A Game-Theoretic Framework for Congestion Control in General Topology Networks [1]

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Abstract

We study control of congestion in general topology communication networks within a fairly general mathematical framework that utilizes noncooperative game theory. We consider a broad class of cost functions, composed of pricing and utility functions, which capture various pricing schemes along with varying behavior and preferences for individual users. We prove the existence and uniqueness of a Nash equilibrium under mild convexity assumptions on the cost function, and show that the Nash equilibrium is the optimal solution of a particular "system problem". Furthermore, we prove the global stability of a simple gradient algorithm and its convergence to the equilibrium point. Thus, we obtain a distributed, market-based, end-to-end framework that addresses congestion control, pricing and resource allocation problems for a large class of of communication networks. As a byproduct, we obtain a congestion control scheme for combinatorially stable ad hoc networks by specializing the cost function to a specific form. Finally, we present simulation studies that explore the effect of the cost function parameters on the equilibrium point and the robustness of the gradient algorithm to variations in time delay and to link failures.

1 Introduction

The congestion control mechanism of the Internet is based on the transfer control protocol (TCP). TCP provides an end-to-end congestion control where each user adjusts its flow rate according to the feedback it receives from the network in the form of lost packets. With the evolution of the Internet in recent years, we are seeing an effort towards improving and modifying the existing flow and congestion control structure. End-to-end congestion control schemes, among others, are widely accepted due to their distributed nature, scalability, and ease in implementation [1].

A variety of distributed congestion control schemes have been suggested in recent studies, attempting to achieve goals like maximizing the user satisfaction quantified as the utility, maintaining fairness among the users based on an adopted criterion, and maximizing network usage. One popular approach advocates using pricing schemes to create the necessary incentive for the users to exercise congestion control in accordance with the goals of the network designer [2]. Subsequent studies further elaborated on this approach following its basic principles [3, 4, 5].

Game theory provides a natural framework for developing pricing, and congestion control mechanisms. Users on the Internet are of noncooperative nature in terms of their demand for network resources, and they have no specific information on other users' flow rates, which makes cooperation among users impossible. Hence, noncooperative game theory provides a suitable framework for flow and congestion control problems [6, 7]. Ref. [6] shows that if an appropriate cost function and pricing mechanism are used, one can find an efficient Nash equilibrium for a multi-user network which is further stable under different update algorithms. Ref. [7] shows existence and uniqueness of a Nash equilibrium under different classes of cost functions for a simple two-node multiple links system. Ref. [8] formulates a combined routing and flow control problem as a Nash game with a large number of players, and obtain nearly-optimal policies with nonconcave objective functions. Ref. [9] incorporates pricing into a flow control game as an active decision variable controlled by the network (service provider), and study the problem as a hieararchical game in a many-users regime. Game theoretic concepts have also been used in [2, 3, 4, 5].

Although network games are widely used in the current literature as a tool for designing congestion control and pricing schemes, the relationship between different types of network games, and various assumptions on the user behavior have not been specifically addressed. Furthermore, uniqueness of the equilibrium, stability and convergence have been investigated in most cases only for specific cost functions. In this paper we focus on congestion control and provide a fairly general framework based on noncooperative game theory. Specifically, we will show the existence and uniqueness of a Nash equilibrium (NE) under reasonable convexity assumptions on the cost function for a general network structure. Moreover, a simple but efficient gradient descent algorithm will be shown to converge to the NE under the same set of assumptions. Finally, a congestion control scheme for an ad hoc network, and two protocols for

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Internet-style networks will implemented, all within the established framework. The organization of the paper is as follows. The underlying network model is given in the next section. In Section 3, the existence and uniqueness of NE is proven for a general cost structure. Section 4 includes global stability and convergence results for a simple gradient descent algorithm. In Section 5, system problem and optimality of NE is investigated, and in Section 6, a specific congestion control scheme is presented for ad hoc networks. The paper ends with the concluding remarks of Section 7.

2 The Model

We consider a general network model based on fluid approximations. Fluid models are widely used in addressing a variety of network control problems such as congestion control [5, 10, 11], routing [10, 7], and pricing [12, 9]. The topology of the network is characterized by a set of nodes $\mathcal{N} = \{1, \ldots, N\}$ and a set of links $\mathcal{L} = \{1, \ldots, L\}$ connecting the nodes. We make the natural assumption of *connectivity*, and let $\mathcal{M} = \{1, \ldots, M\}$ denote the set of active users. For simplicity, each user is associated with a (unique) connection. Hence, the i^{th} $(i \in \mathcal{M})$ user corresponds to a unique connection between the source and destination nodes $s_i, d_i \in \mathcal{N}$. Each link $l \in \mathcal{L}$ has a fixed positive capacity $C_l > 0$, and is associated with a buffer $b_l \geq 0$. The route (path) i^{th} user's connection traverses is determined by a routing algorithm, and corresponds to a subset of links $l \in \mathcal{L}$ connecting the two nodes s_i, d_i. The nonnegative flow, x_i, sent by the i^{th} user over this path R_i satisfies the bounds $0 \leq x_i \leq x_{i,max}$. The upper bound, $x_{i,max}$, on the i^{th} user's flow rate may be a user-specific physical limitation, and cannot exceed the minimum capacity of the links on the route, $\min_l C_l \ , l \in R_i$. It is possible to define a routing matrix, $\mathbf{A}$, as in [2] that describes the relation between the set of routes $\mathcal{R} = \{1, \ldots, M\}$ associated with the users (connections) and links $l \in \mathcal{L}$,

$$A_{l,i} = \begin{cases} 1, & \text{if source i uses link } l \\ 0, & \text{if source i does not use link } l \end{cases} \ ,$$

where $i \in \mathcal{M}$ and $l \in \mathcal{L}$. Using the routing matrix $\mathbf{A}$, the capacity constraints of the links are given by

$$\mathbf{A}\mathbf{x} \leq \mathbf{C} \ , \qquad (2.1)$$

where $\mathbf{x}$ is the $(M \times 1)$ flow rate vector of users and $\mathbf{C}$ is the $(L \times 1)$ link capacity vector. We define the flow rate vector, $\mathbf{x}$, to be feasible if it is nonnegative and satisfies (2.1). Let $\mathbf{x_{-i}}$ be the flow rate vector of all users except the i^{th} one. For a given fixed, feasible $\mathbf{x_{-i}}$, we derive a strict upper-bound $m_i(\mathbf{x_{-i}})$ on flow rate of the i^{th} user, x_i, based on (2.1): $m_i(\mathbf{x_{-i}}) = \min_{l \in R_i} \left(C_l - \sum_{j \neq i} A_{l,j} \, x_j\right) \geq 0$.

We associate an M-player noncooperative game with this problem. Here, the users (players) are noncooperative in the sense that they have no means of communicating with each other about their preferences, and each user wishes to optimize its usage of the network resources independently. A specific cost function, J_i, is assigned to user i, which not only models the user's preferences but also includes a feedback term carrying information about the current network state. The i^{th} user minimizes this cost function by adjusting its flow rate $0 \leq x_i \leq m_i(\mathbf{x_{-i}})$ given the fixed, feasible flow rates of all other users on its path, $\{x_j : j \in (R_j \cap R_i)\}$.

J_i is taken as the difference of a user-specific pricing function, P_i, and a utility function, U_i. The pricing function P_i indicates the current state of the network. This "feedback" term can be interpreted as the price the user pays for using the network resources. There is a variety of approaches in the literature on possible choices for the pricing term, depending on the specific feedback type. For example, studies [2, 3] develop an explicit congestion notification (ECN) mechanism based on packet marking. Another approach [5, 4] makes use of the queueing delays as an indication of congestion level in the network. One advantage of the latter approach is that it is based on measurements of the individual users, and does not require active participation of the network. The prices in this context should be interpreted in terms of network credits, which do not necessarily relate to real money. The pricing structure here, however, does perform one of the main functions of money: measuring and quantifying the resources. This provides a basis for versatile resource allocation schemes.

The utility function of the i^{th} user is taken to be increasing and concave in accordance with elastic traffic as well as with the economic principle, law of diminishing returns. We focus on the bandwidth as the main resource in the system. Therefore, the utility of the i^{th} user depends only on its flow rate. Thus, the cost function is defined as the difference between the pricing and the utility functions:

$$J_i(\mathbf{x}; \mathbf{C}, \mathbf{A}) = P_i(\mathbf{x}; \mathbf{C}, \mathbf{A}) - U_i(x_i) \ , i \in \mathcal{M}. \qquad (2.2)$$

We note that P_i does not have to depend on the flow rates of all other users. It can be structured to depend only on the flow rates of the users who share links on user i's route.

3 Existence and Uniqueness of the Nash Equilibrium

In the context of the network game introduced above, the Nash equilibrium (NE) is defined as a set of flow rates, $\mathbf{x}^*$ (and corresponding set of costs J^*), with the property that no user can benefit by modifying its flow while the other players keep theirs fixed. Furthermore, if the NE, $\mathbf{x}^*$, meets the capacity constraints as well as the positivity constraint with strict inequality, then it is also an *inner* solution. Mathematically speaking, $\mathbf{x}^*$ is in NE, when x_i^* of any i^{th} user is the solution to the following optimization problem given all users on its path have equilibrium flow rates, $\mathbf{x}^*_{-i}$:

$$\min_{0 \leq x_i \leq m_i(\mathbf{x}^*_{-i})} J_i(x_i, \mathbf{x}^*_{-i}, \mathbf{C}, \mathbf{A}) \ , \qquad (3.1)$$

where $\mathbf{x_{-i}} := \{x_j : j \in R_j \cap R_i\}_{j=1,\ldots,M}$. To proceed further, we make the following two assumptions.

A1. $P_i(\mathbf{x})$ is jointly continuous in all its arguments, and twice continuously differentiable, non-decreasing and convex in x_i, i.e. $\partial P_i(\mathbf{x})/\partial x_i \geq 0$, $\partial^2 P_i(\mathbf{x})/\partial x_i^2 \geq 0$.

A2. $U(x_i)$ is jointly continuous in all its arguments and twice continuously differentiable, non-decreasing and uniformly strictly concave in x_i, i.e. $\partial U_i(x_i)/\partial x_i \geq 0$, $\partial^2 U_i(x_i)/\partial x_i^2 < -\epsilon,\ \epsilon > 0,\ \forall x_i$. Moreover, the optimal solution is an inner one, $\sum_j A_{l,j}x_j < C_l\ ,\forall l$, under the additional assumption:

A3. The i^{th} user's cost function has the following properties at $x_i = 0$ ($x_i = m_i(\mathbf{x_{-i}})$) : $J_i(\mathbf{x} : x_i = 0) > J_i(\mathbf{x})\ ,\forall \mathbf{x}\ x_i \neq 0$ ($J_i(\mathbf{x} : x_i = m_i(\mathbf{x_{-i}})) > J_i(\mathbf{x})\ ,\forall \mathbf{x}\ , x_i \neq m_i(\mathbf{x_{-i}})$) respectively. One sufficient condition for the latter case is that P_i has the property: $P_i(\mathbf{x}) \to \infty$ as $\sum_j A_{l,j}x_j \to C_l$ for any link $l \in R_i$.

We will shortly establish that the congestion control game defined by the cost function (2.2) on the network topology ($\mathbf{C}$, $\mathbf{A}$) admits a NE. We will further show that if P_i is taken as the sum of individual prices P_l on links $l \in R_i$ on the path R_i of the user, and the link prices are functions of only the total flow on that link, $P_l(\sum_{j:l\in R_j} x_j)$, then the NE is unique. The following assumption captures the underlying condition:

A4. For each $i \in \mathcal{M}$, $P_i(\mathbf{x})$ is the sum of link price functions $P_l\ , l \in R_i$; i.e., $P_i = \sum_{l\in R_i} P_l(\sum_{j:l\in R_j} x_j)$. The link price, P_l, is a function of the aggregate flow on link, l.

Theorem 3.1. *Under A1-A4, the network game admits a unique inner Nash equilibrium.*

Proof. Let $X := \{\mathbf{x} \in \mathbb{R}^M : \mathbf{Ax} \leq \mathbf{C}\ , \mathbf{x} \geq 0\}$ be the set of feasible flow rate vectors (or strategy space) of the users. The flow rate of a generic i^{th} user is nonnegative and bounded above by the minimum link capacity on its route, $0 \leq x_i < \min_{l\in R_i} C_l$. Hence, X is bounded. Next, we show that X has a nonempty interior and is convex. Define the following flow rate vector: $\mathbf{x}^{max} := \min_l C_l/M$. Clearly, $\mathbf{x}^{max} \in X$ is feasible and positive as $C_l > 0\,\forall l$. Hence, there exists at least one positive and feasible flow rate vector in the set X, which is an interior point. Thus, the set X has a nonempty interior. Let $\mathbf{x}^1$, $\mathbf{x}^2 \in X$ be two feasible flow rate vectors, and $0 < \lambda < 1$ be a real number. We have, for any $\mathbf{x}^\lambda := \lambda\mathbf{x}^1 + (1-\lambda)\mathbf{x}^2$,

$$\mathbf{Ax}^\lambda = \mathbf{A}(\lambda\mathbf{x}^1 + (1-\lambda)\mathbf{x}^2) \leq C$$

Furthermore, $\mathbf{x}^\lambda \geq 0$ by definition. Hence, $\mathbf{x}^\lambda$ is feasible and is in X for any $0 < \lambda < 1$. Thus, the set X is convex.

Let $X_\epsilon := \{\mathbf{x} \in \mathbb{R}^M : \mathbf{Ax} \leq \mathbf{C} - \epsilon\ , \mathbf{x} \geq 0\}$ for a given sufficiently small $\epsilon > 0$. The set X_ϵ is clearly closed, convex and has a nonempty interior. From *A1-A3*, any feasible flow vector $\mathbf{x}_\epsilon \in X_\epsilon$ is bounded, and hence X_ϵ is compact. By a standard theorem of game theory (Thm. 4.4 p.176 in [13]), the network game admits a NE. Furthermore, by *A3*, the solution has to be inner, as the following argument shows. First, $\mathbf{x} \geq 0$, with $x_i = 0$ for at least one i, cannot be an equilibrium point since user i can decrease its cost by increasing its flow rate. Similarly, the boundary points $\{\mathbf{x} \in \mathbb{R}^M : \mathbf{Ax} \leq \mathbf{C}\ , \mathbf{x} \geq 0,$ with $(Ax)_l = C_l$ for at least one link $l\}$ cannot constitute NE, as users whose flows pass through the link with full capacity have infinite cost under *A3*. Since this applies to all players, the NE has to be independent of ϵ for $\epsilon > 0$ sufficiently small. Thus, it provides a NE to the original game on X.

We now prove uniqueness. Differentiating (2.2) with respect to x_i, and using assumptions *A1*,*A2*, we have

$$f_i(\mathbf{x}) := \frac{\partial J_i(\mathbf{x})}{\partial x_i} = \frac{\partial P_i(\mathbf{x})}{\partial x_i} - \frac{\partial U_i(x_i)}{\partial x_i}\ . \tag{3.2}$$

As a simplification of notation, $\mathbf{C}$ and $\mathbf{A}$ are suppressed as arguments of the functions for the rest of this proof.

Differentiating $J_i(\mathbf{x})$ twice with respect to x_i yields

$$\frac{\partial f_i(\mathbf{x})}{\partial x_i} = \frac{\partial^2 J_i(\mathbf{x})}{\partial x_i^2} = \frac{\partial^2 P_i(\mathbf{x})}{\partial x_i^2} - \frac{\partial^2 U_i(x_i)}{\partial x_i^2} > 0$$

Hence, J_i is unimodal and has a unique minimum. Based on $A3$, $f_i(\mathbf{x})$ attains the zero value at $m_i(\mathbf{x_{-i}}) > x_i > 0$ given a fixed feasible $\mathbf{x_{-i}}$. Thus, the optimization problem (3.1) admits a unique positive solution.

To preserve notation, let $\frac{\partial^2 J(\mathbf{x})}{\partial x_i^2}$ be denoted by B_i. Further introduce, for $i, j \in \mathcal{M},\ j \neq i$,

$$\frac{\partial^2 J_i(\mathbf{x})}{\partial x_i \partial x_j} = \frac{\partial^2 P_i(\mathbf{x})}{\partial x_i \partial x_j} =: A_{i,j}\ ,$$

with both B_i and $A_{i,j}$ defined on the space where $\mathbf{x}$ is nonnegative, and bounded by (2.1). Suppose that there are two Nash equilibria, represented by two flow vectors $\mathbf{x}^1$ and $\mathbf{x}^0$, with elements x_i^0 and x_i^1, respectively. Define the pseudo-gradient vector:

$$g(\mathbf{x}) := \left[\nabla_{x_1} J_1(\mathbf{x})^T \cdots \nabla_{x_M} J_M(\mathbf{x})^T\right]^T \tag{3.3}$$

As the Nash equilibrium is necessarily an inner solution, it follows from first-order optimality condition that $g(\mathbf{x}^0) = 0$ and $g(\mathbf{x}^1) = 0$. Define the flow vector $\mathbf{x}(\theta)$ as a convex combination of the two equilibrium points $\mathbf{x}^0$, $\mathbf{x}^1$:

$$\mathbf{x}(\theta) = \theta\mathbf{x}^0 + (1-\theta)\mathbf{x}^1$$

where $0 < \theta < 1$. By differentiating $\mathbf{x}(\theta)$ with respect to θ,

$$\frac{dg(\mathbf{x}(\theta))}{d\theta} = G(\mathbf{x}(\theta))\frac{dx(\theta)}{d\theta} = G(\mathbf{x}(\theta))(\mathbf{x}^1 - \mathbf{x}^0)\ , \tag{3.4}$$

where $G(\mathbf{x})$ is the Jacobian of $g(\mathbf{x})$ with respect to $\mathbf{x}$:

$$G(\mathbf{x}) := \begin{pmatrix} B_1 & A_{12} & \cdots & A_{1M} \\ \vdots & & \ddots & \vdots \\ A_{M1} & A_{M2} & \cdots & B_M \end{pmatrix}_{M\times M} . \tag{3.5}$$

We also note that, by Assumption *A4*:

$$\sum_{l\in(R_i\cap R_j)} \frac{\partial^2 J_l(\mathbf{x})}{\partial x_i \partial x_j} = \sum_{l\in(R_i\cap R_j)} \frac{\partial^2 J_l(\mathbf{x})}{\partial x_i \partial x_j}$$

$$\Rightarrow A(i,j) = A(j,i) \;\; i,j \in \mathcal{M} \; .$$

Hence, $G(\mathbf{x})$ is symmetric. Integrating (3.4) over θ,

$$0 = g(\mathbf{x}^1) - g(\mathbf{x}^0) = [\int_0^1 G(\mathbf{x}(\theta))d\theta](\mathbf{x}^1 - \mathbf{x}^0) \; , \quad (3.6)$$

where $(\mathbf{x}^1 - \mathbf{x}^0)$ is a constant flow vector. Let $\overline{B_i(\mathbf{x})} = \int_0^1 B_i(\mathbf{x}(\theta))d\theta$ and $\overline{A_{ij}(\mathbf{x})} = \int_0^1 A_{ij}(\mathbf{x}(\theta))d\theta$. In view of *A2* and *A4*, $B_i(\mathbf{x}) > A_{ij}(\mathbf{x}) > 0$, $\forall i,j$. Thus, $\overline{B_i(\mathbf{x})} > \overline{A_{ij}(\mathbf{x})} > 0$, for any $\mathbf{x}(\theta)$. In order to simplify the notation, define the matrix $\mathcal{G}(\mathbf{x}^1, \mathbf{x}^0) := \int_0^1 G(\mathbf{x}(\theta))d\theta$, which can be shown to be full rank for any fixed $\mathbf{x}$. Rewriting (3.6) as, $0 = \mathcal{G} \cdot [\mathbf{x}^1 - \mathbf{x}^0]$, since $\mathcal{G}$ is full rank, it readily follows that $\mathbf{x}^1 - \mathbf{x}^0 = 0$. Therefore, the NE is unique. □

4 Global Stability of NE Under a Gradient Algorithm

We consider a simple dynamic model of the network game where each user modifies its flow rate proportional to the gradient of its cost function with respect to its flow rate. We will specifically show that the unique NE of Thm. 3.1 is globally stable under the algorithm

$$\frac{dx_i}{dt} = \dot{x}_i = -\frac{\partial J_i(\mathbf{x})}{\partial x_i} \;\; , i \in \mathcal{M} \; , \quad (4.1)$$

where 't' is the time variable. We first state (without proof, due to space limitations) the following useful result.

Lemma 4.1. *The matrix $G(\mathbf{x})$ in (3.5) is uniformly positive definite, i.e. $G(\mathbf{x}) > \epsilon I$ for some $\epsilon > 0$ and all $x \in X$.*

Now, following the line of proof of Thm. 8 of [14], let us introduce $h(\mathbf{x}) = -g(\mathbf{x})$ where $g(\mathbf{x})$ is given by (3.3). Let $||h(\mathbf{x})||^2 = h^T(\mathbf{x})h(\mathbf{x})$ be a candidate Lyapunov function for (4.1). Since $\dot{h}(\mathbf{x}) = -G(\mathbf{x})\dot{\mathbf{x}} = -G(\mathbf{x})h(\mathbf{x})$,

$$\frac{d}{dt}||h(\mathbf{x})||^2 = -2\varepsilon h^T G h < -\varepsilon ||h(\mathbf{x})||^2 \; , \quad (4.2)$$

where $\epsilon > 0$ was introduced in the statement of Lemma 4.1. Therefore, (4.1) is asymptotically stable.

Theorem 4.2. *The unique NE of the network game is globally stable under the gradient algorithm (4.1).*

5 System Problem and Optimality of Nash Equilibrium

We introduce here a "system problem" for the model studied above, and discuss its relationship with the models of Kelly [2] and subsequent studies [3, 4, 5]. In all these studies a system problem is defined either as a constrained optimization problem given by

$$\begin{array}{c} \max_{\mathbf{x}\geq 0} \sum_{i\in\mathcal{M}} U_i(x_i) \\ \text{subject to } \; \mathbf{A}\mathbf{x} \leq [C_1 \ldots C_L]^T \end{array} \; , \quad (5.1)$$

or as its relaxed version

$$\min_{\mathbf{x}\geq 0} \sum_{l\in\mathcal{L}} P_l(\sum_{i:l\in R_i} x_i) - \sum_{i\in\mathcal{M}} U_i(x_i) \; , \quad (5.2)$$

where P_l is a link price function satisfying *A1-A3*. This system goal is motivated by the fact that the sum of the utilities of users is maximized, whereas aggregate cost at the links is minimized. The cost function at a link may be chosen as the average delay a packet experiences or the percentage of dropped packets with respect to the total flow at the link. The centralized problem (5.1) is solved by introducing a user problem and a network problem [2] which leads to distributed algorithms. The user problem can be seen as a "trivial game", and is defined for the i^{th} user as,

$$\min_{x_i\geq 0}\{\lambda_i x_i - U_i(x_i)\} \; , \quad (5.3)$$

where λ_i represents the price per unit flow rate, and is assumed not to be a function of x_i. The network problem, on the other hand, yields these λ_i's. In order to solve the network problem, however, a centralized knowledge of the user preferences is necessary. This difficulty is circumvented by introducing a system of coupled differential equations for x_i and λ_i. The solutions to these differential equations converge to the optimal solutions of the user and network problems, and hence to the solution of the system problem (5.1).

In our formulation, however, the user problem is not decoupled, and is a genuine game, as defined by (3.1). In spite of that, we show below that the NE of this game (whose existence and uniqueness have already been established) solves the system problem (5.2). Thus, the Nash solution of the network game is efficient regardless of the number of users in the network. Note that, here the users take the effect of their strategies into account when optimizing their costs.

Theorem 5.1. *The unique NE of the game (3.1) solves the following system problem:*

$$\min_{\mathbf{x}\geq 0} \sum_{l\in\mathcal{L}} P_l(\sum_{i:l\in R_i} x_i) - \sum_{i\in\mathcal{M}} U_i(x_i) \; , \quad (5.4)$$

where P_l and U_i satisfy assumptions A1-A4.

Proof. To solve the system problem (5.4), assuming an inner solution, we take the gradient with respect to user flow rates, $\mathbf{x}$, and obtain the first-order necessary condition for optimality:

$$\sum_{l\in\mathcal{L}} \frac{\partial P_l(\sum_{i:l\in R_i} x_i)}{\partial x_i} - \sum_{i\in\mathcal{M}} \frac{dU_i(x_i)}{dx_i}$$

Notice that the partial derivatives of the link costs at the links not on the path of the i^{th} user with respect to x_i yield zero. Likewise, the utility function of each user depends only on that user's flow rate. Hence, the first-order necessary condition of this problem coincides with the one of the user problem given by (3.2). Furthermore, this solution is unique for (5.4) due to the structures of P_l and U_i. Thus, the NE solves the system problem (5.4). □

Remark 5.2. We have thus seen that the NE of the network game also solves the relaxed system problem (5.4), and hence the formulation of the flow control problem as a noncooperative game brings a useful interpretation to the system problem (5.4). The network game also provides a general framework for a variety of game theory based congestion control schemes [4, 15]. It is not motivated by a specific system goal, which may fail to capture some aspects of the target system. Instead, it provides a natural way of capturing the user behavior. It is more comprehensive as it provides a flexible market-based scheme for a fairly general class of pricing and utility functions. Furthermore, the same framework can be applied to a variety of communication networks problems as it will see next.

6 A Congestion Control Scheme for Ad Hoc Wireless Networks

6.1 Ad Hoc Wireless Networks

Ad Hoc wireless networks are gaining increasing popularity due to the fact that they offer unique benefits and versatility for certain applications. They consist of mobile nodes interconnected by multihop communication paths, and require no pre-existing fixed infrastructure like base stations. Hence, they can be created and used anytime, anywhere, without the limitations of a fixed topology [16]. Ad hoc networks have different requirements on congestion control mechanisms when compared to the ones of wireline networks, as a result of effective routing [16, 17]. A basic assumption here is the combinatorial stability of the ad hoc network, i.e., the topology changes occur sufficiently slowly to permit successful propagation of all topology updates as necessary [16]. After a route is established, re-routing becomes a crucial tool in such a setting for preserving the connection, since not only the topology changes in the network, but also the capacity of the existing paths may vary with time. Therefore, it might be desirable to maintain the capacity usage in the links low enough to provide various re-routing schemes with unused capacity.

Current congestion control schemes, like TCP are not adequate for ad hoc wireless networks for several reasons. One important factor is the 'additive increase multiplicative decrease feature of the TCP which leads to wide fluctuations in the flow rate of users, and aggressive usage of the available bandwidth. This behavior does not satisfy the requirements of an ad hoc network as mentioned above. Another point is the usage of packet losses as a congestion indication in TCP, which does not necessarily hold in the case of wireless networks where packet losses may occur randomly [3]. Hence, congestion control algorithms should be devised to meet the specific requirements of ad hoc wireless networks.

6.2 The Model and the Cost Function

We propose here a congestion control game based on the network game studied in Sections 2, 3, and 4, and in light of the discussions in Section 6.1. The user problem is defined again by the cost function (2.2), and is given by (3.1). Specifically, let the utility function of the i^{th} user be

$$U_i(x_i) = u_i \log(x_i + 1) \, , \tag{6.1}$$

where u_i is a user-specific preference parameter. The link price function is given by,

$$P_l(\sum_{i:l\in R_i} x_i) = \frac{\kappa}{C_l - \sum_{i:l\in R_i} x_i} \, , \tag{6.2}$$

where κ is a network-wide constant which depends on factors like the type of the ad hoc network, number of users, etc. Notice that $1/(C_l - \sum_{i:l\in R_i} x_i)$ corresponds to the delay at the link l, if an $M/M/1$ queue model is assumed, and hence, the price P_i is proportional to the aggregate delay on the i^{th} user's path.

The cost function, as a special case of (2.2), is defined as:

$$J_i(\mathbf{x}, u_i, \kappa) = \sum_{l\in R_i} \frac{\kappa}{C_l - \sum_{i:l\in R_i} x_i} - u_i \log(x_i + 1) \tag{6.3}$$

The utility (6.1), price (6.2), and cost (6.3) functions satisfy *A1-A4*, if parameters κ and u_i are chosen appropriately. Hence, by Thm. 3.1 there exists a unique inner NE. Furthermore, by Thm. 5.1 this NE solves the following system problem:

$$\min_{\mathbf{x}\geq 0} \sum_{l\in\mathcal{L}} \frac{\kappa}{C_l - \sum_{i:l\in R_i} x_i} - \sum_{i\in\mathcal{M}} u_i \log(x_i + 1) \tag{6.4}$$

Moreover, the M-dimensional equilibrium flow vector, $\mathbf{x}^*$, satisfies the link capacity constraints (2.1) as a result of the specific structure of link price function P_l in (6.2). The system goal in (6.4) can be motivated by the fact that it maximizes the aggregate utility of the mobiles, and minimizes the sum of delays at the links.

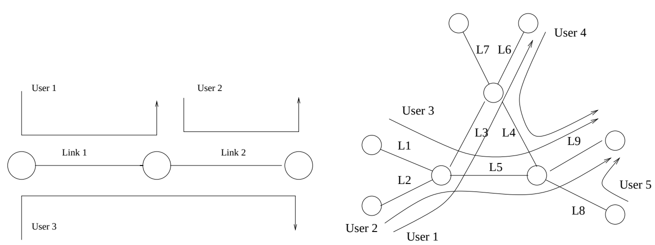
Following (4.1), an update algorithm for the i^{th} user is:

$$\frac{dx_i}{dt} = \dot{x}_i = \frac{u_i}{x_i + 1} - \sum_{l\in R_i} \frac{\kappa}{(C_l - \sum_{i:l\in R_i} x_i)^2}$$

By Thm. 4.2 this algorithm globally converges to the NE. The variables x_i and u_i and the network specific parameter κ are already known to the i^{th} user. Hence, the only feedback a user needs from the network for updating its flow rate x_i is the quantity $\sum_{l \in R_i}(C_l - \sum_{i: l \in R_i})^2$. In any implementation, this value could be delivered to the users either by special maintenance packets or in the packet headers. In an ad hoc network, each mobile has to update its knowledge about the current state of the network not only for congestion control but also for routing purposes. Thus, the proposed feedback mechanism brings little, if any, additional overhead to the network.

6.3 Simulation Studies

Using the congestion control scheme developed above for ad hoc networks, we have studied numerically (using MATLAB) the two specific network configurations shown in Figure 1. The basic network structure in Figure 1 (a) is chosen for its simplicity, as it enables us to observe the effect of different parameters and to compare flow rates of users. The second network in Figure 1 (b), on the other hand, is more general and captures features not covered by the first one. We make the simplifying assumption of predetermined, fixed routes throughout the duration of connections. Furthermore, pricing and utility parameters are fixed for a given simulation. In the simulations, time, and hence the algorithm, are discretized. First, we investigate the effect

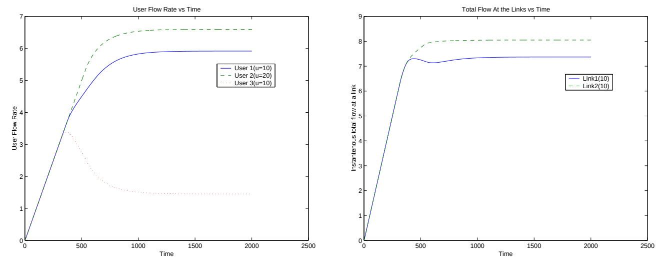


(a) (b)

Figure 1: A basic network configuration (a), and a more complex network with arbitrary routes (b).

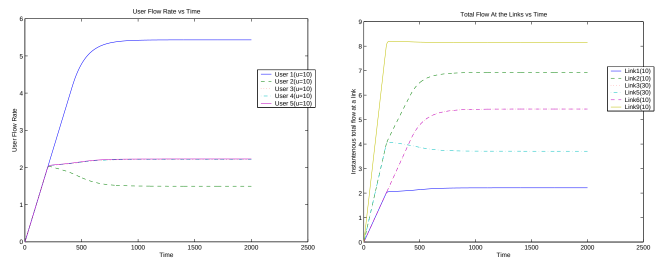
of varying the value of utility parameters, u_i, on the simple network of Figure 1 (a). Link capacities are chosen as 10, and the pricing constant as $\kappa = 10$. Simulation is repeated for utility parameters $\vec{u} = [10\ 10\ 20]$, $[10\ 20\ 10]$, and $[10\ 10\ 10]$. Figure 2 displays results for utility parameters $[10\ 20\ 10]$.

In the first case, although user 3 is charged twice that of users 1 and 2, we have symmetric flow rates due to the doubled utility of user 3. When all users have the same utility as in the third case, however, user 3 ends up with a lower flow rate than others, as it uses both links. When user 2 has $u_2 = 20$ in the second case, he ends up getting a larger share of the link capacity, but less than twice the value of user 1. We also note that links are not necessarily used with full capacity. Although it may seem as a disadvantage in general, this extra capacity contributes to robustness in case of ad hoc networks. The simulation is repeated on the more complex network of Figure 1 (b). Figure 3 shows flow rates of users with symmetric utilities, $u = 10$, and aggregate flow rates on selected links. Another important cost function parameter is the pricing parameter, κ. In



(a) (b)

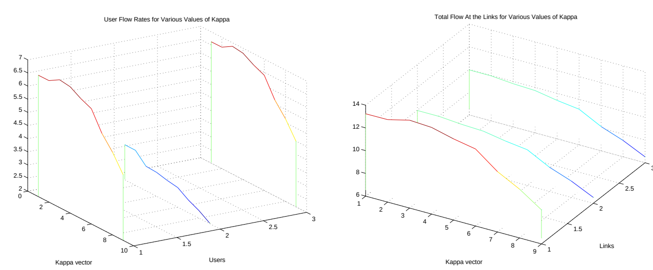
Figure 2: Flow rates of the users and aggregate flow rates at the links on the simple network of Figure 1 for $\vec{u} = [10\ 20\ 10]$ in (a) and (b).



(a) (b)

Figure 3: Flow rates of users with $u = 10$ (a), and aggregate flow rates at selected links (b). Simulation is run on the network of Figure 1 (b).

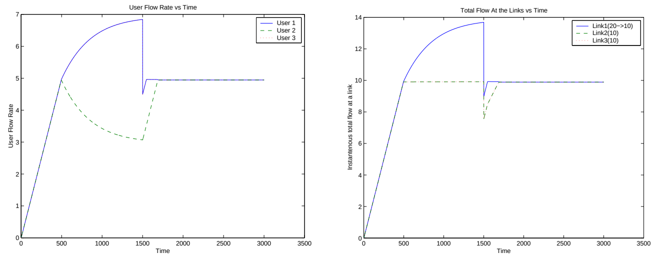
Figure 4, simulation results are shown for various values of $\kappa = 0.001,\ 0.01,\ 0.05,\ 0.1,\ 0.5,\ 1,\ 5,\ 10$, and 20. As expected, increased prices lead to lower flow rates and less capacity usage. Next, robustness of the system is investigated



(a) (b)

Figure 4: Effect of parameter κ on user flow rates (a), and aggregate flow rates at the links (b). The κ values are given in the vector $\kappa = [0.001\ 0.01\ 0.05\ 0.1\ 0.5\ 1\ 5\ 10\ 20]$.

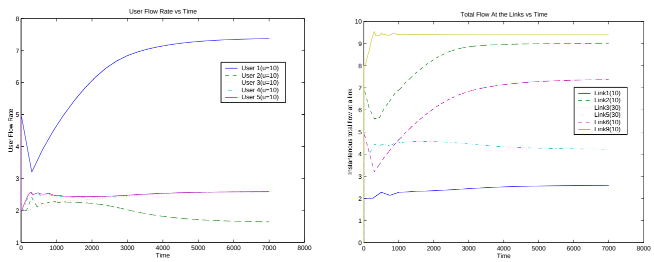
under varying link capacities. Since ad hoc networks are inherently unreliable, robustness of the system is of great importance. The capacity of link 1 on the basic network is halved at $t = 1500$. We observe in Figure 5 that the users respond rapidly to this change, indicating robustness of the scheme. Finally, effect of delay on the proposed scheme is simulated on the network of Figure 1 (b). Users are symmetric in terms of their utility parameters, $u_i = 10\ \forall i$, and the pricing parameter $\kappa = 1$. The delay vector of users is given by $[300\ 150\ 250\ 50\ 20]$. Note that, these delay values are



(a) (b)

Figure 5: Effect of a variation in the capacity of link 1 on user flow rates (a), and aggregate flow rates at the links (b). The capacity of link 1 is halved at $t = 1500$.

vastly exaggerated when typical, local ad hoc networks are considered. It is observed in Figures 6 (a) and (b) that the user flows and aggregate link flow rates converge, although slightly slower than the ones in the ideal case.



(a) (b)

Figure 6: Effect of delay on user (a) and aggregate flow rates (b) at the links. The delay vector of users is chosen as $[300\ 150\ 250\ 50\ 20]$.

7 Conclusion

We have seen that noncooperative game theoretic approach provides an appropriate framework for developing congestion control schemes for communication networks. Moreover, with a suitable choice of cost functions these schemes are easily implementable. There are, however, several open issues and many directions for future research. In the implementation area, the scheme proposed here can be refined and further developed. Specifically, adaptive protocols might be investigated, where cost parameters vary with network conditions. Another possible direction for research is the analytical study of the effect of delays on the system, and conditions for stability on a system with high propagation delays.

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