# A Game-Theoretic Analysis on the Conditions of Cooperation in a Wireless Ad Hoc Network

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## Abstract

*To enable proper functioning of wireless ad hoc networks, all nodes in the network are expected to cooperate in forwarding each other's packet. But relaying other nodes' packets involves spending energy without getting any immediate revenue. Hence, due to the constraints on available energy the nodes usually have in such networks, from an individual node's point of view, the best response is always not to cooperate. We model this problem as a repeated game and show analytically that given a suitable punishment mechanism, individual nodes can be deterred from their selfish behavior, and cooperation can emerge as the best response. We also show that generosity by other nodes cannot be part of a credible deterrent mechanism..*

## 1. Introduction

An ad-hoc network consists of nodes that are wirelessly interconnected without the aid of wired infrastructure or centralized control. In many such networks, the nodes have limited amount of energy. Also such networks do not have specialized nodes that are responsible for routing traffic in the network. All nodes are required to act as routers and forward traffic generated by other nodes in the network to support the basic functioning of the network. Given the constraints on the total energy a node has and the fact that forwarding other nodes' packets consumes energy without immediately benefiting the relay node, a selfish node might not find it to be in best of its interest to relay packets. Simulation based studies [9,10] show that even a small percentage (10-40%) of non-cooperating nodes can bring the network throughput considerably down (16-32 % degradation). It is interesting therefore to find conditions under which cooperation would be the preferred strategy even for selfish nodes. In this paper, we use a game-theoretic analysis to find conditions that would make cooperation a preferred choice for the nodes in ad hoc networks.

Several papers have addressed the issue of cooperation in wireless ad hoc networks. In [3, 4], the authors have proposed the use of a virtual currency called *nuglets* to encourage cooperation among nodes and prevent traffic overload in the network. The basic idea is that a node has to spend *nuglets* to get the relay service from other nodes and any node can earn *nuglets* by forwarding other nodes' packets. They propose heuristic-based algorithms that use virtual currency as a means to stimulate cooperation among nodes in the network. In [6], the authors evaluate the effectiveness of a reputation-based mechanism (proposed in [7]) in enforcing cooperation in a network.

[5] considers a network of $N$ nodes that can be classified into different classes based on the average power constraint they have. They then study the tradeoff between the node lifetime and the fraction of packets successfully transmitted (called normalized acceptance rate (NAR)) by a node. They provide a mathematical framework for finding the Pareto-optimal NARs for each user under the power constraints. They also propose an acceptance algorithm called Generous Tit-For-Tat (GTFT) that accepts or rejects a relay request depending on its past experience. They also show that GTFT converges to the theoretically derived optimal NARs. The GTFT algorithm is simple and scalable. But it is not fair in the sense that all the nodes participating in relay of packets from a source node are punished even if only a single node on the route is denying relay requests

repeatedly and hence bringing down the perceived throughput of the source node.

Researchers have also used game theory for analysis of cooperation of nodes in wireless ad hoc networks [1, 5, 6]. All these papers define the game as an algorithm that dictates the behavior of a node at a given point in time as a response to the behavior of other nodes. Some of them (e.g. [1, 5]) have shown analytically that if all the nodes behave according to that algorithm, it would lead to cooperation among the nodes. To the best of our knowledge, [1] is the only paper that considers network topology in their analysis. Based on their analysis, the authors in [1] conclude that incentives are required to stimulate cooperation. In [8], the authors propose a general framework for analysis of strategies encouraging cooperation in ad hoc networks.

In this paper, we take a different approach. We show analytically in a game-theoretic framework [11] that it is possible to devise suitable punishment mechanisms for errant nodes that are strong enough to deter deviation. While the exact implementations of payoff functions and punishment strategies might differ according to the context of the particular network, the general principle of punishment (with subsequent offers for 'rehabilitation') as a deterrence mechanism for deviant behavior is applicable in a variety of networks.

## 2. Assumptions and network topology

We make the following assumptions in our analysis.

a. There are $n$ nodes in the network.
b. The nodes are self-interested which means that they choose a behavior that maximizes their individual payoff.
c. The time is divided into sessions and in any session each node in the network has one packet to transmit to a particular destination.
d. Each session is long enough for the packets from each node in the network to reach their destination.
e. The network is a multi-hop network which means that packets exchanged between any two nodes (that are not within each other's communication range) are forwarded by other nodes.
f. The routes followed by the packets are such that the load generated in the network in each session is equally distributed among all nodes.
g. The routes taken by the packets do not change in a session.
h. The average number of hops taken by each packet to reach its destination is $h$.
i. A node spends energy in receiving and transmitting packets. The energy spent in processing the packet is negligible (and hence can be ignored in the analysis) as compared to the energy used in transmitting and receiving a packet.
j. Each packet is of same length. The energy spent in transmitting a packet is same for all nodes. Also the energy spent in receiving a packet is same for all nodes.

## 3. Modeling cooperation as a game

We model the interaction among the nodes in the network as a game. Each of the nodes $i, i \in \{1, 2, ..., n\}$ want to transmit one packet each in each session (which we refer to as a stage), and each of them operate in the pure strategy space $S_i = \{C_i, NC_i\}$ within each stage, where C stands for cooperation, i.e. the node decides to transmit every packet it receives, and NC stands for non-cooperation when the node does not transmit any of the packets it receives. In other words, at every stage of the game, each of the nodes can decide independently to either cooperate or not cooperate. We assume that none of the nodes fail during the game.

Let the benefit of sending a packet be $b$, the cost of receiving a packet $c_r$, and the cost of transmitting a packet $c_t$. We also assume that the benefit of receiving a packet for the destination node is zero. To make the problem tractable, we have assumed that each packet goes through $h$ hops on an average before it reaches its destination node. The packet transmission is considered successful – and the benefit b is accrued – only if the packet is sent successfully over these h hops. Let the total number of nodes in the system be n, and the fraction of the nodes that are cooperating be $\alpha$. Hence, the expected number of nodes that are cooperating is $n\alpha$. According our assumptions, each packet travels $h$ hops to reach its destination. Hence, there are on an average $nh$ transmissions and $nh$ receptions in the network to transmit one packet from each node. According to assumption (f), the packet forwarding load is uniformly distributed among the nodes. Hence, in the single stage game, every cooperating node in the system receives $(h/\alpha)$ packets. We assume here that a non-cooperating node in the network can go to sleep mode when it does not want to transmit or receive a packet and hence does not spend any energy in receiving unwanted packets. Each cooperating node transmits $(h/\alpha)$ packets, while each non-cooperating node transmits just its own packet.

### 3.1. Nash equilibrium for the one-shot game

The payoff function for any node (except when $\alpha = 0$ or $\alpha = 1$) can be summarized in the following equations, where the superscript s stands for success (i.e. the node is successful in sending its packet), the superscript f stands for failure (the node is unsuccessful in sending its packet), subscript c stands for cooperation, and subscript n stands for non-cooperation:

$$\pi_c^s = b - c_r\left(\frac{h}{\alpha}\right) - c_t\left(\frac{h}{\alpha}\right) \quad (1)$$

$$\pi_c^f = -c_r\left(\frac{h}{\alpha}\right) - c_t\left(\frac{h}{\alpha}\right) \quad (2)$$

$$\pi_n^s = b - c_t \quad (3)$$

$$\pi_n^f = -c_t \quad (4)$$

When $\alpha = 0$ (i.e. no node is cooperating), the above equations do not hold, and payoff is zero for all the nodes ($\pi_c^f = \pi_c^s = \pi_n^s = \pi_n^f = 0$), and when $\alpha = 1$, the payoff for all nodes is

$$\pi = b - h(c_r + c_t) \quad (5)$$

The payoffs increase monotonically with $\alpha$. It can be readily seen (comparing (1) with (3), and (2) with (4)) that in a one-shot game (i.e. when the nodes play this game only once), regardless of the value of $\alpha$ (except when $\alpha = 1$, which means every node is cooperating), the best response for a node, when any other node is cooperating, is not to cooperate, since that way, it does not incur any receiving cost, and nor does it incur any transmission costs for any of the other packets. This is true regardless the node is successful or not in sending its packet. Similarly, when a node is not cooperating, the best response in a one-shot game of any other node is to not cooperate.

Thus, the best response of any node, regardless of the strategy employed by any other node is not to cooperate. If we imagine an n-dimensional matrix where we write down the payoffs for each of the n nodes in strategic (or normal) form, we can easily verify that for this one-shot game, there is a single Nash equilibrium [11] when all the nodes do not cooperate. This selfish behavior can be considered as an “n-dimensional Prisoner’s Dilemma” after the oft-mentioned problem, since in spite of getting higher payoffs when all nodes decide to cooperate, the nodes end up choosing this myopic strategy of non-cooperation.

### 3.2. Repeated Game Equilibrium

However, if this game is repeated infinitely (which is a practical assumption if millions of packets pass through the system), by devising a suitable “punishment mechanism”, the nodes can be deterred away from this selfish strategy, and towards a strategy of cooperation where all the nodes cooperate for a higher payoff. This punishment mechanism is stated as follows:

As long as node $i$ chooses strategy $C_i$ in each stage of the game, all other nodes choose strategy $C_{-i}$ (in other words, all the nodes continue to cooperate as long as the other nodes do the same). Here the subscript $-i$ denotes any node other than node $i$. If node $i$ ever chooses strategy $NC_i$, and continues choosing it for the next $(p-1)$ stages, the other nodes continue to choose strategy $C_{-i}$. This is essentially a benefit-of-doubt for the errant behavior. However, if node $i$ continues to choose strategy $NC_i$ beyond $p$ stages, the other nodes react in a fashion such that they continue to cooperate with all nodes except node $i$ (the ‘punishment’ phase) for the next $q$ stages. Subsequent to the punishment phase, the errant node is allowed back into the network (the ‘parole’ phase) when it is expected to cooperate for $r$ stages, without getting to send its own packets in return. After cooperating continuously for $r$ stages, node $i$ is allowed to reenter the network (the ‘rehabilitation’ phase) with all its prior privileges, and the stages of the game are repeated with the prior history forgotten.

Thus, the punishment mechanism consists of three phases – at first, the errant node is allowed a benefit of doubt for $p$ stages, when the other nodes continue to cooperate assuming that the non-cooperation behavior will go away. The nodes then enter the punishment phase, where all other nodes decide not to cooperate with the errant node for $q$ stages. Finally, the errant node is allowed to enter the network on the condition that it cooperates for the next r stages without getting to send its packets in return, and on successful completion of the parole stage, the errant node re-enters the network with full privileges.

Proposition: Any value of $p > 1$ will result in non-cooperative behavior.

Proof: This proposition essentially states that a node will be immediately punished on not cooperating, and other nodes will not allow for any benefit of doubt. For if it were not so, the best response for any errant node would have been to not cooperate over $(p-1)$ stages, and then cooperate in the $p^{th}$ stage, and this response would ensure that in every p stages of the game, the node gets higher payoff by not cooperating as compared to cooperating in the first $(p-1)$ stages, and then gets a payoff that is equal to that ensured by cooperating in the $p^{th}$ stage. Thus, this non-cooperating strategy would dominate the cooperating strategy. □

This result is significant; insofar it proves that in any wireless ad hoc network the strategy of cooperation by all nodes is not a Nash equilibrium for any mechanism that allows for 'generosity' on the part of other nodes. This is an interesting departure from the Generous Tit-For-Tat (GTFT) mechanism that is proposed in [5].

As in [1], we assume a known discount factor $\delta$ for the payoffs of the nodes for each stage of the game in future, and it is the same for all the nodes, since the nodes are identical. In financial literature, the discount factor is the time value of money, but in our context, the discount factor can be more realistically interpreted as the premium demanded by the nodes for the risk of non-cooperation of the other nodes in future.

If the potentially errant node deems it better to go through the punishment phase, any non-cooperation on its part is punished for $q$ stages, and then the node is then allowed back on trial into the system for $r$ stages, as per the punishment mechanism outlined above. During the punishment phase, the node, realizing that it does not make sense to send or receive any packet, receives a zero payoff. Finally, in the parole phase, since it knows that none of its own packets will be transmitted, it only transmits the packets of the other nodes it receives. We denote the present value of the payoffs in these $(q+r)$ stages as $V'_{NC}$:

$$V'_{NC} = (b - c_t) - \delta^{q+1}\left[\sum_{m=0}^{r-1} \delta^m (h.c_r + (h-1)c_t)\right] \tag{6}$$

or

$$V'_{NC} = (b - c_t) + \delta^{q+1}\left(\frac{1-\delta^r}{1-\delta}\right)(h.c_r + (h-1).c_t) \tag{7}$$

Note that $\alpha = 1$ in (6) and (7) since we are evaluating the best response of a single errant node when all other nodes are cooperating, and when n is large enough, this is a reasonable approximation.

After these $(q+r)$ stages, the node is allowed to rejoin the network, at which point it again decides to repeat its past behavior (since it has decided not to cooperate, any further cooperation at this point will only reduce its payoff). As the stages of the game are repeated infinitely, the sub-game at this point for the errant node is the original game itself. Therefore, the best response at this stage is the same as it was at the beginning of the game. Let $V_{NC}$ denote the present value of the payoffs of the non-cooperating node. Then,

$$V_{NC} = V'_{NC} + \delta^{q+r+1} V_{NC} \tag{8}$$

or

$$V_{NC} = \frac{V'_{NC}}{1-\delta^{q+r+1}} \tag{9}$$

Any node will decide to cooperate only if the present value of the payoffs for cooperating is equal to or greater than the present value of the payoffs for not cooperating ($V_C \geq V_{NC}$), which leads to the following inequality:

$$\frac{1}{1-\delta}[b-h.c_r-h.c_t] \geq \frac{1}{1-\delta^{q+r+1}}\begin{bmatrix}(b-c_t)\\ +\delta^{q+1}\left(\frac{1-\delta^r}{1-\delta}\right)(h.c_r+(h-1).c_t)\end{bmatrix} \quad (10)$$

Simplifying, we get

$$\delta^{q+r+1}\left[2hc_r+(2h-1)c_t-b\right] - \delta^{q+1}\left[h.c_r+(h-1)c_t\right]+\delta(b-c_t) \geq h.c_r+(h-1)c_t \quad (11)$$

The above inequality defines the bound on $q$ and $r$ and therefore defines the punishment mechanism. With this inequality in place, none of the nodes have any incentive to deviate from the strategy of cooperation, and therefore by definition this is a Nash equilibrium. The values of $b, c_t, c_r, h$ and $\delta$ are known constants. Since we have one inequality, and two unknowns, we will need to place some constraints on the relative magnitudes of $q$ and $r$, to find out the minimum number of time periods for which we need to punish the errant node to effectively deter it from deviating from the cooperative equilibrium. For example, one simplifying assumption might be $q = r$, i.e. the punishment and parole phase continue for the same number of stages. Another special case would be when we do away with the parole phase altogether ($r = 0$).

## 4. Conclusions

This paper shows analytically how cooperation can be achieved by using a punishment mechanism in a wireless ad hoc network. An interesting departure from some existing algorithms is that generosity can not be part of a credible punishment behavior. While the specifics of the punishment mechanism might vary depending on the network characteristics, the general framework of punishment and parole can be applied in a variety of networks.

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