pure strategy Nash equilibrium channel allocation. In Fig. 13 we illustrate the convergence properties for an arbitrarily chosen user, which converges to a mixed strategy allocation: selects channel 1 with probability 0.575 or channel 3

with probability 0.425. The evolutions of the weights for all the users in the network are shown in Fig. 14.

We compare the performance of the proposed algorithms for both cooperative and non-cooperative scenarios. The performance measures considered are the average SIR, average throughput per user, and total average throughput for the network. At the beginning of each time slot, every user will either choose the same equilibrium channel for transmission (in cooperative games with pure strategy Nash equilibrium solutions), or will choose a channel to transmit with some probability given by the mixed strategy equilibrium (i.e., learning using $U1$). In the random channel allocation scheme, every user chooses a channel with equal probability from a pool of four channels.

Figure 15 shows the CDF of Time Average SIR in different games. All learning games and the potential game outperform the random channel allocation
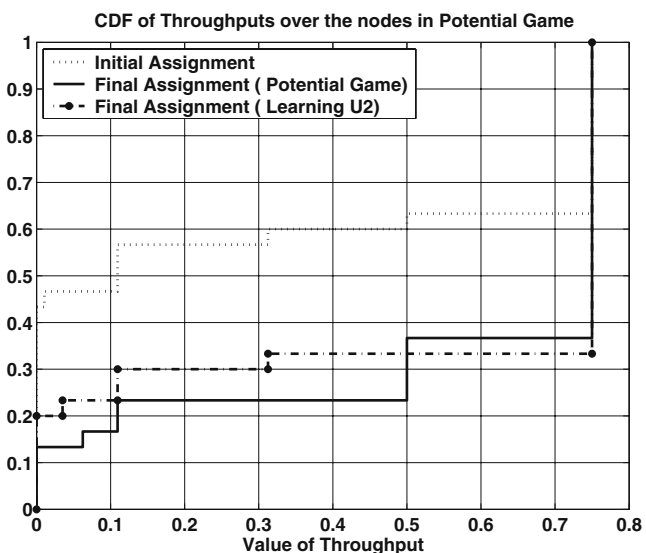

Figure 9 CDF for the achieved throughputs. Initial channel assignment vs. final channel assignment

Table 2 SIR and normalized throughput of all users at initial and final channel assignment

	Total normalized throughput
Initial	9.4
Final (Potential Game)	16.5
Final (Learning $U2$)	15.3

Figure 10 No-regret learning for cooperative users: weights distribution evolution for an arbitrary user
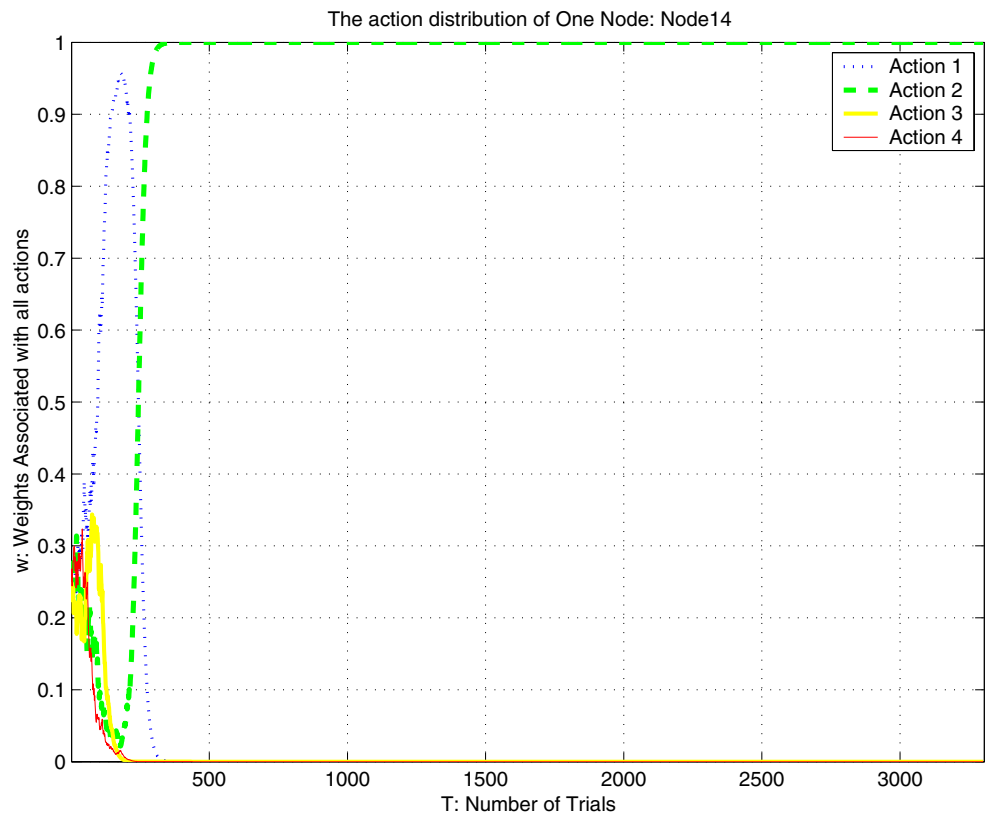

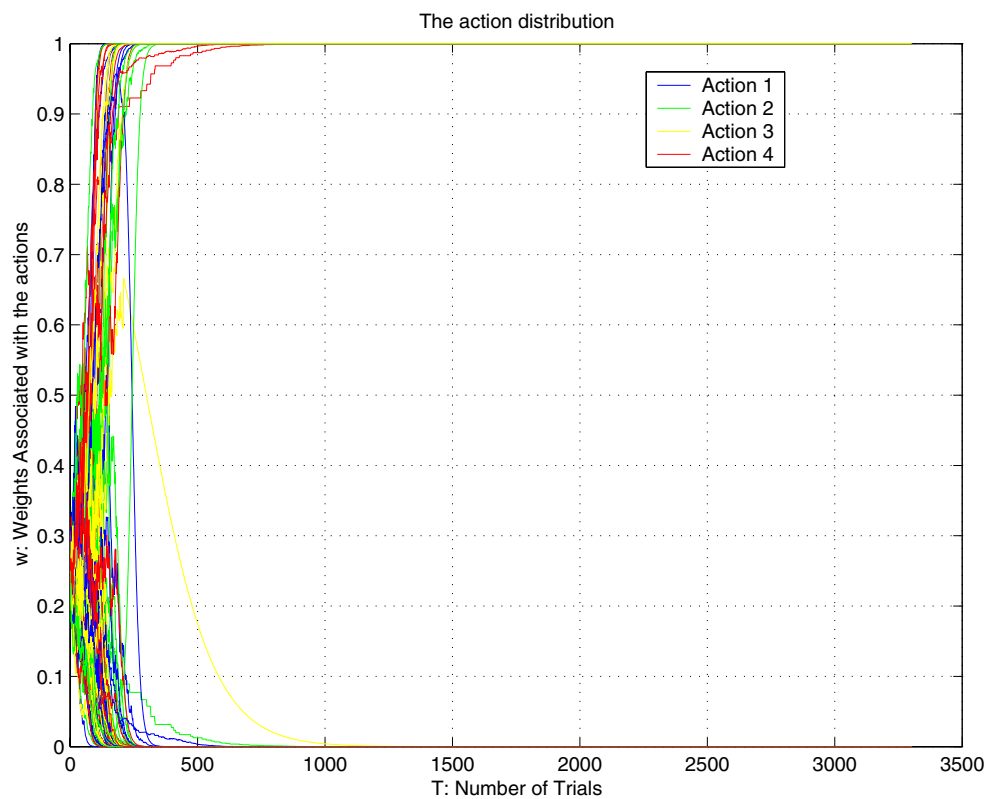**Figure 11** No-regret learning for cooperative users: weights distribution evolution for all users

Figure 12 No-regret learning for cooperative users: SIR of users in different channels at Nash equilibrium
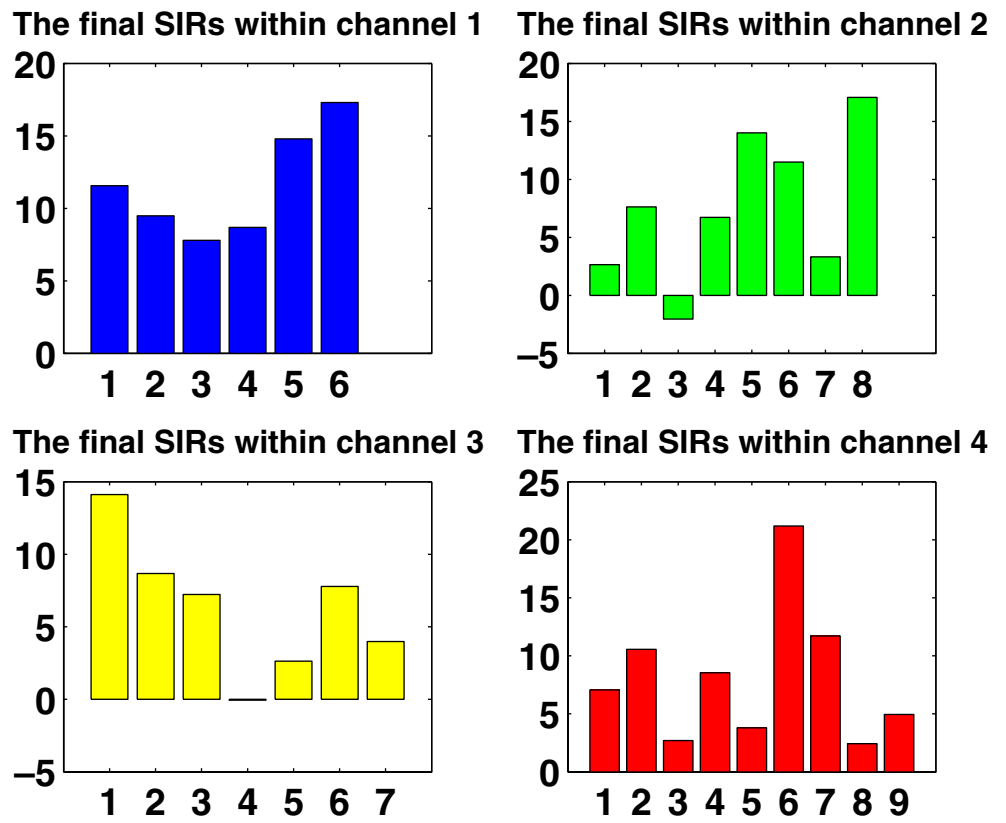

Figure 13 No-regret learning for selfish users: weights evolution for an arbitrary user
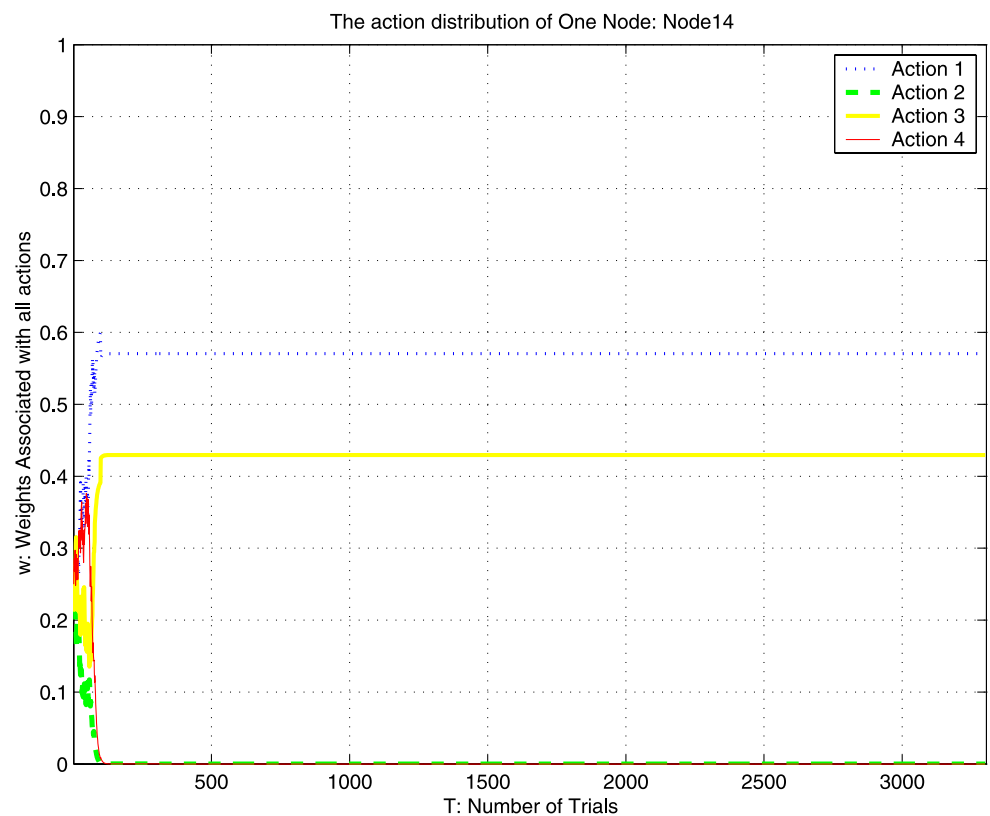
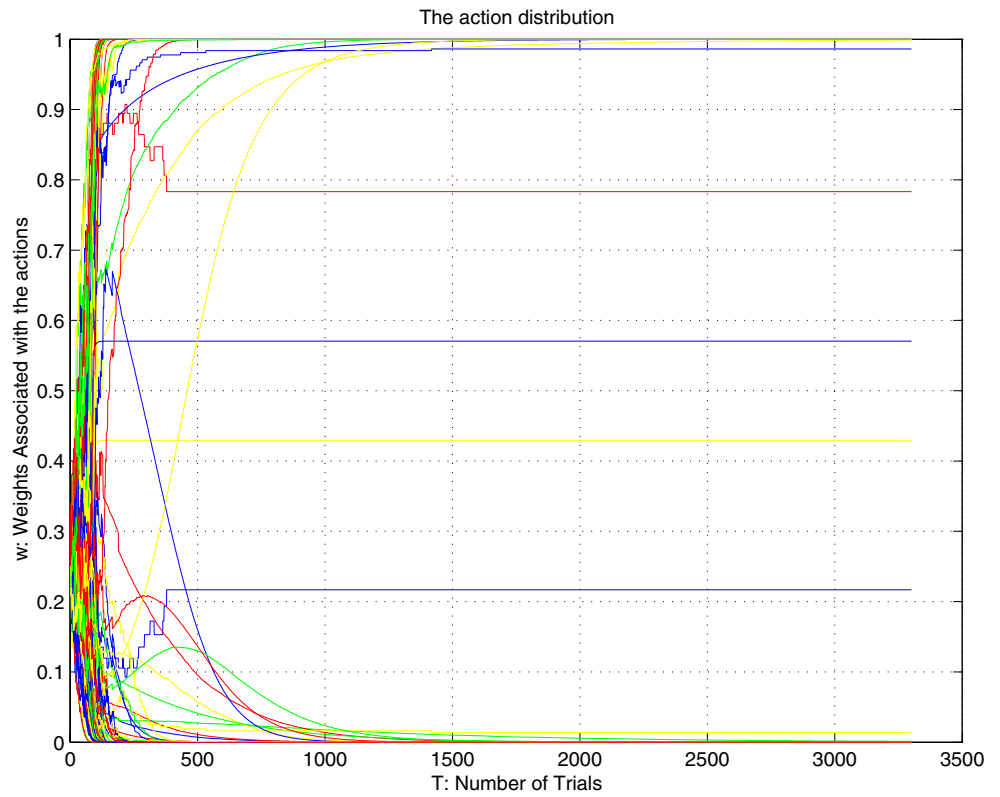**Figure 14** No-regret learning for selfish users: evolution of weights for all users

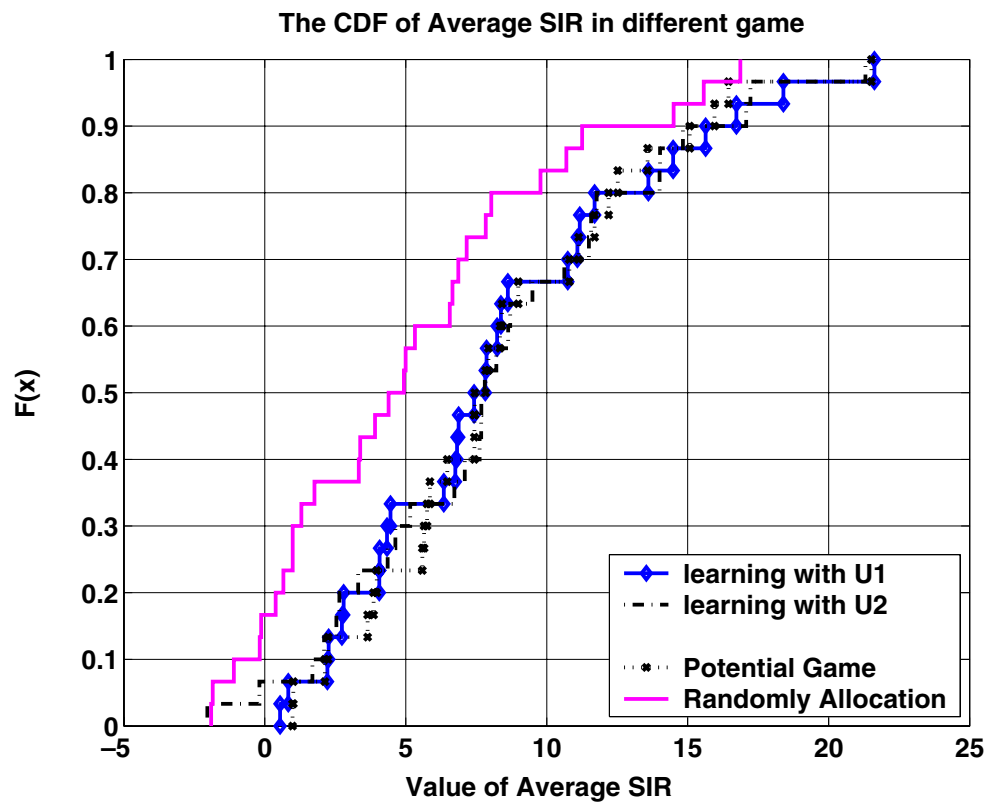**Figure 15** The CDF of time average SIRs

Figure 16 The CDF of
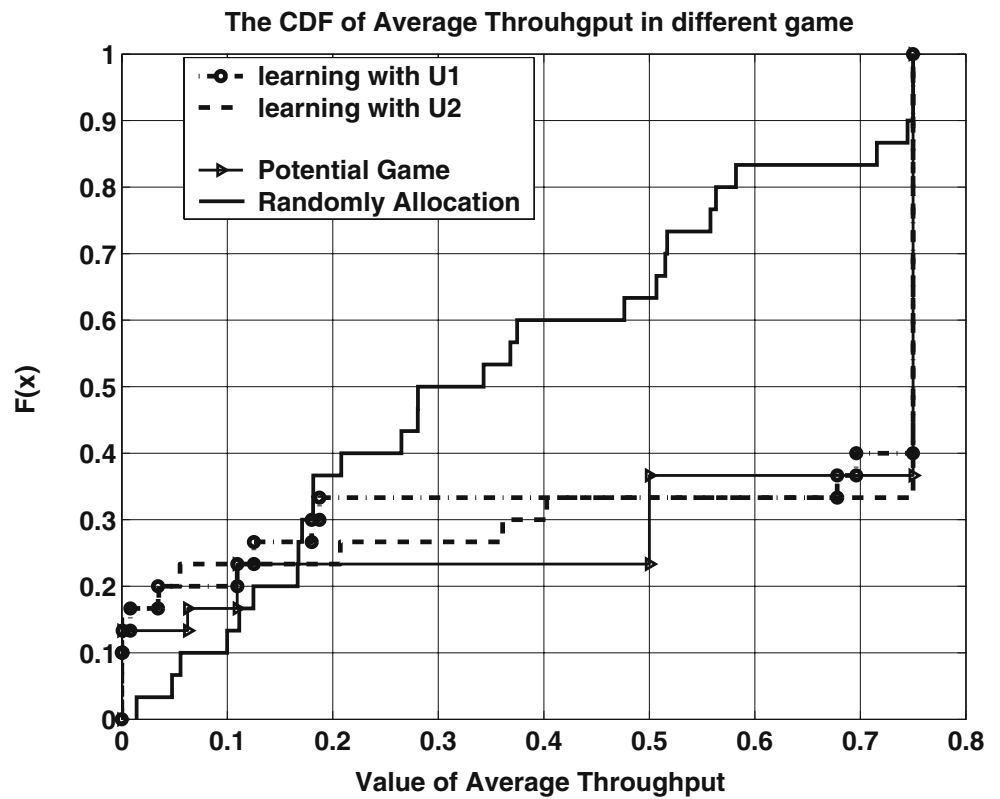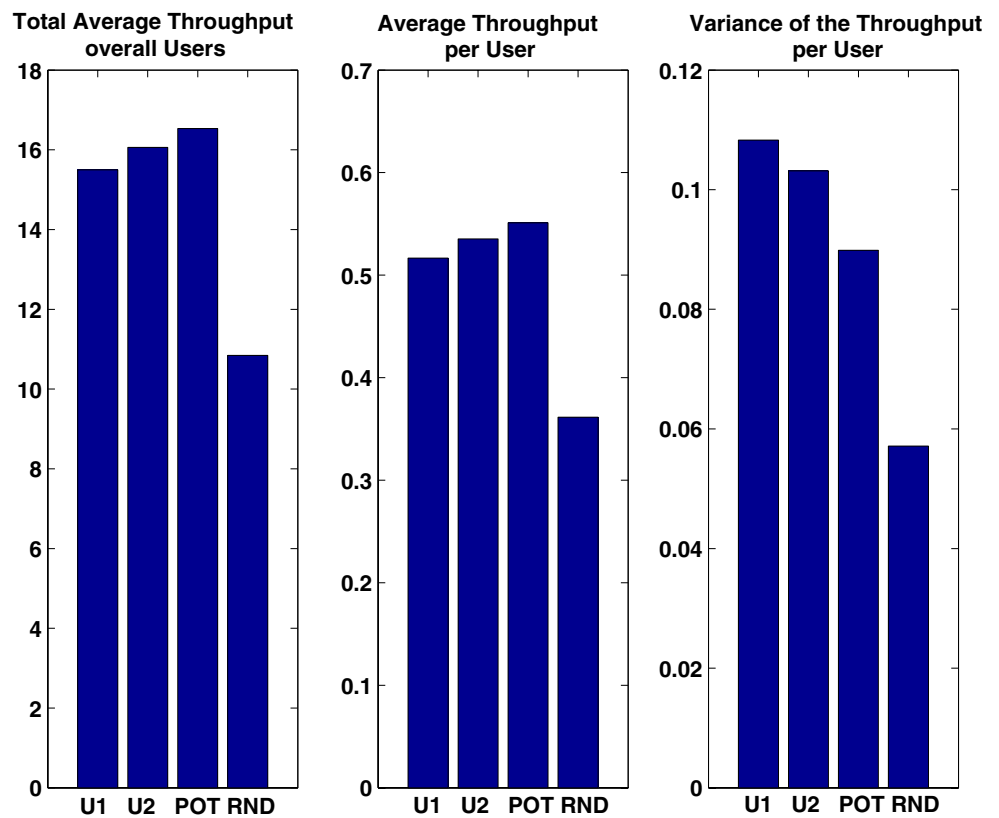average throughput

Figure 17 Total average-
throughput, the mean and the
variance of the throughput
per user

scheme. The potential game has the best throughput performance, followed closely by the cooperative learning scheme. It can be seen in Fig. 16 that half of the users have an average throughput below 0.3 in the random allocation scheme. The percentage of users whose average throughput is below 0.3 is 23% in potential game, 27% for learning using $U2$ and 34% for learning using $U1$, while the fraction is 51% for the random selection.

In Fig. 17 we summarize the performance comparisons among the proposed schemes: total average throughput, average throughput per user, and variance of the throughput per user. The variance performance measure quantifies the fairness, with the fairest scheme achieving the lowest variance. Among all the proposed schemes, potential channel allocation game has the best performance. It is interesting to note that in terms of average obtained throughput per user, the three schemes perform very similar, but differ in the performance variability across users. It seems that even when cooperation is enforced by appropriately defining the utility, the potential game formulation provides a fairness advantage over the no-regret learning scheme.

5 Conclusion

In this work, we have investigated the design of channel sharing etiquette for cognitive radio networks for both cooperative and non-cooperative scenarios. Two different formulations for the channel allocation game were proposed: potential game formulation, and no-regret learning. We showed that all the proposed spectrum sharing policies converge to a channel allocation equilibrium, although a pure strategy allocation can be achieved only for cooperative scenarios. Our simulation results have showed that the average performance in terms of SIR or achievable throughput is very similar for both learning and potential game formulation, even for the case of selfish users. However, in terms of fairness, we showed that both cooperation and allocation strategy play an important role. While the proposed potential game formulation yields the best performance, its applicability is limited to cooperative environments and significant knowledge about neighboring users is required for the implementation. By contrast, the proposed no-regret learning algorithm is suitable for non-cooperative scenarios and requires only a minimal amount of information exchange.

In this work, our focus was on implementing dynamic frequency channel allocation in a distributed fashion, by considering frequency reuse strategies based on SIR measurements. The channel allocation is implemented

at a call level time scale, and cannot respond to lower time scale variations, for example caused by traffic burstiness. While in this work we have considered a worst case scenario for which all users continuosly transmit packets, a more realistic traffic model may prompt to further spectral efficiency gains for the channel allocation if properly combined with an efficient MAC scheme. This is subject of future work.

Appendix

Proof Suppose there is a potential function of game Γ :

$$Pot'(S) = \sum_{i=1}^{N} \left(-a \sum_{j \neq i, j=1}^{N} p_j G_{ij} f(s_j, s_i) - (1-a) \right.$$
$$\left. \sum_{j \neq i, j=1}^{N} p_i G_{ji} f(s_i, s_j) \right) \quad (9)$$

where $0 < a < 1$. Then for all $i \in \{1, 2, ..., N\}$,

$$Pot'(s_i, s_{-i}) = \sum_{i=1}^{N} \left(-a \sum_{j \neq i, j=1}^{N} p_j G_{ij} f(s_j, s_i) \right.$$
$$\left. -(1-a) \sum_{j \neq i, j=1}^{N} p_i G_{ji} f(s_i, s_j) \right)$$
$$= -a \sum_{j \neq i, j=1}^{N} p_j G_{ij} f(s_j, s_i)$$
$$-(1-a) \sum_{j \neq i, j=1}^{N} p_i G_{ji} f(s_i, s_j)$$
$$+ \sum_{k \neq i, k=1}^{N} \left[-a \sum_{j \neq k, j=1}^{N} p_j G_{kj} f(s_j, s_k) \right.$$
$$\left. -(1-a) \sum_{j \neq k, j=1}^{N} p_k G_{jk} f(s_k, s_j) \right]$$
$$= -a \sum_{j \neq ij=1}^{N} p_j G_{ij} f(s_j, s_i)$$

$$-(1-a)\sum_{j\neq i j=1}^{N} p_i G_{ji} f(s_i, s_j)$$

$$+\sum_{k\neq i,k=1}^{N}\left[-ap_i G_{ki} f(s_i, s_k)\right.$$

$$-a\sum_{j\neq k, j\neq ij=1}^{N} p_j G_{kj} f(s_j, s_k)$$

$$-(1-a)p_k G_{ik} f(s_k, s_i)$$

$$\left.-(1-a)\sum_{j\neq k, j\neq i, j=1}^{N} p_k G_{jk} f(s_k, s_j)\right]$$

$$=-a\sum_{j\neq i, j=1}^{N} p_j G_{ij} f(s_j, s_i)$$

$$-(1-a)\sum_{j\neq i, j=1}^{N} p_i G_{ji} f(s_i, s_j)$$

$$+\sum_{k\neq i,k=1}^{N}\left(-ap_i G_{ki} f(s_i, s_k)\right)$$

$$+\sum_{k\neq i,k=1}^{N}\left(-(1-a)p_k G_{ik} f(s_k, s_i)\right)$$

$$+\sum_{k\neq i,k=1}^{N}\left(-a\sum_{j\neq k, j\neq i, j=1}^{N} p_j G_{kj} f(s_j, s_k)\right.$$

$$\left.-(1-a)\sum_{j\neq k, j\neq i, j=1}^{N} p_k G_{jk} f(s_k, s_j)\right)$$

$$=-a\sum_{j\neq i, j=1}^{N} p_j G_{ij} f(s_j, s_i)$$

$$-(1-a)\sum_{j\neq i, j=1}^{N} p_i G_{ji} f(s_i, s_j)$$

$$-a\sum_{k\neq i,k=1}^{N} p_i G_{ki} f(s_i, s_k)$$

$$-(1-a)\sum_{k\neq i,k=1}^{N} p_k G_{ik} f(s_k, s_i)$$

$$+\sum_{k\neq i,k=1}^{N}\left(-a\sum_{j\neq k, j\neq ij=1}^{N} p_j G_{kj} f(s_j, s_k)\right.$$

$$\left.-(1-a)\sum_{j\neq k, j\neq ij=1}^{N} p_k G_{jk} f(s_k, s_j)\right)$$

Let

$$Q(s_{-i}) = \sum_{k\neq i,\ k=1}^{N}\left(-a\sum_{j\neq k, j\neq i, j=1}^{N} p_j G_{kj} f(s_j, s_k)\right.$$

$$\left.-(1-a)\sum_{j\neq k, j\neq i, j=1}^{N} p_k G_{jk} f(s_k, s_j)\right),$$

Then,

$$Pot'(s_i, s_{-i}) = -a\sum_{j\neq i,\ j=1}^{N} p_j G_{ij} f(s_j, s_i)$$

$$-(1-a)\sum_{j\neq i, j=1}^{N} p_i G_{ji} f(s_i, s_j)$$

$$-a\sum_{k\neq i,k=1}^{N} p_i G_{ki} f(s_i, s_k)$$

$$-(1-a)\sum_{k\neq i,k=1}^{N} p_k G_{ik} f(s_k, s_i) + Q(s_{-i})$$

$$=-(a+(1-a))\sum_{j\neq i, j=1}^{N} p_j G_{ij} f(s_j, s_i)$$

$$-(a+(1-a))\sum_{j\neq i, j=1}^{N} p_i G_{ji} f(s_i, s_j)$$

$$+Q(s_{-i})$$

If user i changes its strategy from s_i to s_i', we can get:

$$Pot'(s_i', s_{-i}) = -a\sum_{j\neq i,\ j=1}^{N} p_j G_{ij} f(s_j, s_i')$$

$$-(1-a)\sum_{j\neq i, j=1}^{N} p_i G_{ji} f(s_i', s_j)$$

$$-a\sum_{k\neq i,k=1}^{N} p_i G_{ki} f(s_i', s_k)$$

$$-(1-a)\sum_{k\neq i,k=1}^{N} p_k G_{ik} f(s_k, s_i') + Q(s_{-i})$$

$$=-(a+(1-a))\sum_{j\neq i, j=1}^{N} p_j G_{ij} f(s_j, s_i')$$

$$-(a+(1-a))\sum_{j\neq i, j=1}^{N} p_i G_{ji} f(s_i', s_j)$$

$$+Q(s_{-i})$$

Here $Q(s_{-i})$ is not affected by the strategy changing of user i. Hence,

$$
\begin{aligned}
Pot'(s_i', s_{-i}) - Pot'(s_i, s_{-i}) = & -(a + (1-a)) \sum_{j \neq i, j=1}^{N} p_j G_{ij} f(s_j, s_i') - (a + (1-a)) \sum_{j \neq i, j=1}^{N} p_i G_{ji} f(s_i', s_j) \\
& - \left(-(a + (1-a)) \sum_{j \neq i, j=1}^{N} p_j G_{ij} f(s_j, s_i) - (a + (1-a)) \sum_{j \neq i, j=1}^{N} p_i G_{ji} f(s_i, s_j) \right) \\
= & - \sum_{j \neq i, j=1}^{N} p_j G_{ij} f(s_j, s_i') - \sum_{j \neq ij=1}^{N} p_i G_{ji} f(s_i', s_j) - \left(- \sum_{j \neq ij=1}^{N} p_j G_{ij} f(s_j, s_i) - \sum_{j \neq i, j=1}^{N} p_i G_{ji} f(s_i, s_j) \right)
\end{aligned}
$$

From Eq. 5,

$$
\begin{aligned}
U_i(s_i', s_{-i}) - U_i(s_i, s_{-i}) = & - \sum_{j \neq ij=1}^{N} p_j G_{ij} f(s_j, s_i') - \sum_{j \neq ij=1}^{N} p_i G_{ji} f(s_i', s_j) \\
& - \left(- \sum_{j \neq ij=1}^{N} p_j G_{ij} f(s_j, s_i) - \sum_{j \neq ij=1}^{N} p_i G_{ji} f(s_i, s_j) \right) \forall i = 1, 2, ..., N,
\end{aligned}
$$

$$
U_i(s_i', s_{-i}) - U_i(s_i, s_{-i}) = Pot'(s_i', s_{-i}) - Pot'(s_i, s_{-i}) \forall i = 1, 2, ..., N,
$$

So, $Pot'(S)$ in Eq. 9 is an exact potential function of game Γ. If we set a to $\frac{1}{2}$ in Eq. 9, $Pot'(S)$ is the same as $Pot(S)$ defined in Eq. 7, and we prove that Eq. 7 is an exact potential function of game Γ.

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