# A Utility-Based Joint Power and Rate Adaptive Algorithm in Wireless Ad Hoc Networks

Jialing Zheng and Maode Ma, *Member, IEEE*

***Abstract*—In this paper, a utility-based joint power and rate adaptive algorithm (UPRA) is proposed to alleviate the influence of channel variation by using softened signal-to-interference-and-noise ratio (SINR) requirements. The transmission power and rate of each user will be adjusted to maximize its own net-utility according to the states of channel, which results in a balance between the effective transmission rate and power consumption. Although the UPRA works based on a non-cooperative game, cooperation among users can be achieved so that the throughput of the whole network will be improved. The convergence of the algorithm has also been studied in both feasible and infeasible cases.**

***Index Terms*—Joint power and rate adaptive, non-cooperative game, effective transmission rate, wireless ad hoc networks.**

## I. Introduction

TRANSMISSION power and rate have significant impacts on the performance of wireless networks. On one hand, high transmission rate requires high Signal-to-Interference-and-Noise Ratio (SINR). On the other hand, high transmission power means more energy consumption and interference to other users. Although the issue of joint power and rate control is challenging, it offers more flexibility in radio resource management thus the possibility of better network performance.

Non-cooperative game model was employed for power control in wireless networks in previous work. One characteristic of non-cooperative games is "all players for themselves" [1]. Each node tries to maximize its own net-utility, which is defined as utility minus cost. With distinct optimizing objectives, different utility functions were formulated in prior work, such as throughput per unit power consumption [2], frame outage [3], Shannon capacity [4] and sigmoid-like softened SINR requirement [5]. The cost was often modeled as a linear function of transmission power. In this way, cooperation was introduced to the non-cooperative game. It should be pointed out that all the above algorithms were designed for cellular networks where power control schemes can be implemented at base stations. However, power control in wireless ad hoc networks can only work based on local measurements. In [6], a joint transmission power and rate control algorithm based on game theory, called Distributed Power and Rate Control based on Step-up Pricing Game (DPRC/SPG), was proposed for wireless ad hoc networks. Its net-utility function is defined as the normalized transmission rate minus the cost of power. However, variations of channel condition between two consecutive iterations were not considered in the algorithm. Moreover, the proposed utility function was just on transmission rate while the probability of successful transmission was not taken into account.

To improve the performance of wireless ad hoc networks, a utility-based joint power and rate adaptive algorithm for wireless ad hoc networks, called UPRA, is proposed in this paper. The beauty of this scheme is that the utility function is designed with consideration of both the transmission rate and the probability of successful transmission. Specifically, the utility is defined as effective transmission rate, which is the product of the transmission rate and the probability of successful transmission. Each node selects the power-rate pair to achieve the highest net-utility according to its historical SINRs by measurements. Instead of setting SINR to track the hard threshold like in [7]–[10], a dynamic and optimized margin is reserved for the instant SINR threshold and the balance can be achieved between effective transmission rate and power conservation, which in turn improves the net-utility of each node and the performance of the whole network.

The rest of the paper is organized as follows. In Section II, we present the system model, reformulate the power-control problem as a non-cooperative game, which leads to a power and rate control algorithm. We also discuss the convergence of both feasible and infeasible cases. In Section III, numerical evaluation is given. Finally, Section IV concludes the paper.

## II. Utility-based Joint Rate and Power Control

### A. System Model

We consider a wireless ad hoc network consisting of $N$ transmitter-receiver pairs (links), where all nodes are equipped with identical multi-rate half-duplex transceivers. The SINR of the $i$th link is given by

$$SINR_i = \alpha_i P_i \tag{1}$$

with

$$\alpha_i = \frac{G_{ii}}{\sum_{j=1,j\neq i}^{N} G_{ij}P_j + \eta_i}. \tag{2}$$

Here $G_{ij}$ denotes the channel gain from the transmitter of the $j$th link to the receiver of the $i$th link. $P_i$ is the transmission power of the $i$th link which is tunable between 0 and $P_{\max}$. $\eta_i$ is the power of the white noise at the receiver of the $i$th link. In the rest of this paper, "interference" refers to the sum of the interference caused by other links and white noise.

In this multiple rate system, let $\gamma_i$ denote the SINR requirement of the $i$th link for the lowest transmission rate $R_1$. The

Paper approved by S. A. Jafar, the Editor for Wireless Communication Theory and CDMA of the IEEE Communications Society. Manuscript received March 13, 2006; revised May 1, 2007 and October 9, 2007. This paper was presented in part at the IEEE GLOBECOM'07, Washington, November 2007.

The authors are with the School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore (email: emdma@ntu.edu.sg).

Digital Object Identifier 10.1109/TCOMM.2009.0901.060524

0090-6778/09$25.00 © 2009 IEEE

SINR requirements of the $i$th link for the transmission rates $b_2R_1, b_3R_1, \cdots, b_MR_1$ $(1 = b_1 < b_2 < b_3 < \cdots < b_M)$ are $h_2\gamma_i, h_3\gamma_i, \cdots, h_M\gamma_i$ $(1 = h_1 < h_2 < h_3 < \cdots < h_M)$, respectively. Since in Shannon capacity increasing SINR provides diminishing returns in rate, it is assumed that [6]

$$\frac{h_M - h_{M-1}}{b_M - b_{M-1}} > \frac{h_{M-1} - h_{M-2}}{b_{M-1} - b_{M-2}} > \cdots > \frac{h_2 - h_1}{b_2 - b_1} > 1 \quad (3)$$

### B. Power and Rate Selection in UPRA

In the wireless ad hoc network discussed in this paper, all the users are peers and compete for limited radio resources selfishly, thus no explicit cooperation is possible. Such a problem is a typical non-cooperative game [1]. We define the net-utility of user $i$ as

$$NU_i = U_i - C_i \quad (4)$$

where $U_i$ and $C_i$ are the utility and cost of user $i$, respectively. Each user tries to maximize its own net-utility by jointly adjusting its transmission power $P_i$ and transmission rate $r_i$. In UPRA, $C_i$ is defined as a linear function of transmission power

$$C_i(P_i) = c_iP_i \quad (5)$$

where the coefficient $c_i$ is a positive constant. The utility function $U_i$ is defined as the effective transmission rate, which is the product of the transmission rate and the probability of successful transmission. If $SINR_i > h_k\gamma_i$, applicable data rates could be $R_1, R_1b_2, \cdots, R_1b_k$. The lower is the data rate, the more likely data is successful received. In fact, any rates lower than the threshold can be a candidate.

The variation pattern of the channel condition was studied previously [11]–[14]. Let $\alpha_i(k) = G_{ii}(k)/\left(\sum_{j=1,j\neq i}^{N} G_{ij}(k)P_j(k) + \eta_i\right)$ denote $\alpha_i$ at discrete time instant $k$. Let $\overline{x}$ denote the decibel value of a variable $x$, namely, $\bar{x} = 10\log x$. $\overline{\alpha}_i(k)$ can be assumed to vary according to the random-walk model [13].

$$\overline{\alpha}_i(k+1) = \overline{\alpha}_i(k) + \overline{n}_i(k) \quad (6)$$

where $\overline{n}_i(k)$ is a normal distributed random variable representing the variation of environment. The mean value of $\overline{n}_i(k)$ is zero and its variance is $\sigma^2$. If link $i$ transmits with rate $b_mR_1$ at time index $k$ and power $P_i$ is fixed from time instant $k$ to $k+1$, the probability of $SINR_i$ staying above the threshold $h_m\gamma_i$, referred to as SINR guarantee, for time index $k+1$ is represented by

$$S(SINR_i, m) = \frac{1}{2}\left\{1 + erf\left[\frac{\ln(SINR_i/h_m\gamma_i)}{\sigma\sqrt{2}}\right]\right\} \quad (7)$$

with $m = 1, 2, \cdots, M$. $S(SINR_i, m)$ is the cumulative distribution function of lognormal distribution. $\sigma = 0$ means that $\alpha_i$ is constant. When $\sigma$ increases, higher $SINR_i$ is needed to achieve the same SINR guarantee.

In order to ensure that $SINR_i$ is greater than or equal to the threshold, a suitable transmission rate will be selected under the constraint of channel conditions. Let $r_i$ denote the ratio of candidate transmission rate to the lowest data rate. Then $r_i$ is a function of $SINR_i$, denoted as

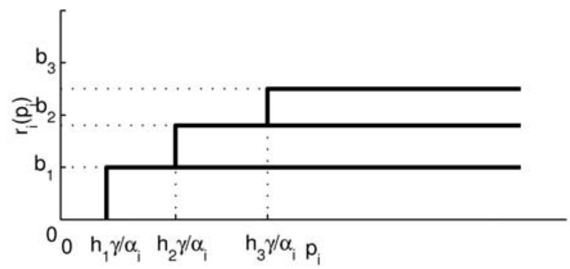


Fig. 1. Candidate data rates and the transmission power under the condition of $\alpha_i$ in a three-rate system.

$$r_i(SINR_i) = \begin{cases} b_M, b_{M-1}, \cdots, \text{or } b_1 & SINR_i \geq h_M\gamma_i \\ b_{M-1}, \cdots, \text{or } b_1 & h_M\gamma_i > SINR_i \geq h_{M-1}\gamma_i \\ \vdots & \vdots \\ b_1, & h_2\gamma_i > SINR_i \geq h_1\gamma_i \\ 0, & h_1\gamma_i > SINR_i \end{cases} \quad (8)$$

which is plotted in Fig. 1. It should be noted that the relationship between $r_i$ and $SINR_i$ is not one-to-one relationship.

Thus far, we have formulated the relationship between transmission rate $r_i$, SINR guarantee, and $SINR_i$ respectively. We integrate transmission rate and SINR guarantee into the utility function $U_i$ as (9).

$$U_i(SINR_i, m) = r_i(SINR_i)S_i(SINR_i, m) = \frac{b_m}{2}\left\{1 + erf\left[\frac{\ln(SINR_i/h_m\gamma_i)}{\sigma\sqrt{2}}\right]\right\} \quad m = 1, 2, \cdots, M \quad (9)$$

$U_i$ is an increasing function of $SINR_i$ which satisfies: $U_i(0, m) = 0$ ; $U_i(\infty, m) = b_m$.

By (1), (4) and (5), the net utility of user $i$ is

$$NU_i(SINR_i, m, P_i) = U_i(SINR_i, m) - C_i(P_i) = r_i(SINR_i)S_i(SINR_i, m) - \frac{c_i}{\alpha_i}SINR_i \quad (10)$$

The utility and the cost versus SINR of a 3-rate system are depicted in Fig. 2. The curves are utilities using different transmission rates. The straight lines 1, 2, 3 and 4 denote cost at different channel condition $\alpha_i$. The slope of the cost lines is $c_i/\alpha_i$. Since $\alpha_i = G_{ii}/\left(\sum_{j=1,j\neq i}^{N} G_{ij}p_j + \eta_i\right) \leq G_{ii}/\eta_i$. The cost line has a positive lower bound as illustrated by line 1. Let $\underline{K}_i$ be the slope of line 1, i.e. $\underline{K}_i = c_i(\eta_i/G_{ii})$. For a given transmission rate $b_m$, when the slope is small enough $c_i/\alpha_i < 0.5b_m/h_m\gamma_i$, the cost line has at most one intersection with the utility curve. When the slope increases to $0.5b_m/h_m\gamma_i$, the intersection is at the point $(h_m\gamma_i, 0.5b_m)$ as shown by line 4. $\overline{K}_i$ is the slope of line 4, i.e., $\overline{K}_i = 0.5b_m/h_m\gamma_i$. When we continue to increase the slope, there is no intersection any more and it means the cost is very high.

The joint rate and power control problem for user $i$ can be formulated as

$$\max_{\substack{0 \leq P_i \leq P_{\max}, \\ m=1,2,\cdots,M}} NU_i \quad (11)$$

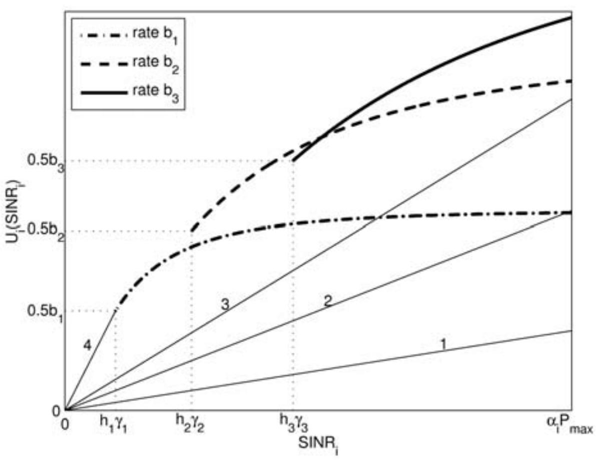



Fig. 2. Utility (and cost) versus *SINR* for user $i$.

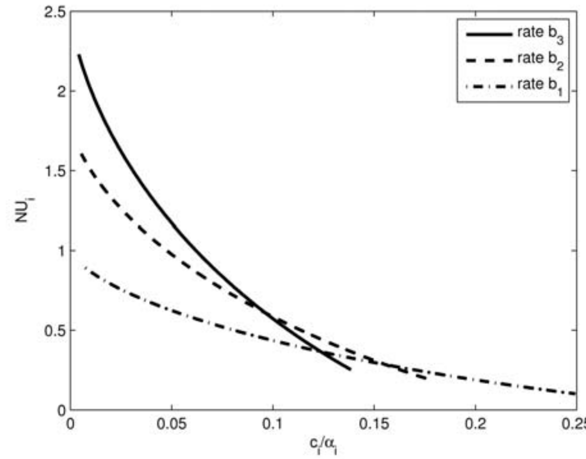



Fig. 3. Net utility using $SINR_{iTAR}$ versus $c_i/\alpha_i$ for user $i$.

Taking the derivative of $NU_i$ with regard to $SINR_i$, we have:

$$\frac{d\left[U_i(SINR_i, m_i) - \dfrac{c_i}{\alpha_i} SINR_i\right]}{dSINR_i} = 0 \tag{12}$$

Solving this function for $\sigma > 0$, it has a unique solution

$$SINR_i = \min\left(h_m\gamma_i e^{\sqrt{2\sigma^2 \ln(b_m\alpha_i/\sqrt{2\pi}\sigma c_i h_m \gamma_i)+\sigma^4}-\sigma^2}, \alpha_i P_{\max}\right)$$

for $c_i/\alpha_i \le \min\left(b_m/(\sqrt{2\pi}\sigma h_m\gamma_i), 0.5b_m/h_m\gamma_i\right)$. For a given rate $b_m$, there exists one optimal target SINR, denoted as $SINR_{iTAR,m}$, with which the net utility could be maximized.

If $\alpha_i P_{\max} > h_m\gamma_i$, $SINR_{iTAR,m} = 0$. Otherwise, $SINR_{iTAR,m}$ is denoted in (13),

If $\sigma = 0$, it means that the channel condition remains constant and the best strategy is to set the target SINR as the SINR threshold $h_m\gamma_i$. The resultant utility is equal to the relative transmission rate.

So far, we have got the best target SINR for a given $m$.With the aim to maximize the net utility, the overall optimum power and rate among different rates are denoted by $SINR_{iOPT}$ and $m_{OPT}$ respectively as

$$(SINR_{iOPT}, m_{iOPT}) = \underset{\substack{0\le P_i \le P_{\max},\\ m=1,2,\cdots,M}}{\arg\max} \left[NU_i\left(SINR_i, m_i\right)\right] \tag{14}$$

From (1), the optimum power is given as

$$P_{iOPT} = SINR_{iOPT}/\alpha_i \tag{15}$$

It should be noted that if $SINR_{iOPT} = 0$, the best strategy for a node is not to transmit.

In our model, if an iteration occurs at time index $k$, the iteration index is defined as $k$. Apply the iteration index to (15), we obtain

$$P_{iOPT}(k+1) = \frac{SINR_{iOPT}(k+1)}{SINR_i(k)} P_i(k). \tag{16}$$

### C. UPRA Algorithm

Initially, each link uses the predetermined transmission rate and power. In each of the following iterations, link $i$ operates as following steps.

1) Calculate $\alpha_i$ by (2) and $\sigma$ from historical values of $\alpha_i$, respectively. Then, $SINR_{iTAR}$ and net utility for each transmission rate are obtained according to (13) and (10), respectively. The $(SINR_{iOPT}, m_{iOPT})$ pair that has the highest net utility can be found. If $SINR_{iOPT} = 0$, give up the transmission in this iteration and wait for a random time to start its next iteration. Otherwise go to step 2.
2) Data is transmitted using the transmission rate $b_{m_{iOPT}} R_1$ and the transmission power $P_{iOPT}$ obtained by (16).

The net utility of higher data transmission rates decreases faster than that of lower data transmission rates when $c_i/\alpha_i$ increases, as shown in Fig. 3. This property has been proven analytically in Appendix. Therefore, according to (14), lower data transmission rate is preferred when $c_i/\alpha_i$ is large as illustrated in Fig. 4(b). Since large $c_i/\alpha_i$ indicates a worse channel condition, links will not further exacerbate the already deteriorated channel by increasing transmission power greedily. In this way, cooperation is realized.

### D. Feasibility and Convergence

By UPRA, a system is defined as feasible if a power assignment vector $P = [P_1, P_2, \cdots, P_N]^T$ exists while $SINR_i \ge b_m\gamma_i$ for all links. Such a power vector is a feasible solution.

In a feasible system, an iterative power control algorithm $\mathbf{P}(k+1) = I(\mathbf{P}(k))$ is said to be standard if for all $\mathbf{P} \ge 0$ the following properties are satisfied [15].

- Positivity: $I(\mathbf{P}) > 0$;
- Monotonicity: If $\mathbf{P} \ge \mathbf{P}'$, then $I(\mathbf{P}) \ge I(\mathbf{P}')$;
- Scalability: For all $a > 1$, $aI(\mathbf{P}) > I(a\mathbf{P})$.

$I(\mathbf{P})$ is referred as the *interference function*. It can be proven that UPRA is standard as shown in Appendix. Using the results of [15] concerning convergence, we get the following

$$SINR_{iTAR,m} = \begin{cases} \min\left(h_m\gamma_i e^{\Psi_m}, \alpha_i P_{\max}\right) & \frac{c_i\eta_i}{G_{ii}} \le \frac{c_i}{\alpha_i} \le \min\left(\frac{b_m}{\sqrt{2\pi}\sigma h_m\gamma_i}, \frac{0.5b_m}{h_m\gamma_i}\right) \\ h_m\gamma_i & \frac{b_m}{\sqrt{2\pi}\sigma h_m\gamma_i} < \frac{c_i}{\alpha_i} < \frac{0.5b_m}{h_m\gamma_i} \\ 0 & \frac{c_i}{\alpha_i} \ge \frac{0.5b_m}{h_m\gamma_i} \end{cases} \quad (13)$$
$$\text{with } \Psi_m = \sqrt{2\sigma^2\ln(b_m\alpha_i/\sqrt{2\pi}\sigma c_i h_m\gamma_i) + \sigma^4} - \sigma^2$$

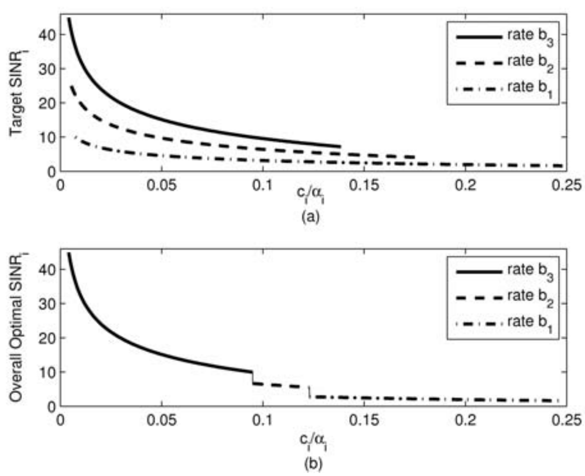



Fig. 4. $SINR_{iTAR}$ and $SINR_{iOPT}$ versus $c_i/\alpha_i$ for user $i$ of UPRA.

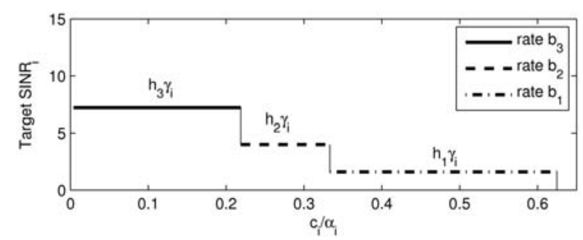



Fig. 5. $SINR_{iTAR}$ versus $c_i/\alpha_i$ for user $i$ of DPRC/SPG.

properties of our scheme when it is applied to a feasible system.

- If the proposed scheme has a fixed point, it is unique.
- For any initial power vector $\mathbf{P}$, the iteration converges to a unique fixed point $\mathbf{P}^*$.

When the system is infeasible, there is a problem of "opting out", which is not special to UPRA but rather arises whatever algorithm is used [9]. By UPRA, links that find $SINR_{iOPT} = 0$ will give up transmission for a random period as described in Step 1, which means that these links quit from the game. Some other links may select the slow rates rather than the highest one.

## III. Performance Evaluation

### A. Numerical Evaluation for a Link

In this subsection, we demonstrate how a user reacts to the variations of channel conditions by adjusting transmission rate and power by the UPRA algorithm. From (2) and the fact that $c_i$ is a constant, large $c_i/\alpha_i$ means large interference and hostile environment. We show the power and rate selection procedure of a 3-rate system where $h_3 = 4.5$, $h_2 = 2.5$, $h_1 = 1$, $b_3 = 2.5$, $b_2 = 1.8$ and $b_1 = 1$. SINR requirement for the lowest transmission rate $\gamma_i = 1.6$. Cost coefficient $c_i = 1$. The transmission power is bounded by $P_{\max} = 1$. We set the standard variance $\sigma$ to be 1.

Fig. 4(a) shows the changes of $SINR_{iTAR}$ for 3 rates when $c_i/\alpha_i$ varies between $0.03b_m/(\sqrt{2\pi}\sigma h_m\gamma_i)$ and $b_m/(\sqrt{2\pi}\sigma h_m\gamma_i)$. It can be seen that as $c_i/\alpha_i$ increases, $SINR_{iTAR}$ decreases. Moreover although $SINR_{iTAR}$ of a higher transmission rate is always greater than that of a lower transmission rate, it decreases faster than that of a lower one.

Fig. 3 shows how much net utility can be achieved if $SINR_{iTAR}$ in Fig. 4(a) is used. As $c_i/\alpha_i$ increases, the net utility decreases. When $c_i/\alpha_i$ is very small, the net utility of rate $b_3$ is the greatest. The net utility of a higher transmission rate decreases faster than that of a lower transmission rate when $c_i/\alpha_i$ increases. When $0.095 < c_i/\alpha_i < 0.123$, the net utility of rate $b_2$ becomes the greatest. When $c_i/\alpha_i > 0.123$, rate $b_1$ has the greatest net utility. Accordingly, the curve of $SINR_{iOPT}$ will be composed of 3 parts as shown in Fig. 4(b), each corresponding to one transmission rate. Without loss of generality, it can be proved that due to the monotonic $SINR_{iOPT}$, the optimum transmission rate will be decreasing when $c_i/\alpha_i$ increases. From Fig.3-4, we can find that a link will be able to slow down its transmission rate and reduce target SINR when the environment becomes hostile, which is in consistent with other rate-adaptive MAC algorithms in ad hoc networks (e.g., [16]). Implicit social cooperation can be produced by the UPRA algorithm though each link tries to maximize its own net utility selfishly in the non-cooperative game. $SINR_{iTAR}$ versus $c_i/\alpha_i$ for DPRC/SPG is also shown in Fig. 5 for comparison, in which $SINR_{iTAR}$ remains constant and equal to a threshold value in each segment and does not vary with the channel conditions.

### B. Simulation Results

To illustrate the advantage of the proposed mechanism, we compare the UPRA algorithm with DPRC/SPG algorithm in [6]. The latter sets the SINR threshold as target SINR.

We consider 8 one-hop links randomly located in a 1.5-km-square area. Maximum power is 1W. Background receiver noise power within the user's bandwidth of $\eta_i = 8\times10^{-13}$W is used in the simulation. The path gain $G_{ij}$ is modeled as $G_{ij} = A_{ij}/d_{ij}^4$, where $d_{ij}$ is the distance between the transmitter of link $j$ to the receiver of link $i$, and $A_{ij}$ is the attenuation factor due to fading. We assume that all $A_{ij}$ are independent and identically log-normally distributed random variables with

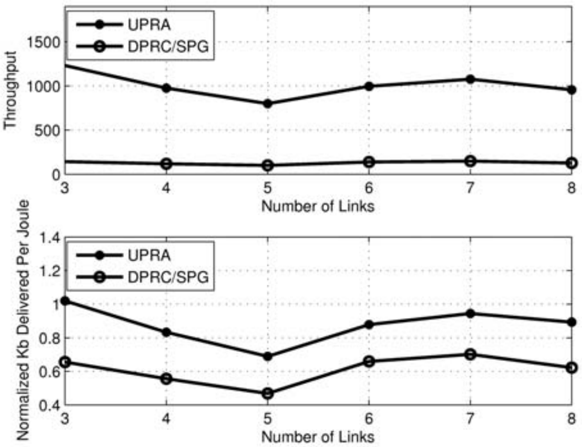



Fig. 6. Throughput and power versus the number of links.

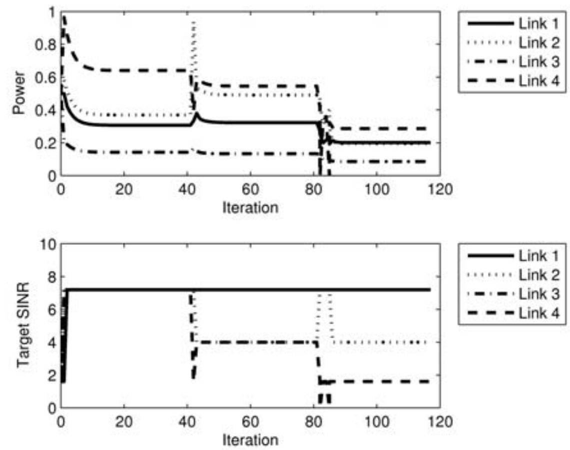



Fig. 7. Evolution of power and target *SINR* in a feasible case, using DPRC/SPG.

0-dB expectation and 8-dB log-variance. The fading values of links change asynchronously. When $SINR_{iOPT} = 0$, link $i$ holds its transmission for a random period which is uniformly distributed between $[1 \sim N]$ iterations.

First, we verify the capacity improvement via simulation with different numbers of links. The cost coefficient for DPRC/SPG starts from the lower bound $c_{min} = \frac{h_M(b_M - b_{M-1})}{(h_M - h_{M-1})P_{\max}}$, increased by $c = c\delta$. The step size is set to be $\delta = 1.2$ in the comparison. Simulation results show that the UPRA algorithm can achieve an obvious improvement on throughput and power conservation over the DPRC/SPG algorithm as shown in Fig. 6. It is because of the fact that the latter is vulnerable since actual SINR maybe readily goes below the target threshold due to variation of channel conditions, causing the cost coefficient to increase step by step and eventually making some links be refrained from transmission. The UPRA algorithm can also alleviate the influence of SINR variation. When channel condition fluctuates intensively, the target SINR will be set higher than the SINR threshold so as to leave a margin to cover the variation to some extent. When channel condition is stable, higher transmission rates are used and the target SINR is close to the SINR threshold, which leads to thrift in power.

Next, convergence is shown using a 4-link 3-rate system. The path gain matrix G of the simulated system is shown in (17).

$$G = \begin{bmatrix} 0.0707 & 0.0060 & 0.0104 & 0.0008 \\ 0.0151 & 0.1692 & 0.0071 & 0.0035 \\ 0.0120 & 0.0041 & 0.3203 & 0.0005 \\ 0.0015 & 0.0140 & 0.0005 & 0.0731 \end{bmatrix} \times 10^{-9} \qquad (17)$$

It is easy to verify that the system is feasible when $\gamma_i = 1.6$, $h_3 = 4.5$, $h_2 = 2.5$, $h_1 = 1$, $b_3 = 2.5$, $b_2 = 1.8$ and $b_1 = 1$. The evolution curves of power and SINR for the DPRC/SPG and the UPRA are shown in Fig. 7 and Fig. 8, respectively. Before the 40th iteration, the target SINR and power converge to the same point in the two schemes because their power vector and rate vector are identical in feasible condition. At the 40th and the 80th iterations, fading values of the four links change and they cause jumps in target SINR and power. After the 80th iteration, by the DPRC/SPG, link 1 and 3 select the highest rate; link 2 selects the middle rate and link 4 selects the lowest rate. While by the UPRA, link 1, 2 and 3 select the highest rate and link 4 selects the lowest rate.

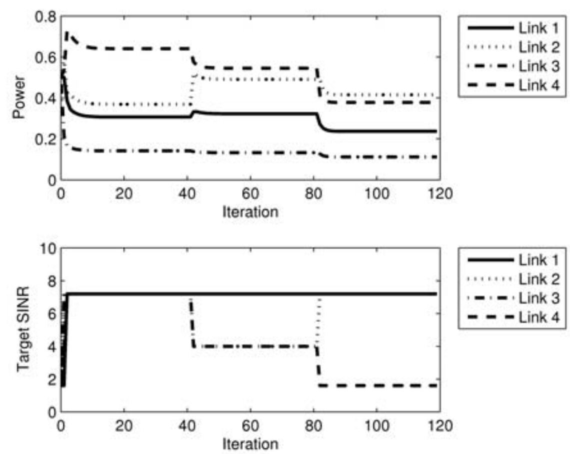



Fig. 8. Evolution of power and target *SINR* in a feasible case, using UPRA with cost coefficient $c = 1$.

The system is infeasible when $\gamma_i = 2$. The evolution of power and target SINR for DPRC/SPG and UPRA are shown in Fig. 9 and Fig. 10, respectively, which show that both schemes are convergent. Before the 40th iteration, by the DPRC/SPG, link 1, 2, and 3 select the highest rate while link 4 selects the lowest rate. At the same time, by the UPRA, link 2 and link 3 select the highest rate while link 1 and link 4 refrain from transmission. This is because $G_{22}$ and $G_{33}$ are much greater than $G_{11}$ and $G_{44}$ in (17). It means that the channel condition favors link 2 and link 3.

## IV. Conclusion

In this paper, a joint power and rate adaptive algorithm, called UPRA, has been proposed to deal with the variation of channel conditions in wireless ad hoc networks. The operation of the network is modeled as a non-cooperative game and a net-utility function based on effective transmission rate

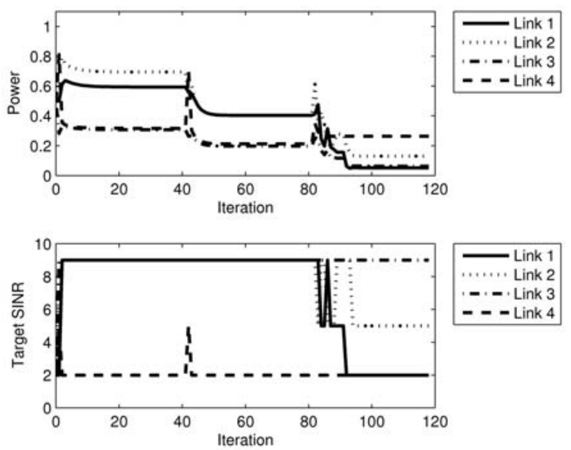



Fig. 9. Evolution of power and target *SINR* in an infeasible case, using DPRC/SPG.

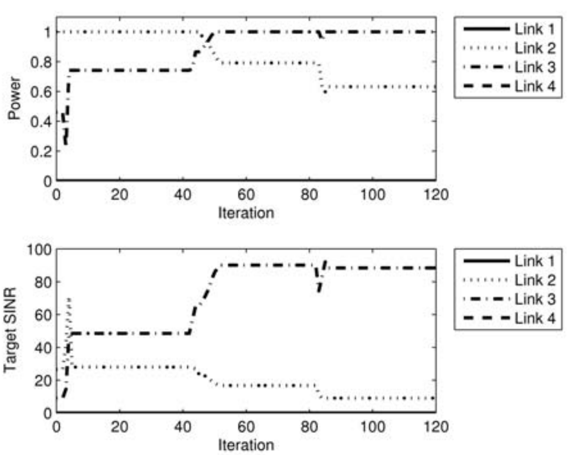



Fig. 10. Evolution of power and target *SINR* in an infeasible case, using UPRA with cost coefficient $c = 1$.

and power consumption has been designed. By softening the SINR requirement, a margin can be obtained so that data is more likely to be successfully delivered in unstable wireless channels, thus increase the overall network throughput. Moreover, the UPRA algorithm is proven to be convergent under both feasible conditions and infeasible conditions. Simulation results show that by using the UPRA algorithm, higher throughput can be obtained and more energy can be saved.

## APPENDIX

*Lemma 1*: $SINR_{iTAR}(c_i/\alpha_i)$ is a non-increasing function of $c_i/\alpha_i$ on $[\underline{K}_i, \bar{K}_i]$ for all $i$.

*Proof*:

Let $\lambda_m = \sqrt{2\ln(\frac{b_m \alpha_i}{\sqrt{2\pi}\sigma c_i h_m \gamma_i}) + \sigma^2}$.

For $\frac{c_i}{\alpha_i} \le \min(\frac{b_m}{\sqrt{2\pi}\sigma h_m \gamma_i}, \frac{0.5 b_m}{h_m \gamma_i})$,

$$SINR_{iTAR,m} = \min(h_m \gamma_i e^{\Psi_m}, \alpha_i P_{\max}) \ge \min(h_m \gamma_i, \alpha_i P_{\max}),$$

$$\frac{\partial SINR_{iTAR,m}}{\partial (c_i/\alpha_i)} = -\frac{\alpha_i \sigma}{c_i \lambda_m} SINR_{iTAR,m} < 0.$$

For $\frac{b_m}{\sqrt{2\pi}\sigma h_m \gamma_i} < \frac{c_i}{\alpha_i} < \frac{0.5 b_m}{h_m \gamma_i}$,

$$SINR_{iTAR,m} = h_m \gamma_i \le \min(h_m \gamma_i e^{\Psi_m}, \alpha_i P_{\max}).$$

For $\frac{c_i}{\alpha_i} \ge \frac{0.5 b_m}{h_m \gamma_i}$, $SINR_{iTAR,m} = 0 < h_m \gamma_i$.

Q.E.D.

*Lemma 2*: For $c_i/\alpha_i$ on $[\underline{K}_i, \bar{K}_i]$ and two rates $m > n$, $SINR_{iTAR,m} \ge SINR_{iTAR,n}$.

*Proof*:

For $\frac{c_i}{\alpha_i} \le \min\left(\frac{b_m}{\sqrt{2\pi}\sigma h_m \gamma_i}, \frac{b_m}{\sqrt{\pi}}, \frac{b_n}{\sqrt{2\pi}\sigma h_n \gamma_i}, \frac{b_n}{\sqrt{\pi}}\right)$, $SINR_{iTAR,m} = h_m \gamma_i e^{\Psi_m}$, $SINR_{iTAR,n} = h_n \gamma_i e^{\Psi_n}$.

$$\frac{\partial SINR_{iTAR,n}}{\partial b_n} = \frac{SINR_{iTAR,n}}{b_n} \frac{\sigma}{\lambda_n} > 0$$
$$\frac{\partial SINR_{iTAR,n}}{\partial h_n} = \frac{SINR_{iTAR,n}}{h_n} \left[1 - \frac{\sigma}{\lambda_n}\right] > 0$$

and if $m > n$, we have $b_m > b_n$ and $h_m > h_n$. The directional derivative of $SINR_{iTAR,n}$ along the vector $(b_m - b_n, h_m - h_n)$ is

$$\nabla(SINR_{iTAR}) \cdot (b_m - b_n, h_m - h_n) = \left(\frac{\partial SINR_{iTAR}}{\partial b}, \frac{\partial SINR_{iTAR}}{\partial h}\right) \cdot (b_m - b_n, h_m - h_n) > 0$$

For $\max\left(\frac{b_m}{\sqrt{2\pi}\sigma h_m \gamma_i}, \frac{b_n}{\sqrt{2\pi}\sigma h_n \gamma_i}\right) < \frac{c_i}{\alpha_i} < \min\left(\frac{b_m}{\sqrt{\pi}}, \frac{b_n}{\sqrt{\pi}}\right)$, $SINR_{iTAR,m} \ge SINR_{iTAR,n}$.

Q.E.D.

*Lemma 3*: $SINR_{iTAR}(c_i/\alpha_i)/\alpha_i$ increases with increasing $c_i/\alpha_i$ on $[\underline{K}_i, \bar{K}_i]$ for all $i$.

*Proof*:

For $c_i/\alpha_i \le \min\left(\frac{b_m}{\sqrt{2\pi}\sigma h_m \gamma_i}, \frac{b_m}{\sqrt{\pi}}\right)$,

$$SINR_{iTAR}(c_i/\alpha_i)/\alpha_i = \min\left(h_m \gamma_i e^{\Psi_m}/\alpha_i, P_{\max}\right),$$

$$\frac{\partial \left[SINR_{iTAR}(\frac{c_i}{\alpha_i})\frac{1}{\alpha_i}\right]}{\partial \left(\frac{c_i}{\alpha_i}\right)} = \frac{1}{c_i} SINR_{iTAR}\left(\frac{c_i}{\alpha_i}\right)\left[1 - \frac{\sigma}{\lambda}\right] > 0.$$

Q.E.D

*Lemma 4*: $SINR_{iOPT}$ is a non-increasing function of $c_i/\alpha_i$.

*Proof*:

Let $U'$ denote the slope of $U_i(c_i/\alpha_i)$ with regard to $c_i/\alpha_i$.

$$U' = \frac{\partial U_i(\frac{c_i}{\alpha_i})}{\partial(\frac{c_i}{\alpha_i})} = -\frac{\alpha_i b}{\sqrt{2\pi} c_i \lambda} e^{\frac{\Psi^2}{2\sigma^2}} < 0$$

Let $F = \frac{\alpha_i b}{\sqrt{2\pi} c_i \lambda^2} e^{\frac{\Psi^2}{2\sigma^2}} > 0$.

We get

$$\frac{\partial U'}{\partial b} = -\left[\sigma - \frac{1}{\lambda}\right]\frac{F}{b} \text{ and } \frac{\partial U'}{\partial h} = -\frac{F}{h}\left[\lambda - \sigma + \frac{1}{\lambda}\right]$$

The directional derivative of $U'$ along the vector $(b_m - b_n, h_m - h_n)$ is

$$\begin{aligned} &\nabla U' \cdot (b_m - b_n, h_m - h_n) \\ =&\left(\frac{\partial U'}{\partial b}, \frac{\partial U'}{\partial h}\right) \cdot (b_m - b_n, h_m - h_n) \\ \le& -F\frac{\Delta b}{b_n}\left[\sigma - \frac{1}{\lambda_n}\right] - F\frac{\Delta b}{b_n}\left[\lambda_n - \sigma + \frac{1}{\lambda_n}\right] \\ =& -F\frac{\Delta b}{b_n}\lambda_n < 0. \end{aligned}$$

Q.E.D

For $\frac{\Delta h}{h_n} \geq \frac{\Delta b}{b_n}$, the inequality holds. This condition is reasonable since the SINR requirement is faster than the increase of the data rate. Hence, for two rates $m$ and $n$ $(m > n)$, the utility $U_i$ of $m$ decreases faster than that of $n$. By *Lemma 3* and (3), the cost of $m$ increases faster than that of $n$. By (9), the net utility $NU_i$ of $m$ falls faster than that of $n$. According to UPRA, we always select the rate which has the largest net utility, the optimal data rate will decrease when $c_i/\alpha_i$ increases. By *Lemma 2*, $SINR_{iOPT}$ decreases with increasing $c_i/\alpha_i$.

*Proposition 1*: UPRA is standard on $[\underline{K}_i, \bar{K}_i]$ for all $i$.

*Proof*:

From (15), the interference function of UPRA is

$$I(\mathbf{P}) = [I_1(\mathbf{P}), I_2(\mathbf{P}), \cdots, I_N(\mathbf{P})]^T$$

where $\mathbf{P} = [P_1, P_2, \cdots, P_N]^T$ and $I_i(\mathbf{P}) = SINR_{iOPT}/\alpha_i$.

- Positivity: Since background noise $\eta_i > 0$, $I(\mathbf{P}) > 0$.
- Monotonicity: $I(\mathbf{P}) = \frac{SINR_{iOPT}}{SINR_i} P_i = \frac{SINR_{iOPT}}{\alpha_i} = SINR_{iOPT}(\frac{c_i}{\alpha_i})\frac{c_i}{\alpha_i}\frac{1}{c_i}$. Since $\alpha_i = G_{ii}/\left(\sum_{j=1, j\neq i}^{N} G_{ij}P_j + \eta_i\right)$, we get $\alpha_i(\mathbf{P}) \leq \alpha_i(\mathbf{P}')$ for $\mathbf{P} \geq \mathbf{P}'$. From *Lemma 3*, $I(\mathbf{P})$ is increasing with $c_i/\alpha_i$. Hence, for a fixed price coefficient $c_i$, $I(\mathbf{P}) \geq I(\mathbf{P}')$.
- Scalability: For all $a > 1$, we have

$$\begin{aligned} I(a\mathbf{P}) &= SINR_{iOPT}\left(c_i/\alpha_i(a\mathbf{P})\right)/\alpha_i(a\mathbf{P}) \\ &\leq SINR_{iOPT}\left(c_i \frac{\sum_{j=1, j\neq i}^{N} G_{ij}P_j + \eta_i}{G_{ii}}\right) \\ &\quad \cdot \frac{\sum_{j=1, j\neq i}^{N} aG_{ij}P_j + \eta_i}{G_{ii}} \\ &< aSINR_{iOPT}\left(c_i/\alpha_i(\mathbf{P})\right)/\alpha_i(\mathbf{P}) = aI(\mathbf{P}) \end{aligned}$$

*Lemma 4* tells us $SINR_{iOPT}$ is decreasing with $c_i/\alpha_i$, so the first inequality holds.

Q.E.D.

## References


[1] A. J. Jones, *Game Theory: Mathematical Models of Conflict*. Horwood Publishing, p. 47, 2000.

[2] C. U. Saraydar, N. B. Mandayam, and D. J. Goodman, "Efficient power control via pricing in wireless data networks," *IEEE Trans. Commun.*, vol. 50, no. 2, pp. 291-303, Feb. 2002.

[3] S. Dey and J. Evans, "Optimal power control in wireless data networks with outage-based utility guarantees," in *Proc. IEEE Conf. Decision and Control*, vol. 1, pp. 570-575, Dec. 2003.

[4] S. Gunturi and F. Paganini, "Game theoretic approach to power control in cellular CDMA," in *Proc. IEEE Veh. Technol. Conf.*, vol. 58, no. 4, pp. 2362-2366, 2003-Fall.

[5] M. Xiao, N. B. Shroff, and E. K. P. Chong, "A utility-based power-control scheme in wireless cellular systems," *IEEE/ACM Trans. Networking*, vol. 11, no. 2, pp. 210-221, Apr. 2003.

[6] Q. Chen and Z. Niu, "Joint power and rate control for wireless ad hoc networks: a game-theoretical approach with step-up pricing," in *Proc. IEEE Veh. Technol. Conf.*, vol. 2, pp.695-699, 2005-Fall.

[7] G. J. Foschini and Z. Miljanic, "A simple distributed autonomous power control algorithm and its convergence," *IEEE Trans. Veh. Technol.*, vol. 42, no. 4, pp. 641-646, Nov. 1993.

[8] A. Subramanian and Ali H. Sayed, "Joint rate and power control algorithms for wireless networks," *IEEE Trans. Signal Processing*, vol. 53, no. 11, pp. 4204-4214, Nov. 2005.

[9] S. Koskie and Z. Gajic, "A Nash game algorithm for SIR-based power control in 3G wireless CDMA networks," *IEEE/ACM Trans. Networking*, vol. 13, no. 5, pp. 1017-1026, Oct. 2005.

[10] S. Sorooshyari and Z. Gajic, "Optimal SIR and interference-based distributed power control for wireless networks," in *Proc. IEEE Conf. Decision Control*, vol. 5, pp. 5128-5133, Dec. 2004.

[11] K. Shoarinejad, J. L. Speyer, and G. J. Pottie, "A distributed scheme for integrated predictive dynamic channel and power allocation in cellular radio networks," in *Proc. GLOBECOM*, vol. 6, pp. 3623-3627, Nov. 2001.

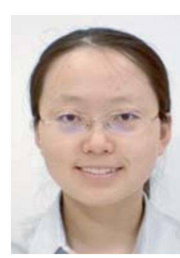
[12] S. Ulukus and R. D. Yates, "Stochastic power control for cellular radio systems," *IEEE Trans. Commun.*, vol. 46, no. 6, pp. 784-794, June 1998.

[13] K. Leung, "Power control by interference predictions for wireless packet networks," *IEEE Trans. Wireless Commun.*, vol. 1, no. 2, pp. 256-265, Apr. 2002.

[14] K. Leung, "A Kalman-filter method for power control in broadband wireless networks," in *Proc. IEEE INFOCOM*, vol. 2, pp. 948-956, Mar. 1999.

[15] R. D. Yates, "A framework for uplink power control in cellular radio systems," *IEEE J. Select. Areas Commun.*, vol. 13, no. 7, pp. 1341-1347, Sept. 1995.

[16] Z. Li, A. Das, A. K. Gupta, and S.Nandi, "Full auto rate protocol for wireless ad hoc networks," *IEE Proc. Commun.*, vol. 152, no. 3, pp. 311-319, June 2005.

**Jialing Zheng** received her B.S. degree in communications engineering and her M.S. degree in electrical engineering from Xidian University, Xi'an, China, in 2000 and 2003, respectively. She is currently working for her Ph.D. degree in the School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore. Since January 2007, she has been working with Satellite Engineering Center, Singapore, as a Senior R&D engineer. Her current research interests include wireless networks and real-time embedded systems.

**Maode Ma** received his Ph.D. degree in computer science from Hong Kong University of Science and Technology in 1999. Dr. Ma is an Associate Professor in the School of Electrical and Electronic Engineering at Nanyang Technological University in Singapore. He has extensive research interests in wireless networking and optical networking. He has published more than 100 international academic research papers. He currently serves as an Associate Editor for IEEE COMMUNICATIONS LETTERS and other six International Journals.