A GAME-THEORETIC FRAMEWORK FOR INTERFERENCE AVOIDANCE IN AD-HOC NETWORKS

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ABSTRACT

It is shown in this paper that direct extensions of distributed greedy Interference Avoidance (IA) techniques for networks with centralized receivers to networks with multiple uncoordinated receivers (as in ad-hoc networks) do not always lead to convergence and some channel conditions that lead to non-convergence are identified. A framework based on potential game theory is presented which could be used to construct convergent IA games in these de-centralized networks. Example waveform adaptation games for IA are formulated according to this framework. It is shown that these convergent games lead to the maximization of global network objectives.

KEYWORDS

Interference Avoidance, Game Theory, Ad-hoc Network

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***Abstract* - It is shown in this paper that direct extensions of distributed greedy Interference Avoidance (IA) techniques for networks with centralized receivers to networks with multiple uncoordinated receivers (as in ad-hoc networks) do not always lead to convergence and some channel conditions that lead to non-convergence are identified. A framework based on potential game theory is presented which could be used to construct convergent IA games in these de-centralized networks. Example waveform adaptation games for IA are formulated according to this framework. It is shown that these convergent games lead to the maximization of global network objectives.**

1. INTRODUCTION

Networks are becoming less structured and increasingly involve distributed decision making. Nodes are required to independently adapt in a way such that the interference in the network is minimized. This paper investigates such distributed waveform adaptation strategies which reduce interference and consequently facilitate multi-user communication in ad-hoc networks. Greedy IA algorithms by waveform adaptation (wherein users choose waveforms that increase their own SINR or utility) for networks with a centralized receiver have been extensively investigated in [1], [2], [3], [4] and [5]. These algorithms are shown to converge to a set of waveforms which maximize the sum capacity of the multiple access channel. However, in wireless systems where users talk to multiple uncoordinated receivers (as in an Ad-hoc network), direct applications of the same greedy IA by waveform adaptation techniques might not to lead to a stable fixed point ([6] and [7]). This is caused by the asymmetry of the mutual interference between users at different receivers, leading the users to adapt their sequences in conflicting ways.

In this paper we enumerate some channel conditions under which any greedy IA game in which users try to maximize their performance by waveform adaptation does not lead to convergence. We then design a framework based on potential game theory that could lead to distributed and convergent waveform adaptation games that maximize some network welfare function. In these games, the utility function of a user incorporates some measure of the influence caused by a particular user's actions on the other users in the network, to ensure convergence.

The paper is organized as follows. The system model for the network scenario under consideration is described in Section 2. Section 3 describes some channel conditions under which greedy IA games do not converge in ad-hoc networks. Section 4 gives a brief overview of game-theory and potential games. Section 5 presents the game-theoretic framework for IA games in a decentralized network. Section 6 presents example formulations of waveform adaptation games based on the framework. Section 7 summarizes the paper and presents directions for future research.

2. SYSTEM MODEL

The ad-hoc network analyzed here is made up of a cluster of transmit and receive node pairs. Each transmit node is assumed to have a separate receive node. Figure 1 shows the system model with transmit-receive node pairs indicated by arrows. The model is a generalization of a network with co-located or centralized receivers and the results presented here are applicable to the centralized network scenario as well. The transmit power levels of nodes are assumed to be fixed by a process independent of waveform adaptation. Interference is caused at a receive node by transmissions from user nodes different from the one associated with the particular receive node. The interference caused is influenced by the correlation between the waveforms of user nodes, transmit power levels and the channel characteristics. A signal space characterization is used to represent the waveforms of nodes [4]. This signal space representation specifies the waveform of a node in orthogonal signal dimensions (time or frequency) and is referred to as the signature sequence of the node.

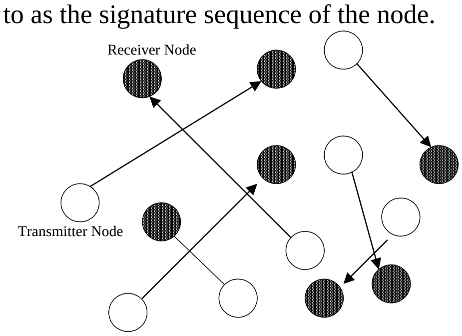

Figure 1: Network with multiple un-coordinated receivers

Let N be the number of signal dimensions available for transmission and K be the number of transmitting nodes in the network. Let $s_i \in \mathbb{C}^{N\times 1}$ be the signature sequence associated with transmitting node *i*. The signature sequences are allowed to have real values (as opposed to bi-polar sequences). Without loss of generality, the signature sequences are assumed to have unit norm $\left(\|s_i\| = 1\right)$. Let p_i be the transmit power level of the i^{th} transmit node and h_{ij} be the fading coefficient of the channel between the i^{th} transmit node and the j^{th} receive node. The channel is assumed to be constant over all signal dimensions and also constant over the time required for the adaptation process. The data bit to be transmitted from the i^{th} transmit node is denoted by b_i. The received signal at the j^{th} receive node is given by

$$r_j = \sum_{i=1}^{K} \sqrt{p_i} h_{ij} b_i s_i + z \qquad (1)$$

where, $r \in \mathbb{C}^{N\times 1}$. The vector, $z \in \mathbb{C}^{N\times 1}$, models additive white Gaussian noise with zero mean and unit variance.

3. NON-CONVERGENCE OF GREEDY BEST RESPONSE GAMES

An example scenario is used to show that greedy adaptation procedures do not always converge in decentralized networks (as is the case in ad-hoc networks). Greedy adaptation is used to refer to algorithms in which each user myopically tries to maximize its own Signal to Interference Ration (SIR) or some other measure of link capacity in response to adaptations by other users in the network (e.g. the iterative water-filling algorithm).

Consider the cluster of three transmit-receive node pairs shown in Figure 2. The transmit power of all user nodes is assumed to be equal. Also, the nodes are assumed to have only two signal dimensions available for transmission. Let the channel gains in the network be ordered as follows,

$$\begin{aligned} &h_{21} > h_{11} > h_{31} \\ &h_{13} > h_{33} > h_{23} \\ &h_{32} > h_{22} > h_{12} \end{aligned} \qquad (2)$$

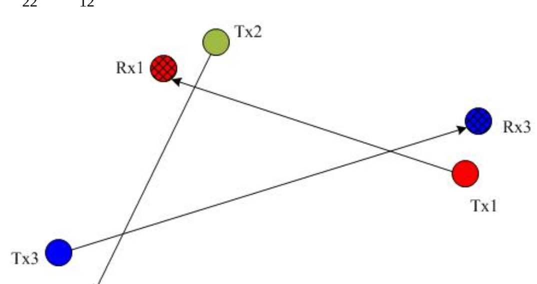

Figure 2: Network scenario to illustrate non-convergence

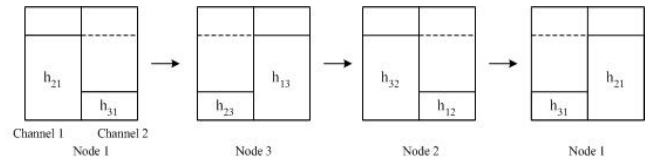

Figure 3: Distribution of power at the receive node corresponding to the adapting transmit node

Consider an adaptation process in which each user tries to maximize its SINR by water-filling over the interference and noise it sees, in a round-robin fashion. Assume that at the start of the adaptation process, transmit node *2* uses dimension *1* and transmit node *3* uses dimension *2*. Transmit node *1* chooses dimension *2* as the interference seen from node *3* is smaller than that from node *2* at receive node *1*. However, at receive node *3*, more interference is seen from transmit node *1* than transmit node *2* and hence transmit node *3* moves to dimension *1*. Receive node *2* sees more interference from transmit node 3 than transmit node *2*. Hence transmit node *2* shifts to dimension *2*. However, as the interference seen from node *3* is smaller than that from node *2*, transmit node *1* chooses dimension *1* now. This cycle thus continues with each node choosing the two dimensions alternately. This process is illustrated in Figure 3, which shows the distribution of interference power in the two signal dimensions at the receiver corresponding to the transmit node making decisions.

It can easily be shown that such allocation cycles or non-convergence of greedy resource allocations occur when the channel gains between multiple users are ordered cyclically, similar to the ordering for three users given in Equation (2).

This section thus illustrated that greedy IA algorithms in which users are concerned only with maximizing their own performance do not necessarily converge in networks with multiple uncoordinated receivers. Therefore we incorporate some measure of the influence of a user's action on the other users in the system, into the utility function of the user, to aid convergence of IA algorithms.

4. GAME THEORY AND POTENTIAL GAMES

Consider a normal form game represented ([8]) as $\Gamma = \left\langle M, \{A_i\}_{i\in M}, \{u_i\}_{i\in M} \right\rangle$, where Γ is a game and $M = \{1, 2, \ldots, K\}$, is the set of players of the game. The set of actions available for player i is denoted by A_i and the utility function associated with each player *i* by u_i. If the set of all available actions for all players is represented by $A = \times_{i\in M} A_i$, then $u_i : A \to \mathbb{R}$. The utility function for each player is thus a function of the actions in the game. Players select actions that maximize their utility functions. A Nash Equilibrium (NE) for a game is an action profile from which no player can increase his utility by unilateral deviations. An action profile $a \in A$ is a NE if and only if

$$u_i(a) \geq u_i(b_i, a_{-i}) \forall i \in M, b_i \in A_i \quad (3)$$

where, (b_i, a_{-i}) refers to the action profile in which the action of user *i* is changed from a_i to b_i, while the actions of all the other players in the game remain the same. Nash equilibria form the steady states of the game.

Suppose that a normal form game is played repeatedly. At each stage of the game, players choose actions that improve their utility functions in a round-robin fashion. The criteria for a particular choice of action gives rise to the best and better response dynamics defined below.

Best Response Dynamic: At each stage, a player *i* is permitted to deviate from $a_i \in A_i$ to some action $b_i \in A_i$ iff $u_i(b_i, a_{-i}) \geq u_i(c_i, a_{-i}) \forall c_i \in A_i$ and $u_i(b_i, a_{-i}) > u_i(a)$.

Better Response Dynamic: At each stage, a player *i* is permitted to deviate from $a_i \in A_i$ to some action $b_i \in A_i$ iff $u_i(b_i, a_{-i}) > u_i(a)$.

A potential game ([9] and [10]) is a normal form game such that any changes in the utility function of any player in the game due to a unilateral deviation by the player is also correspondingly reflected in a global function. A function $P : A \to \mathbb{R}$, is called an exact potential function if $u_i(a_i, a_{-i}) - u_i(\hat{a}_i, a_{-i}) = P(a_i, a_{-i}) - P(\hat{a}_i, a_{-i})$, $\forall i$ and $a_i, \hat{a}_i \in A_i$. A game that has an exact potential function is called an exact potential game. A function $P : A \to \mathbb{R}$, is called an ordinal potential function, if $u_i(a_i, a_{-i}) \geq u_i(\hat{a}_i, a_{-i}) \Leftrightarrow P(a_i, a_{-i}) \geq P(\hat{a}_i, a_{-i}), \forall i$ and $a_i, \hat{a}_i \in A_i$. A game that has an ordinal potential function is called an ordinal potential game.

Exact and ordinal potential games converge to a NE while following a best response dynamic. It has also been established that potential games with finite action spaces converge to a NE while following a better response dynamic, and it is believed that these results will hold (with minor modifications) in continuous action spaces

5. FRAMEWORK FOR CONVERGENT IA GAMES

A game theoretic model for interference avoidance games in ad-hoc networks based on potential game theory is suggested here. Potential games are chosen as these are easy to analyze and give a framework where users maximize a global network function by only trying to maximize their own utilities leading to simple game formulations.

Let the user node-pairs be the players of the game. The signature sequences (or waveforms) of users constitute the actions for the players in the game. Let the utility associated with a particular user be given by the following,

$$\begin{aligned} u_i(s_i, s_{-i}) = {} & f_1(s_i) - \sum_{j \neq i, j=1}^{K} f_2\left(I(s_j, s_i), p_j, p_i, h_{ji}, h_{ii}\right) \\ & - \sum_{j \neq i, j=1}^{K} \gamma_{ij} f_3\left(I(s_i, s_j), p_i, p_j, h_{ij}, h_{jj}\right) \end{aligned} \quad (4)$$

Function f_1 quantifies the benefit associated with a particular choice of signature sequence and power. Function f_2 is a measure of the interference due to the other users present in the system perceived at the receive node for user node *i*. Function *I* is some function of two signature sequences s_i and s_j (e.g. the correlation between the sequences). Function f_3 is a measure of interference caused by a particular user at the receivers associated with other users in the network and coefficient γ_{ij} is a weighting factor.

A simple formulation of a candidate potential function is given by,

$$P(s) = \sum_{i=1}^{N} \begin{pmatrix} f_1(s_i) - \alpha \sum_{j \neq i, j=1}^{K} f_2\left(I(s_j, s_i), p_j, p_i, h_{ji}, h_{ii}\right) \\ -\beta \sum_{j \neq i, j=1}^{K} \gamma_{ij} f_3\left(I(s_i, s_j), p_i, p_j, h_{ij}, h_{jj}\right) \end{pmatrix} \quad (5)$$

This function consists of the weighted sum of the utilities of all users. Coefficients α and β are weighting factors and matrix $s = [s_1, s_2, \ldots, s_N]$. Separating all the terms involving the i^{th} user,

$$\begin{aligned} P(s_i, s_{-i}) = {} & f_1(s_i, p_i) - \alpha \sum_{j \neq i, j=1}^{K} f_2\left(I(s_j, s_i), p_j, p_i\right) \\ & - \beta \sum_{j \neq i, j=1}^{K} \gamma_{ij} f_3\left(I(s_i, s_j), p_i, p_j\right) \\ & - \alpha \sum_{j \neq i, j=1}^{K} f_2\left(I(s_i, s_j), p_i, p_j\right) \\ & - \beta \sum_{j \neq i, j=1}^{K} \gamma_{ji} f_3\left(I(s_j, s_i), p_j, p_i\right) \\ & \underbrace{- \sum_{k \neq i, k=1}^{K} \begin{pmatrix} f_1(s_k, p_k) - \\ \alpha \sum_{j \neq k, j \neq i, j=1}^{K} f_2\left(I(s_j, s_k), p_j, p_k\right) \\ -\beta \sum_{j \neq k, j \neq i, j=1}^{K} \gamma_{kj} f_3\left(I(s_k, s_j), p_k, p_j\right) \end{pmatrix}}_{Non-contributing\ Terms} \end{aligned} \quad (6)$$

Exact Potential Game: The above game is an exact potential game if the following condition (Equation (7)) is satisfied.

$$u_i(\hat{s}_i, s_{-i}) - u_i(s_i, s_{-i}) = P(\hat{s}_i, s_{-i}) - P(s_i, s_{-i}) \qquad (7)$$

Here, $\hat{s}_i$ is a signature sequence for user *i* different from s_i. Examining Equation (6), it is seen that the condition (Equation (7)) is satisfied and Equation (5) forms an exact potential function under the two scenarios listed below,

Scenario 1:

$$\begin{gathered} f_2\left(I\left(s_j, s_i\right), p_j, p_i, h_{ji}, h_{ii}\right) = f_2\left(I\left(s_i, s_j\right), p_i, p_j, h_{ij}, h_{jj}\right) \\ f_3\left(I\left(s_j, s_i\right), p_j, p_i, h_{ji}, h_{ii}\right) = f_3\left(I\left(s_i, s_j\right), p_i, p_j, h_{ij}, h_{jj}\right) \\ \alpha = \beta = \frac{1}{2},\ \gamma_{ij} = \gamma_{ji}\ \forall i, j \end{gathered} \qquad (8)$$

or

Scenario 2:

$$\begin{gathered} f_2(\bullet) = f_3(\bullet) \\ \alpha = \beta = \frac{1}{2}, \gamma_{ij} = 1\ \forall i, j \end{gathered} \qquad (9)$$

When Scenario 2 holds, the potential function reduces to the following,

$$P(s) = \sum_{i=1}^{K} \begin{pmatrix} f_1(s_i, p_i) - \\ \sum_{j \neq i, j=1}^{K} f_{pot}\left(I\left(s_i, s_j\right), p_i, p_j, h_{ji}, h_{ii}\right) \end{pmatrix} \qquad (10)$$

where,

$$f_{pot}(\bullet) = f_3(\bullet) = f_2(\bullet)$$

Ordinal Potential Game: Formulation of the game as an ordinal potential game requires the following condition (Equation (11)) to be satisfied.

$$u_i(\hat{s}_i, s_{-i}) \geq u_i(s_i, s_{-i}) \Leftrightarrow P(\hat{s}_i, s_{-i}) \geq P(s_i, s_{-i}) \qquad (11)$$

This is possible when $f_{2i}(\bullet) = f_{3i}(\bullet) = f_{ui}(\bullet)$, where $f_{ui}(\bullet)$ is an ordinal transformation of $f_{pot}(\bullet)$ and the utility function of each user is given by,

$$\begin{aligned} u_i(s_i, s_{-i}) = f_1(s_i, p_i) &- \sum_{j \neq i, j=1}^{K} f_{2i}\left(I\left(s_j, s_i\right), p_j, p_i\right) \\ &- \sum_{j \neq i, j=1}^{K} f_{3i}\left(I\left(s_i, s_j\right), p_i, p_j\right) \end{aligned} \qquad (12)$$

The ordinal potential function for the game is given by,

$$P(s) = \sum_{i=1}^{K} f_1(s_i, p_i) - \sum_{j \neq i, j=1}^{K} f_{pot}\left(I\left(s_i, s_j\right), p_i, p_j\right) \qquad (13)$$

Under this game formulation it is possible to construct convergent adaptation games with each user trying to maximize a different utility function, as long as the utility functions are ordinal transformations of each other.

6. EXAMPLE CONVEREGENT IA GAMES

1) Sum Inverse Signal to Interference Ratio Game

A candidate interference function for $f_2(\bullet)$ in the framework could be the Inverse Signal to Interference Ratio (ISIR) at a particular receiver defined as follows,

$$f_2\left(I\left(s_j, s_i\right), p_j, p_i, h_{ji}, h_{ij}\right) = -\frac{s_i^H s_j s_j^H s_i p_j h_{ji}^2}{p_i h_{ii}^2} \qquad (14)$$

Here, function *I* is the correlation between the two sequences. An exact potential game could be formed with $f_2(\bullet) = f_3(\bullet)$. The utility of a user *i* as a function of its signature sequence is then given by,

$$\begin{aligned} u_i(s_i, s_{-i}) = &-\frac{s_i^H \left(\sum_{j \neq i, j=1}^{K} \frac{s_j s_j^H p_j h_{ji}^2}{h_{ii}^2} \right) s_i}{p_i} \\ &- p_i s_i^H \left(\sum_{j \neq i, j=1}^{K} \frac{s_j s_j^H h_{ij}^2}{p_j h_{jj}^2} \right) s_i \end{aligned} \qquad (15)$$

Each user tries to minimize its ISIR and in addition, also tries to reduce the interference it causes at receivers corresponding to other users. Thus each user's utility function incorporates a measure of the influence of its actions on the other users in the system as opposed to utility functions for users in greedy IA games.

From the framework, the potential function for this game is the negative sum of the ISIRs of users at their respective receivers (Equation (16)).

$$P(s) = -\sum_{i=1}^{N} s_i^H \left(\sum_{\substack{j=1 \\ j \neq i}}^{N} \frac{s_j s_j^H p_j h_{ji}^2}{p_i h_{ii}^2} \right) s_i \qquad (16)$$

Hence the game results in the minimization of the interference in the network.

The game is played as follows. Each user iteratively chooses a waveform that maximizes its utility function. Let,

$$X = \frac{\left(\sum_{j \neq i, j=1}^{K} \frac{s_j s_j^H p_j h_{ji}^2}{h_{ii}^2} \right)}{p_i} + p_i \left(\sum_{j \neq i, j=1}^{K} \frac{s_j s_j^H h_{ij}^2}{p_j h_{jj}^2} \right) \qquad (17)$$

Then the utility function for each user can be re-written as,

$$\begin{aligned} u_i(s_i, s_{-i}) &= -s_i^H (X) s_i \\ &= \frac{-s_i^H (X) s_i}{s_i^H s_i} \end{aligned} \qquad (18)$$

where, the second line follows from the fact that $\left(\|s_i\| = s_i^H s_i = 1\right)$. The utility function is maximized by the Eigenvector corresponding to the minimum Eigenvalue of *X*, since *X* is positive semi-definite matrix. This constitutes the best response of the user to the current state of the network.

Convergence: Since exact potential games exhibit best response convergence to a NE of the game, the ISIR game converges. The potential function is continuously differentiable. Also, the utility function is a concave function of the signature sequence of the user and hence has a unique maximum. These properties can be used to show that the game converges to a maxima of the potential function. The game hence finds waveforms (or signature sequences), that minimize the sum interference in the network for a given power vector.

Simulation Results: The negative of the potential function (the sum of ISIRs for the users in the network) is shown in Figure 4 for waveform adaptation in a network with six users and three signal space dimensions. The power levels of the users are fixed at 1 watt each. Multiple simulation runs illustrate the convergence of the algorithms from different random initial choice of waveforms. It is seen that the algorithm iteratively decreases the interference in the network.

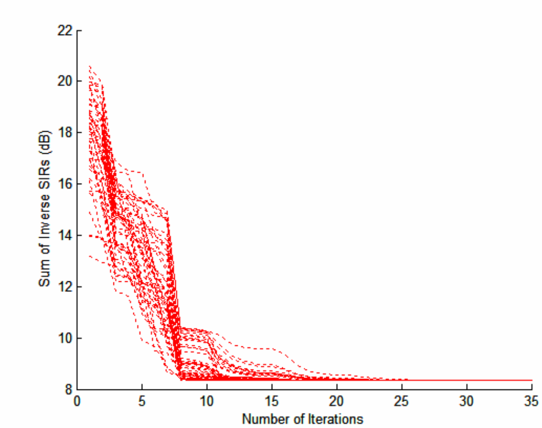

Figure 4: Best-response convergence for a network with multiple un-coordinated receivers

2) Weighted Interference Game

Another choice for function $f_2(\bullet)$ is the interference seen at a receiver corresponding to a particular user weighted by the received power of the user. This function is given by,

$$f_2\left(I\left(s_j, s_i\right), p_j, p_i, h_{ji}, h_{ij}\right) = -s_i^H s_j s_j^H s_i \left(p_j h_{ji}^2\right)\left(p_i h_{ii}^2\right) \quad (19)$$

Function I is again the correlation between the two sequences. The utility function for user *i*, with $f_2(\bullet) = f_3(\bullet)$ is given by,

$$u_i(s_i, s_{-i}) = -s_i^H \left(\sum_{j\neq i, j=1}^{K} s_j s_j^H p_j h_{ji}^2 p_i h_{ii}^2 \right) s_i - s_i^H \left(\sum_{j\neq i, j=1}^{K} s_j s_j^H p_i h_{ij}^2 p_j h_{jj}^2 \right) s_i \quad (20)$$

The corresponding exact potential function is given by,

$$P(s) = -\sum_{i=1}^{N} s_i^H \left(\sum_{\substack{j=1 \\ j\neq i}}^{N} s_j s_j^H p_j h_{ji}^2 p_i h_{ii}^2 \right) s_i \quad (21)$$

As in the ISIR game, it can be shown that the NE for the weighted interference game are the minimizers of the sum of weighted interferences (Equation (21)).

In [6], a waveform adaptation game is described for a network where each transmit node has multiple collaborating receivers (multi-base networks). Each user iteratively finds sequences that maximizes the sum of interferences at all its receivers weighted by the received power of the user at these receivers. This game is similar to the weighted interference game. Hence the framework can be directly used to construct waveform adaptation games for multiple base networks as well.

7. SUMMARY

Some conditions under which myopic greedy interference avoidance algorithms do not lead to convergence in ad-hoc networks are identified in this paper. A framework based on Potential game theory is developed which could be used to construct convergent IA games in these networks. Example waveform adaptation games are formulated that minimize some measure of the interference in the network.

Another approach to develop distributed adaptation strategies for these networks is to develop games in which users try to achieve a feasible target performance (and not maximize their performance). In [11] a greedy waveform and power adaptation scheme for the centralized receiver scenario is presented where users can achieve a target feasible SINR. However the identification of a feasible target performance in ad-hoc networks (de-centralized networks) remains an open problem.

The practical implementations of the adaptation strategies described in this paper could involve considerable feedback and could substantially increase the overhead in the network. Hence more efficient feedback mechanisms need to be investigated. Alternately, properties of games, such as the better response convergence properties of potential games could be used to design reduced feedback schemes ([12]).

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