A Game Theoretical Approach to Distributed Relay Selection in Randomized Cooperation

Simone Sergi and Giorgio M. Vitetta, *Senior Member, IEEE*

***Abstract*—In this paper, the problem of node management in the presence of randomized cooperation is tackled. First, game theory is exploited to model the problem of setting up a cluster of cooperative nodes in a wireless network as a multiplayer noncooperative game. In this game the set of players is made of all the nodes belonging to a potential relay cluster an d the set of actions for each player consists of two options only (characterized by different payoffs), namely transmitting a data packet or remaining silent. Then, a novel strategy for the management of node participation to a distributed cooperative link is derived. The proposed solution is fully distributed, is characterized by autonomous choices made by each potential relay and is of significant practical interest since it guarantees the participation of a proper number of nodes to a virtual antenna array (so avoiding an energy waste associated with an excessive number of cooperating nodes) without requiring any overhead for node management.**

***Index Terms*—Game theory, distributed randomized - orthogonal space time coding, selfish nodes, ad hoc networks, cooperative communication.**

I. Introduction

In wireless ad hoc and sensor networks data communications usually require the *cooperation* of their nodes; for instance, data transmission from a given source node to a far destination node can involve other nodes acting as relays to establish a reliable and energy efficient *multihop link* [1]. To enhance link performance in each hop, relays can be also grouped to form *clusters* of cooperative transmitters; when this occurs, the nodes of each cluster coordinate their data transmissions according to a specific strategy, i.e. according to a given *distributed cooperative transmission technique*. An important example of this approach is offered by the so called *distributed space-time coding schemes*, like *distributed orthogonal space-time coding* (D-OSTC) [2]. Unluckily, the implementation of distributed transmission methods is hindered mainly by their significant complexity and by the large overhead required for node management. This is due to the need of identifying the nodes that can potentially join each cluster and of assigning the available codewords to cluster nodes in a proper fashion [2], [3].

Recently, a solution to the problem of codeword assignment for D-OSTC has been proposed in [4], [5]. According to this solution, known as *distributed randomized – orthogonal space-time coding* (DR-OSTC), each node transmits a linear combination of multiple codewords, which are randomly selected from a code matrix shared by all the network nodes. In principle, the DR-OSTC scheme does not require an a priori knowledge of the number of nodes contributing to data transmission in each cluster; in practice, however, an accurate code design and a proper number of nodes in each distributed transmission are required to ensure a given outage probability. These problems are analysed in detail in [6], where the minimum number of nodes required to accomplish a cooperative transmission and the code size needed to satisfy given performance requirements are derived. However, as fas as we know, what is still missing in the technical literature about DR-OSTC is an efficient and low overhead strategy for the selection of a proper set of nodes for cooperative transmissions.

In this paper a novel distributed strategy for managing the node participation in a DR-OSTC based cooperative link is derived resorting to a game theoretical modelling of the considered problem [7]. In any scenario characterized by a limited *signal-to-noise ratio* (SNR)[1], the proposed strategy is able to reach the needed diversity order on a cooperative link and, at the same time, to avoid an energy waste deriving from an excessive number of cooperative nodes. In addition, it is characterized by the following relevant features: a) each node is allowed to manage, in a completely autonomous fashion, its contribution within a cluster of potential relays; b) no transmission overhead is required for node management; c) no prior information about the distribution of neighbouring nodes or the channel statistics are needed for the distributed management of network nodes. For the sake of completeness, our analysis considers two different cases of node behavior: in the first case, all the networks nodes are *selfish*, whereas in the second one they are *prone to cooperation*.

The following part of this manuscript is organized as follows. The network model is described in Section II. In Section III some basic rules about node cooperation are given and the participation dilemma of each node is formalized as a noncooperative multiplayer game. Then, a learning strategy for network nodes is described in Section IV and is exploited in Section V to derive an optimal stochastic transmission strategy. Performance results are illustrated in Section VI. Finally, Section VII offers some conclusions.

II. Network Model

To ease the derivation of the proposed strategy for the management of node transmission and avoid diverting the reader's attention from the main contribution of this work, a

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The authors are with the Department of Information Engineering, University of Modena e Reggio Emilia (e-mail: {simone.sergi, giorgio.vitetta}@unimore.it).

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[1]Note that this situation is of relevant interest when dealing with energy constrained distributed networks.

two-hop link, i.e. a relay network, is analysed in the following. In our scenario a *source* node is expected to communicate with a *destination* through a set of N_T hierarchically equivalent and *rational* potential relay nodes, each endowed with a single antenna and operating in a *decode and forward* fashion. Each node is also equipped with a battery having a finite stored energy; therefore it needs to limit its power consumption for data transmission so that its lifetime is extended as long as possible. Each data packet sent by the source can be correctly detected by a number of potential relays (correct detection depends on both transmission power and channel state); then each relay can decide if forwarding the packet toward the destination or not. For energy saving it is advisable to let a proper number of relay nodes contribute to packet forwarding. As shown in the following, this goal can be achieved adjusting the *transmission probability* P_{tx} of the potential relay nodes, so that a subset of nodes (selected in the set of N_T available nodes) actually plays, on the average, an active role in the task of cooperative forwarding. The active nodes adopt the DR-OSTC scheme proposed in [4] for data transmission; this means that each node uses a single codeword randomly selected from a common code matrix of proper size L for data relaying[2]. It is well known that, in this scenario, the achievable performance on a link in the presence of a limited SNR mainly depends on the number M of distinct codewords employed by at least one node [6], [4]. For this reason, in the following the performance enhancement deriving from the exploitation of the same codeword from multiple transmitting nodes of the same cluster is neglected. Moreover, to simplify our analysis, the errors due to channel impairments (hence fading and noise) are also neglected in the relays to destination *multiple input – single output* (MISO) link, so that the only figure of merit taken into account when assessing link quality is represented by the achieved degree of diversity [6]. In our model the following assumption are made: a) The packets transmitted by the source contain a known preamble which can be exploited by all the potential relays to achieve a rough synchronization for their transmission [4]. This imperfect synchronization can be deemed sufficient when considering delay tolerant ST codes [8], [9]; however, a comprehensive discussion about this topic is beyond the scopes of this paper (see [4] for a deeper analysis). b) The whole set of potential relays is always of adequate size, so that a reliable data transmission can be accomplished[3] [6]. c) The actual outcome of any cooperative transmission attempt towards the destination is sent to the potential relay nodes through a single bit ACK/NAK feedback[4]. If a fixed and known transmission power is assumed for each node, the same feedback can be also exploited to estimate its mean channel attenuation towards the destination itself.

[2]In the technical literature, more complex random rules, useful to achieve a higher degree of diversity in the presence of few relay nodes, have been proposed (e.g., see [5]); however, they are not taken into consideration here for simplicity.

[3]In other words, it is assumed that the needed degree of diversity can be certainly achieved by the whole set of nodes able to both decode the transmission from the source and transmit towards the destination.

[4]In this work, for the sake of simplicity, the transmission of a single bit feedback for each transmitted packet is considered. However, the learning strategy proposed in Section IV can be easily generalized to the case of cumulative ACK.

Two different scenarios for the behavior of relay nodes are considered in the following. In the first scenario (dubbed *scenario* #1) all the nodes are *selfish*, whereas in the second one (denoted *scenario* #2) they are *prone to cooperation*. If we take into consideration the general case of an ad hoc network consisting of peer nodes, the first scenario refers to the situation in which each user owns its terminal and aims only at carrying out its own data communications; in this case, in the eyes of every potential relay, any cooperation effort is perceived as a waste of personal resources so that, generally speaking, it has to be properly stimulated. On the contrary, the second scenario is well suited to describe, for example, sensor networks, where the nodes are under the control of the same central authority. In fact, in the last situation the only goals of each node are the efficiency and the effectiveness of the network it belongs to. It is important to note that in scenario #1, proper policies for cooperation enforcement are typically needed [10], [11]; in this case, all the solutions proposed in the technical literature rely on the simple rationale that a node acquires some *credits* (usually represented by virtual money or a reputation level) when it cooperates with other nodes and, in turn, can exploit these credits to obtain cooperation. On the contrary, a simpler approach can be adopted for scenario #2. In fact, in this case any concept of credit or reputation is no more necessary, since, if a node is able to assess the benefits brought to the network by its actions, it will make its choices without expecting a personal reward.

III. Game Description

A. Rules and description of the game

In the derivation of our strategy the following assumptions are made:

1) The nodes belonging to the same potential relay cluster do not exchange information, but are able to listen to a common signal, originating from the destination node, and carrying information about the status of the last transmission attempt (ACK/NAK feedback).
2) When the nodes of a cluster of potential relays receive a data packet, each of them autonomously decides whether to forward it or not, provided that it is able to properly decode the conveyed information.
3) The time axis is divided in slots to ease the modellization of node actions. The slot length T_u is equal to the duration of a data packet transmitted by network nodes. Note that, generally speaking, the duration of the slot period can appreciably influence system performance in the presence of a time varying wireless channel.

Our main goal is achieving a degree of diversity equal to R at the destination node, so that data decoding will be carried out correctly without involving an excessive number of active relays (and a consequent energy waste). To reach our goal, we need to devise a node management strategy such that, if a packet is received by a potential relay cluster consisting of N_T nodes, an adequate number $N(t)$ of them decides to contribute to data relaying in the t-th slot exploiting a DR-OSTC scheme. Moreover, we are interested in devising a *distributed and noncooperative strategy*, so that any explicit information sharing among the N_T nodes is avoided. To achieve this target

TABLE I
REPRESENTATION OF THE PARTICIPATION GAME FOR THE n-TH NODE IN SCENARIO #1

	$N_{-n} \leq R-2$	$N_{-n} \geq R-1$
TX	c_n	$\Pr\{M \geq R\} a_n + \Pr\{M < R\} c_n$
$NO\text{-}TX$	0	0

we model the *participation dilemma* (i.e., joining or not the set of nodes that accomplish a cooperative transmission) as a *multiplayer game*; in such a game the set of players consists of the N_T nodes belonging to the potential relay cluster and the action set of each player is made of two distinct options, namely *transmitting* or *remain silent*. The payoffs associated with different node actions depend on the node behavior. Considering at first scenario #1, which refers to a network populated by selfish nodes, it is reasonable to assume that the benefit acquired by the n-th node is related to its active contribution within a successful cluster transmission. In our game model, such a benefit is also inversely proportional to the number of cooperating relays; in addition, a cost related to the energy spent for packet transmission (so depending on the currently experienced channel condition) is charged to each node irrespectively of the transmission success (further details about the payoff definition are provided in Paragraph III-B). For these reasons the following roles are established for the payoffs:

- If the n-th node decides to take part to a cooperative transmission of a data packet towards the destination (TX action) and the cluster it belongs to carries out this task correctly, the associated payoff for this node is equal to a_n.
- If the n-th node decides to take part to a cooperative transmission of a data packet towards the destination (TX action), but the cluster it belongs to does not carry out this task correctly, its payoff is equal to c_n (this denotes a waste of resources).
- If the n-th node decides not to join the cooperative transmission of a data packet towards the destination (NO_TX action) its payoff is equal to 0 regardless of the actual transmission outcome.

The derivation of the optimal transmission strategy for the n-th node requires analysing the node point of view on its participation dilemma and, in particular, its *subjective* vision of the multiplayer game described above. This game can be usefully represented in *normal form*, so that the earnable payoffs can be easily identified on the basis of the actions selected by the n-th node and its opponents: such a representation is provided in Table I. In this table, the (random) parameter $N_{-n}(t)$ represents the number of nodes, different from the n-th one, that decide to take part to a cooperative transmission of a given data packet in the t-th time slot, whereas M denotes the rank of the matrix generated stacking the codewords randomly selected by the involved nodes (hence, the degree of diversity achieved by the transmission). Unluckily, the game we are analysing is characterized by a set of *incomplete information* about $N_{-n}(t)$; this prevents us from computing the expected payoff for the two different choices the n-th node can make. In particular, it is worth nothing that in scenario #1 the payoff expected by the n-th node for a packet transmission is given by

$$\rho(N_{-n}(t)) = \Pr\{M(t) \geq R | N_{-n}(t)\} a_n + \Pr\{M(t) < R | N_{-n}(t)\} c_n. \quad (1)$$

In Section IV it is shown that the opponent strategy can be estimated by the n-th node, so that the above mentioned problem about the parameter $N_{-n}(t)$ can be solved modelling it as a *game of imperfect information* (see [16, Sect. 3.7]). This means that the n-th node is required to optimize its response to the probability mass function $\Pr\{N_{-n}(t)\}$ as well as its expected payoff (1). Actually, this is equivalent to playing a strategy that optimally handles the case in which $N_{-n}(t)$ takes a certain value on the basis of the probability of this event. To ease the modelling, the same strategy can be obtained considering the single payoff metric

$$\alpha(N_{-n}(t)) = \Pr\{N_{-n}(t)\} \rho(N_{-n}(t)), \quad (2)$$

which represents a weighted version of the expected payoff (1). Then, the goal of the n-th node is to behave according to a strategy (hence, in our case, to choose its transmission probability $P_{tx}(n,t)$ in the t-th time slot), such that its mean expected payoff

$$E(TX) = \sum_{N_{-n}} \alpha(N_{-n}(t)) \quad (3)$$

is maximized. However, the dependency on the number of nodes involved in a cooperative transmission makes the derivation of an equilibrium point for the game a non trivial task. To tackle this problem, it is possible to consider the participation dilemma as a *repeated game*. In fact, the game is continuously repeated during the life of a given wireless link since the n-th node has to take a decision about cooperating or remaining silent in each time slot. If all the payoffs can be deemed constant over a few turns of the game itself[5], the game can be deemed *stationary* [12] and this allows the n-th node to acquire information about the behavior of its opponents from their past moves. Note also that the payoffs shown in Table I depend on the probability of the events $\{M(t) < R\}$ and $\{M(t) \geq R\}$, i.e. on the probability that the available degree of diversity is smaller than that needed for a correct transmission or not, respectively. It is easy to understand that these probabilities depend both on the overall number $N(t)$ of nodes involved in a cooperative transmission and on the randomization rule adopted for the codeword assignment [5]. For the randomization rule here considered, closed form expressions to evaluate these probability are provided in Appendix A.

When considering scenario #2, the rationale described so far still holds. However, when the network is populated by nodes prone to cooperation, it is reasonable to assume that each node earns a constant reward for a correct packet transmission of the cluster it belongs to, independently of its actual cooperation.

[5]The validity of this hypothesis depends on the rate of change of the communication channel characterizing a given wireless link. In the following we assume that the channel rate of change is so small to guarantee the effectiveness of the learning strategy proposed in Section IV. The effects of channel variations are assessed in Section VI via computer simulations.

TABLE II
REPRESENTATION OF THE PARTICIPATION GAME FOR THE n-TH NODE IN SCENARIO #2

	$N_{-n} \leq R-2$	$N_{-n} = R-1$	$N_{-n} \geq R$
TX	$\acute{c}_n$	$\Pr\{M=R\}\acute{a}_n$ $+\Pr\{M<R\}\acute{c}_n$	$\Pr\{M \geq R\}\acute{a}_n$ $+\Pr\{M<R\}\acute{c}_n$
$NO\text{-}TX$	0	0	$\Pr\{M \geq R\}\acute{b}_n$

In this circumstance, a node earns a positive payoff (given by b_n) also when it decides not to join a cooperative transmission of a data packet towards the destination (NO_TX action), but the active cooperating nodes correctly carry out this task. The representation of this modified game is provided in Table II. Note that, in this case, the *expected differential payoff*

$$\begin{aligned}\acute{\rho}(N_{-n}(t)) = &\Pr\{M(t) \geq R|N_{-n}(t)\}\,\acute{a}_n\\ &+\Pr\{M(t) < R|N_{-n}(t)\}\,\acute{c}_n\\ &-\Pr\{M(t) \geq R|N_{-n}(t)\}\,\acute{b}_n \qquad (4)\end{aligned}$$

has to be employed in place of $\rho(N_{-n}(t))$ (1) in the derivation of the node participation strategy (and, consequently, in eqs. 2 and 3).

B. Payoff evaluation

1) Payoffs in scenario #1:

Evaluation of a_n - In our model the payoff a_n represents the gain that the n-th cooperating node can earn through the correct transmission of a data packet; it depends on two relevant parameters, namely the estimate of the energy needed for this transmission and the amount of credits originating from this action [7]. In the following we assume that the overall amount of credits earned by the n-th node thanks to its transmissions until the end of the t-th slot over a specific link is equal to

$$P_{ov}(n,t) = B\frac{F_w(n,t)}{\sum_{j\in C_T} F_w(j,t)}, \qquad (5)$$

where B is the overall amount of credits made available by the source node to reward the whole potential relay set, $F_w(n,t)$ is the number of packets that the node has contributed to forward until the end of the t-th slot and $\sum_{j\in C_T} F_w(j,t)$ is the overall number of packets sent by the whole potential relay set C_T, which the n-th node belongs to, over the same time interval[6]. This choice ensures that the source node can define a priori the amount of credits B to be assigned to a relay stage to gain its cooperation, independently of the fraction of nodes that will really contribute to a packet transmission on the second hop. Moreover, it will contribute to limit the overall number of potential relays cooperating in a successful transmission, so that the efficiency of the link itself is ensured.

[6]An objective control of the effective number of transmissions carried out by each node is not trivial since it is not easy for an external observer to understand which nodes have actually participated to a specific transmission. In this work, this issue is not discussed further; it is assumed, however, that the nodes manage their transmission counter without cheating or that an effective distributed control system is realizable. For example, a simple policy cancelling any credit exchange if the sum of the potential relay requests exceeds the credits assigned to them by the source discourages dishonest behaviors in the case of rational - non malicious nodes.

For all the assumptions made above, if $E_{ov}(n,t)$ denotes an estimate of the overall energy consumed by the n-th node until the end of the t-th slot, the overall benefit acquired by the node over the given time interval can be expressed as

$$f\left(P_{ov}(n,t)\right) - g\left(E_{ov}(n,t)\right), \qquad (6)$$

where $f(\cdot)$ and $g(\cdot)$ are monotonic increasing functions having the specific purpose of making the two terms appearing in the right-hand side of (6) *homogeneous* and characterized by similar ranges (so that they play comparable roles in the evaluation of the payoffs). If one or more packet is successfully forwarded by the n-th node in the $(t+1)$-th slot, $F_w(i)$ and $\sum_i F_w(i)$ will increase by 1 and by the number $F_{w,slot}(t+1)$ of nodes participating to the cooperative transmission respectively (an estimation technique for this parameter is illustrated in Sec. VI), so that (see (5)) $P_{ov}(n,t+1) = B\left(F_w(n,t)+1\right)/\left[\sum_{k\in C_T} F_w(k,t) + F_{w,slot}(t+1)\right]$. Moreover, we have that $E_{ov}(n,t+1) = E_{ov}(n,t) + E_{slot}(n,t+1)$, where $E_{slot}(n,t+1)$ represents an estimate of the energy needed for the transmission in the $(t+1)$-th slot; such an estimate can be easily computed if the channel state in the previous slot is known.

Since the overall benefit for the n-th node after a correct transmission in the $(t+1)$-th slot is given by $f\left(P_{ov}(n,t+1)\right) - g\left(E_{ov}(n,t+1)\right)$, the payoff a_n can be defined as the difference between the last quantity and (6), i.e. as

$$\begin{aligned}a_n \triangleq &\left[f\left(P_{ov}(n,t+1)\right) - g\left(E_{ov}(n,t+1)\right)\right]\\ &-\left[f\left(P_{ov}(n,t)\right) - g\left(E_{ov}(n,t)\right)\right], \qquad (7)\end{aligned}$$

since this expresses the additional benefit acquired by the n-th relay node for the transmission of the new packet. This definition deserves the following comments: a) it implies that $a_n > 0$ ($a_n < 0$) corresponds to an actual reward for the node (a damage for it); b) it does not take into account the willingness of the n-th node to spend its residual resources for data transmission (hence, to earn credits for its future needs). To circumvent the last problem, (7) can be generalized as

$$\begin{aligned}\hat{a}_n \triangleq &\left[w_{tx}(n)\cdot f\left(P_{ov}(n,t+1)\right) - w_{en}(n)\cdot g\left(E_{ov}(n,t+1)\right)\right]\\ &-\left[w_{tx}(n)\cdot f\left(P_{ov}(n,t)\right) - w_{en}(n)\cdot g\left(E_{ov}(n,t)\right)\right], \qquad (8)\end{aligned}$$

where the weight $w_{tx}(n)$ ($w_{en}(n)$) measures the willingness of the n-th node to cooperate (to save energy). Note that, since the functions $f(\cdot)$ and $g(\cdot)$ are generic, the real influence of these weights on the payoffs depends on their ratio, i.e. on the parameter

$$K_n \triangleq \frac{w_{tx}(n)}{w_{en}(n)}, \qquad (9)$$

which can be interpreted as a *risk affinity* [7] for the n-th node. A large risk affinity pushes the n-th node to cooperate with the aim of gaining credits to acquire the neighbour's support in the near future; a small risk affinity, instead, can be interpreted as an appreciable energy avidity, which pushes the node to cooperate scarcely and only when its channel conditions are favourable.

Evaluation of c_n - When a set of relays takes part to a transmission attempt without reaching the diversity order R

needed to ensure a correct detection of the forwarded packet, the transmission fails and the energy spent by each involved node is wasted. The payoff c_n assigned to the n-th node for an unsuccessful transmission can be evaluated resorting to the approach described in the previous Paragraph for a_n; the only difference is that, in the new situation, there is no reward deriving from the node action. Therefore, the payoff c_n can be expressed as (see (7))

$$c_n \triangleq g\left(E_{ov}(n,t)\right) - g\left(E_{ov}(n,t+1)\right) \tag{10}$$

or, introducing the above mentioned weights, as

$$\hat{c}_n \triangleq w_{en}\left(n\right) \cdot \left[g\left(E_{ov}(n,t)\right) - g\left(E_{ov}(n,t+1)\right)\right]. \tag{11}$$

Note that the payoff $\hat{c}_n$ (11) is insensitive to the risk affinity factor K_n (9).

2) Payoffs in scenario #2:

In scenario #2, since the nodes aim at contributing to the effectiveness of the network, the concepts of credit exchange and cooperation market is no longer needed. Therefore, it is reasonable to assume that in the t-th slot each node will earn the same reward $\Gamma(t)$ after a correct transmission of the cluster of potential relays which it belongs to, irrespective of its actual contribution. In the following it is assumed that $\Gamma(t) = \gamma$ for a correct transmission and $\Gamma(t) = 0$ for a failure. The value of the parameter γ influences the node behavior and, in particular, its willingness to contribute to packet transmissions (it plays a similar role as the coefficient K_n (9)); in fact, generally speaking, a node will get more favourable to transmitting as γ increases. The cost charged to a node is related to the energy spent for packet transmissions in the same way as in the previous case. Therefore, the distinct payoffs earnable by the n-th node of a cluster are 3 and are associated with the following events (all referring to the $(t+1)$-th slot):

1) Correct transmission of a packet in the presence of a real contribution from the n-th node - Each element of the cluster earns a reward equal to γ for the achievement of this goal, but this payoff is decreased by a quantity proportional to the consumed energy. Then, following the rationale illustrated in Paragraph III-B1, the payoff for the n-th node is defined as[7]

$$\acute{a}_n \triangleq f\left(\gamma\right) - \left(g\left(E_{ov}(n,t+1)\right) - g\left(E_{ov}(n,t)\right)\right). \tag{12}$$

2) Correct transmission of a packet in the absence of a real contribution from the n-th node - In this situation the n-th node does not consume energy, so that its payoff can be defined as

$$\acute{b}_n \triangleq f\left(\gamma\right). \tag{13}$$

3) Transmission failure in the presence of real contribution from the n-th node - In this case the network does not earn any benefit, so that this event represents a resource waste for n-th node. For this reason, the node payoff can be expressed as

$$\acute{c}_n \triangleq -\left(g\left(E_{ov}(n,t+1)\right) - g\left(E_{ov}(n,t)\right)\right). \tag{14}$$

[7]Note that, generally speaking, the functions $f(\cdot)$ and $g(\cdot)$ are different from the those appearing in (6)-(11) and referring to scenario #1. Morover, even in this case, for the sake of simplicity and generality, the precise structure of these functions is not defined.

IV. Strategy Played by the Opponents

As illustrated in the next Section, the derivation of the optimal transmission strategy to be played by the n-th node of a relay cluster in the t-th slot requires an estimate of the probability $\Pr\{N_{-n}(t)\}$. In the following we show how this knowledge can be acquired through a proper learning strategy that takes advantage of the repetitiveness of the game itself; in our derivation it is assumed that the degree of diversity R needed on a wireless link is perfectly known to the potential relay nodes (the effects of an imprecise estimation of R will be analysed in Section VI).

To begin, we note that the probability $\Pr\{N(t)\}$ can be evaluated as

$$\Pr\{N(t)\} = \sum_{m=1}^{L} \Pr\{N(t)|M(t)\} \Pr\{M(t)\}, \tag{15}$$

provided that $\Pr\{M(t)\}$ and $\Pr\{N(t)|M(t)\}$ are available. The probability $\Pr\{M(t)\}$ can be easily estimated through the observation of the last transmission outcomes and adopting a Poisson model[8] for $M(t)$, so that

$$\Pr\{M(t)\} = F(M; \lambda_M(t)), \tag{16}$$

where

$$F(x; \lambda_M(t)) \triangleq \frac{\lambda_M^x(t) e^{-\lambda_M(t)}}{x!}. \tag{17}$$

In Appendix B it is shown that $\lambda_M(t)$ can be computed as

$$\lambda_M(t) = Q^{-1}\left(R, \frac{k(t)}{k(t)+1}\right), \tag{18}$$

where $Q^{-1}(R,\lambda)$ is the inverse of the *regularized upper incomplete gamma function* (see [19, Sect. 2.7]) and

$$k(t) \triangleq \frac{\Pr\{M \geq R\}}{\Pr\{M < R\}} \tag{19}$$

represents the ratio between the probability of a correct packet transmission and that of a transmission failure; note that this parameter represents the only information available to the n-th node about the behavior of its opponents (hence, about their transmission probability P_{tx}) and can be easily estimated from the observation of the last T_0 transmission attempts made by the relay cluster (generally speaking, in a time varying scenario, the optimal value of T_0 depends on the channel coherence time).

The problem of estimating $\Pr\{N(t)|M(t)\}$ is more complicated and is solved resorting to the Bayes theorem [14]; this leads to the estimate

$$\Pr\{N(t)|M(t)\} \simeq \frac{\Pr\{M(t)|N(t)\} \tilde{P}r\{N\}}{\sum_l \Pr\{M(t)|N(t)=l\} \tilde{P}r\{N=l\}}, \tag{20}$$

where $\tilde{P}r\{N\}$ represents the prior knowledge we can acquire about the number of transmitting nodes from what is expected to be the regime in a normal functioning of our system; note that, in this case, prior beliefs acquire an operational significance (see [14, Chap. 5]). In our scenario the developed

[8]This choice is motivated by the fact that this model provides a statistical description of the number of discrete and independent events with known mean [13].

strategies for the management of node participation aim at keeping the number of cooperating nodes close to the one needed to ensure a degree of diversity equal to R and a Gaussian shaped probability mass function $\tilde{P}r\{N\}$ is deemed to be a reasonable model[9]. This function is fully described by two parameters, namely the standard deviation σ_N of $N(t)$ and its mean value $\bar{N}(R)$ (the problem of the evaluation of $\bar{N}(R)$ is tackled in Appendix C). Then, from (20) the estimate

$$\Pr\{N(t)|M(t)\} = \frac{\Pr\{M(t)|N(t)\}\, e^{\left(\frac{-(N-\bar{N}(R))^2}{2\sigma_N^2}\right)}}{\sum_l \Pr\{M(t)|N(t)=l\}\, e^{\left(\frac{-(l-\bar{N}(R))^2}{2\sigma_N^2}\right)}} \quad (21)$$

is easily inferred.

The probability mass function $\Pr\{N(t)\}$ (15) refers to the overall number of nodes $N(t)$ (including the n-th one) involved in a cooperative transmission. However, the random parameter of real interest in our game is the number $N_{-n}(t)$ of opponents only. If we refer to the last T_0 turns of the game, the n-th node contribution can be deemed *decisive* for the cluster it belongs to if the overall number of transmissions n_{TX} accomplished by the node itself is not smaller than $T_0/2$; otherwise it is *marginal*. For this reason the estimate

$$\Pr\{N_{-n}(t)\} = \begin{cases} \Pr\{N(t) = N_{-n}(t)+1\} & \text{if } n_{TX} \geq T_0/2 \\ \Pr\{N(t) = N_{-n}(t)\} & \text{otherwise} \end{cases} \quad (22)$$

is adopted. Thanks to the hypothesis of stationarity, this result can be deemed stable in the time interval of a few consecutive game turns; therefore, it can be exploited to estimate the number of opponents that will transmit in the next repetition of the game.

V. Derivation of the Node Participation Strategy

A. Strategy based on the estimation of the overall number of cooperating nodes

Generally speaking, whatever the scenario, the goal of the n-th node is to adjust its transmission probability $P_{tx}(n,t)$ in the t-th slot in order to maximise its mean expected payoff (3). In the following, however, we retain the dependency on $N_{-n}(t)$, so that (2) is exploited in place of (3); in fact, this dependency will allow us to perform a continuous adaptation of the transmission probability of the node itself. Therefore the goal of the n-th node becomes the maximization of the ratio between the positive and the negative areas of the weighted expected payoff curve of (2). To achieve this target, starting from an initial strategy that establishes a transmission probability equal to $P_{tx}(n, t=0)$, the node is induced to adapt its transmission probability, in order to contribute to modify the probability $\Pr\{N(t)\}$, which currently describes the global strategy played by the relay stage. This approach relies on the assumption that the weighted expected payoff curve does not substantially differ from node to node[10], so that all the potential relays are expected to favour a modification of their strategies with the common aim of increasing or decreasing the population of active transmitters. For this reason, it is advisable for the n-th node to contribute to a change of $\Pr\{N(t)\}$ proportionally to the gain it expects from this variation; this suggests to update its transmission probability as (see (3))

$$P_{tx}(n,t) = P_{tx}(n,t-1) + \beta\left(\sum_{N_{-n}\in C_1} |\alpha(N_{-n}(t))| - \sum_{N_{-n}\in C_2} |\alpha(N_{-n}(t))|\right). \quad (23)$$

Here C_1 represent the set $\{N_{-n}(t) < \bar{N}(R) | \alpha(N_{-n}(t)) < 0\}$ (a cluster having one of these sizes is likely not to reach the needed diversity order), C_2 represents the set $\{N_{-n}(t) > \bar{N}(R) | \alpha(N_{-n}(t)) < 0\}$ (the negative metric is due to the excessive number of cooperating nodes sharing the payoff) and β is a step size.

The rationale behind (23) is that the payoff earnable by the n-th node is positive within a certain (and dependent on its subjective channel experience) range of $N(t)$ around $\bar{N}(R)$; the values of $N(t)$ in this range allow the cluster to reach with high probability the needed degree of diversity and, at the same time, the n-th node to earn a fraction of the credits at disposal sufficient to cover the consumed energy. Therefore, given an imperfect estimate of $N(t)$, the goal of each node is to maximize its mean achievable payoff or, from a different perspective, to minimize the mean negative payoff that could arise from its transmission. In fact, the transmission probability in (23) is adjusted in a way to minimize the probability of the occurrence of the cases producing a negative value of the payoff (2) for the given node.

Finally, the proposed participation strategy for the n-th node can be summarized in the following steps:

1) Computation of the expected payoff resulting from a transmission attempt of the n-th node for all the possible values of $N_{-n}(t)$. This requires: a) Evaluating the payoffs a_n and c_n ($\acute{a}_n$, $\acute{b}_n$ and $\acute{c}_n$) via (8) and (11) ((12), (13) and (14)) in scenario #1 (scenario #2); b) Evaluating the expected payoff or the differential payoff for a transmission attempt; this can be done combining the game payoffs according to Table I (for scenario #1) or Table II (for scenario #2) (see (1) or (4), respectively).
2) Estimation of the number of nodes transmitting in the following turn. This requires: a) Tracking the last T_0 transmission outcomes; b) Computing the probability $\Pr\{M(t)\}$ (16); c) Estimating the probability $\Pr\{N(t)|M(t)\}$ via (20); d) Estimating the average number of nodes participating to the last transmissions via (15) and (22).
3) Computation of the metric (2) representing the expected payoff weighted by the probability mass function of the number of transmitting nodes.
4) Derivation of the node participation strategy based on (23).

B. Strategy based on the estimation of the overall number of selected codewords

In scenario #2 a different solution for the game can be developed taking as unknown parameter the degree of diversity

[9]This choice is motivated by the fact that this model describes a situation of maximum uncertainty about this random parameter [13].

[10]The weighted expected payoff curves of distinct nodes can exhibit only different vertical offsets due to the difference in the experienced channels.

TABLE III
REPRESENTATION OF THE PARTICIPATION GAME BASED ON THE NUMBER OF SELECTED CODEWORDS FOR THE n-TH NODE IN SCENARIO #2

	$M_{-n} \leq R-2$	$M_{-n} = R-1$	$M_{-n} \geq R$
TX	$\acute{c}_n$	$P_{(R-1)\to R}\, \acute{a}_n + (1 - P_{(R-1)\to R})\, \acute{c}_n$	$\acute{a}_n$
$NO\text{-}TX$	0	0	$\acute{b}_n$

achieved by the involved link instead of the number of transmitting nodes. This approach substantially simplifies the game model at the price of loosing some details, like the codeword selection procedure. Its adoption is made possible by the nature of the payoffs employed in scenario #2; in fact these quantities are not related to the actual number of transmitting nodes, so that this parameter can be neglected.

The new model of the game is described in normal form by Table III, where $M_{-n}(t)$ denotes the number of codewords selected by at least one transmitting node, other than the n-th one. All the payoffs appearing in this table are defined except that referring to the event of transmission with $\{M_{-n}(t) = R-1\}$; in this case the participation of the considered node can be decisive to make the cluster transmission successful, provided that it does not select a codeword already used by other cluster nodes; this occurs with probability

$$P_{(R-1)\to R} = \frac{(L-(R-1))}{L}. \tag{24}$$

Given this result, a transmission strategy for the n-th node can be derived adopting a standard approach (see [7], [17, Pag. 52]). Therefore, the n-th node transmits a data packet if the expected payoff coming from this action is larger than that it can earn if it remains silent. These expected payoffs are given by

$$\begin{aligned} E(TX) &= \acute{c}_n \Pr\{M_{-n}(t) \leq R-2\} \\ &\quad + \acute{a}_n \Pr\{M_{-n}(t) \geq R\} \\ &\quad + \begin{pmatrix} \acute{a}_n P_{(R-1)\to R} + \\ \acute{c}_n (1 - P_{(R-1)\to R}) \end{pmatrix} \Pr\{M_{-n}(t) = R-1\} \end{aligned} \tag{25}$$

for the case of transmission and

$$E(NO\_TX) = \acute{b}_n \Pr\{M_{-n}(t) \geq R\} \tag{26}$$

in the opposite case. At the equilibrium point these two quantities have to be equal, hence $E(TX) - E(NO\_TX) = 0$. Substituting (25) and (26) in the last equality and taking into account that $M_{-n}(t) \in [1, L]$ and that $\Pr\{M_{-n}(t) \leq R-2\} + \Pr\{M_{-n}(t) = R-1\} + \Pr\{M_{-n}(t) \geq R\} = 1$ yields, after some manipulation,

$$\acute{a}_n P_{(R-1)\to R} \Pr\{M_{-n}(t) = R-1\} + \acute{c}_n = 0. \tag{27}$$

Therefore, the equilibrium point for the game is characterized by the probability

$$\Pr\{M_{-n}(t) = R-1\} = -\frac{\acute{c}_n}{\acute{a}_n P_{(R-1)\to R}}. \tag{28}$$

The n-th node strategy is related to what it can infer from the current actions of its opponents. An estimate $\hat{\Pr}\{M_{-n}(t) = R-1\}$ of the probability $\Pr\{M_{-n}(t) = R-1\}$ can be easily obtained from the observation of the last T_0 transmission attempts and applying the same rationale of (22) to (16). In principle, if the expected differential payoff between the choice of transmitting and that of remaining silent

$$D \triangleq E(TX) - E(NO\_TX) \tag{29}$$

is defined, the *best response* for the n-th node is represented by cooperating if $D > 0$ (hence, if $\hat{\Pr}\{M_{-n}(t) = R-1\} > \frac{-\acute{c}_n}{|\acute{a}_n| P_{(R-1)\to R}}$) and remaining silent if $D < 0$ (so, if $\hat{\Pr}\{M_{-n}(t) = R-1\} < \frac{-\acute{c}_n}{|\acute{a}_n| P_{(R-1)\to R}}$). This simple strategy can be directly played by the node even if, in order to avoid a discontinuous behavior of the players, the adoption of a stochastic version of this solution is typically recommended; this can be defined by means of a *smoothed best response* as

$$\Pr(TX) = \begin{cases} 1 - \frac{e^{-\gamma D}}{2} & \text{if } \hat{\Pr}\{M_{-n}(t) = R-1\} > \frac{-\acute{c}_n}{|\acute{a}_n| P_{(R-1)\to R}} \\ \frac{e^{\gamma D}}{2} & \text{if } \hat{\Pr}\{M_{-n}(t) = R-1\} \leq \frac{-\acute{c}_n}{|\acute{a}_n| P_{(R-1)\to R}} \end{cases}, \tag{30}$$

which represents two exponential curves connecting at the indifference point [18]; note that γ is a constant useful to adjust the continuous approximation to the discrete response function.

Briefly, the proposed participation strategy for the n-th node consists of the following steps:

1) Computation of the coefficients $\acute{a}_n$, $\acute{b}_n$ and $\acute{c}_n$ using eqs. (12), (13) and (14) respectively and of the probability $P_{(R-1)\to R}$ (24).
2) Estimation of the number of different codewords that will be selected by at least one node in the next turn. This operation requires: a) Tracking the last T_0 transmission outcomes; b) Generating the estimate $\hat{\Pr}\{M_{-n}(t) = R-1\}$.
3) Computation of the probability $\Pr\{M_{-n}(t) = R-1\}$ (28) at the equilibrium point of the game.
4) Computation of the node participation strategy expressed by (30).

VI. Numerical Results

The performance of the proposed transmission strategies has been assessed via computer simulations. It has been assumed that:

1) The game repetition is run by the packet transmission of a given source node or, in the case of a failed transmission attempt, by the NAK signalling coming from the destination node.
2) Each link between a couple of nodes is affected by time-selective Rayleigh fading (the well known Jakes'model has been adopted with normalized Doppler bandwidth $B_d T_u$), which, however, can be deemed static during the transmission of each packet. Distinct wireless links are affected by statistically independent fading.
3) The empirical choices $T_0 = 5$, (see eq. (19)), $\beta = 1/5$ (see eq. (23)) and $\gamma = 1/10$ (see eq. (30)) have been made.

4) The linear models $f(x) = x$ and $g(x) = kx$ have been adopted for the functions introduced in Paragraph III-B. The value selected for the parameter k in our simulations ensures that the two terms appearing in the right-hand side of (6) range over similar intervals and, consequently, influence the payoffs in a comparable fashion.
5) The adopted DR-OSTC is characterized by $L = 15$ and a packet transmission is deemed correct if the involved relay stage reaches a minimum degree of diversity $R = 6$ (all the nodes are supposed to be aware of this value when not differently stated).
6) The transmission power of the source node can be adjusted to reach, on the average, a variable number of potential relays. More precisely, assuming a population of 40 potential relays, the power levels $P_{src} = 1$, 2 and 3 allow the source to reach an average number of 15, 30 and 40 nodes, respectively. On the contrary, the transmission power of the relay nodes is inversely proportional to the channel attenuation (see (31)).

The different strategies we propose are 3; the first (second) one, denoted *strat.* #1 (*strat.* #2) in the following, is based on the estimation of the number of cooperating nodes in scenario #1 (scenario #2); the third strategy (*strat.* #3), instead, is based on the estimation of the number of selected codewords in scenario #2. The results acquired for these three strategies have been compared with those generated for the trivial approach to relay node management proposed in [6] (and dubbed *always transmit*, AT, in the following); in this approach all the nodes, which are able to correctly decode the packets to be relayed, always forward them towards the destination.

Following [7], numerical simulations have been carried out to assess, for each transmission strategy, the following quantities: 1) its *average throughput*

$$th \triangleq \frac{N(TX)}{T},$$

where $N(TX)$ is the number of packets correctly transmitted by the whole relay stage in the considered T consecutive time units; 2) its *energy efficiency* eff, defined as the mean value, accomplished over the whole set of relay nodes, of the ratio between the overall number of transmission attempts (independently of their success or failure) and the energy spent by the nodes themselves; in other words,

$$eff \triangleq E_{n \in CL}\left(\frac{N(TX_{all}(n))}{E(n,\bar{t})}\right),$$

where n is the index selecting the node in the cluster CL of potential relays, $N(TX_{all}(n))$ the number of transmission attempts accomplished by the n-th node and $E(n,\bar{t})$ the overall energy spent by the same node until the link closure at the end of the $\bar{t}$-th slot; this energy is evaluated as

$$E(n,\bar{t}) = \sum_{t=0}^{\bar{t}} \frac{1}{|h_{n,t}|^2}, \quad (31)$$

where $|h_{n,t}|^2$ is the complex channel gain experienced by the n-th node towards the destination over the t-th time slot. It is worth noting that the energy efficiency allows to assess the ability of a transmission strategy in exploiting both a limited number of active relays and the best channels within all those available at the potential relays.

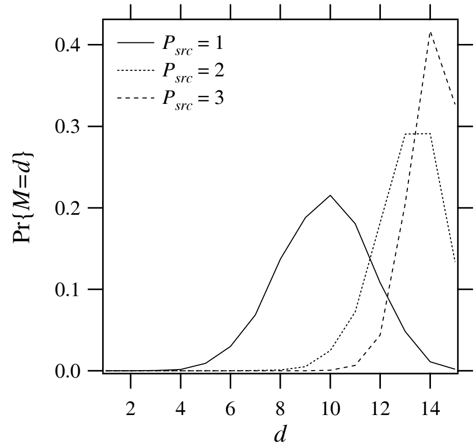

Fig. 1. Estimated probability mass function of the diversity degree achieved by packet transmissions over a link managed in the absence of a control of the cooperating nodes (AT strategy).

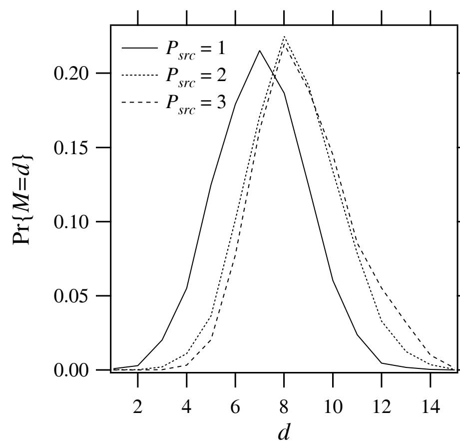

Fig. 2. Estimated probability mass function of the diversity degree achieved by packet transmissions over a link managed in the presence of a control of the cooperating nodes.

Before analysing some performance results for the proposed strategies, it is interesting to gain an insight on how efficiently they can manage the node participation to packet transmissions. Figs. 1 and 2 illustrate the estimated probability mass function of the degree of diversity reached on a specific link managed by the AT solution and by strat. #1, respectively[11], when $B_d T_u = 10^{-3}$. The results shown in Fig. 1 evidence that, if the AT strategy is adopted, the number of active relays can become definitely too large (so worsening the network efficiency) with respect to the link needs since such a number appreciably depends on the source transmission power. On the contrary, strat. #1 is able to control the number of transmitting relays, making it virtually independent on the number of nodes able to decode the source transmission (see Fig. 2) and avoiding an energy waste associated with an excessive number of transmissions.

[11] Simulation results referring to strat. #2 and strat. #3 have been omitted because they are similar to the ones referring to strat. #1.

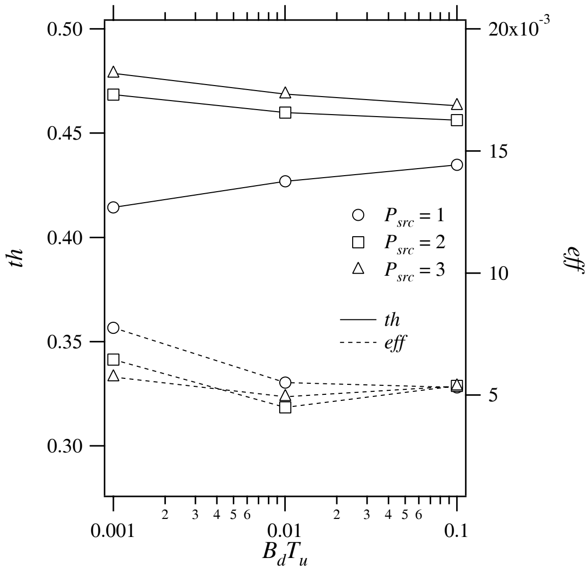

Fig. 3. Average throughput and average energy efficiency provided by strat. #1.

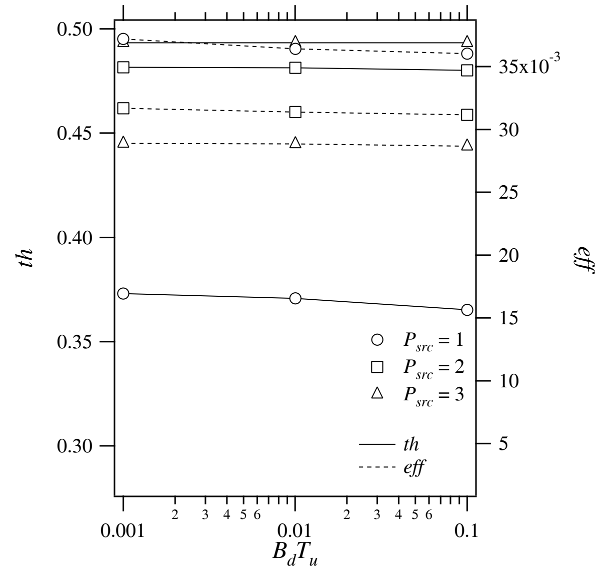

Fig. 5. Average throughput and average energy efficiency provided by strat. #3.

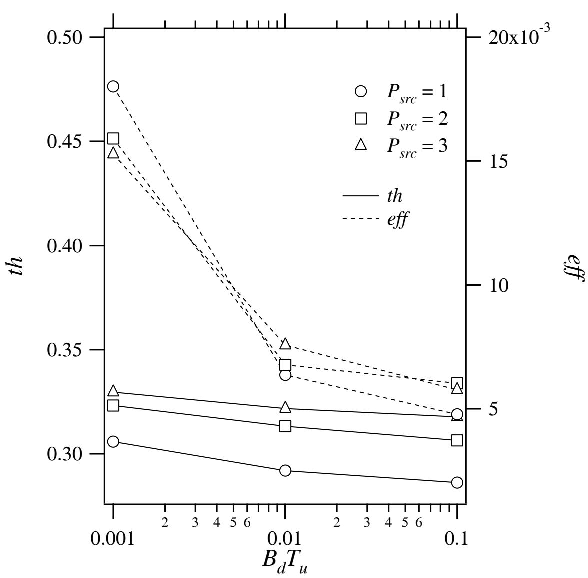

Fig. 4. Average throughput and average energy efficiency provided by strat. #2.

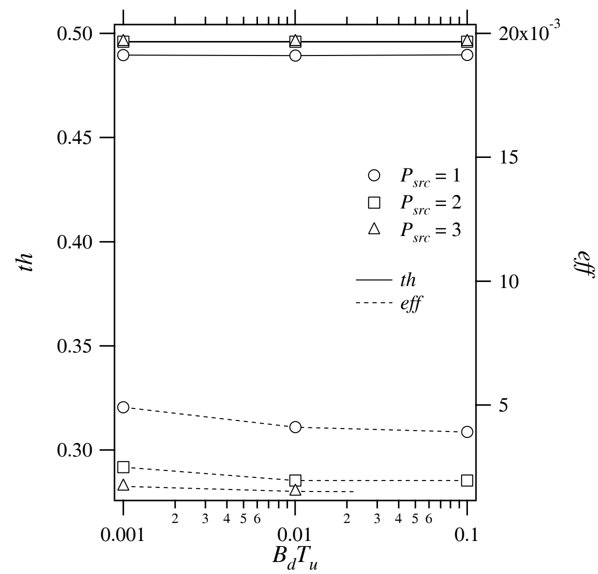

Fig. 6. Average throughput and average energy efficiency provided by the AT strategy.

Numerical results concerning the achievable throughput and the energy efficiency versus the normalized Doppler bandwidth B_dT_u of the channel are illustrated in Figs. 3, 4, 5 and 6 for strat. #1, strat. #2, strat. #3 and AT respectively. The results about the achievable throughput show that strat. #2 is outperformed by the other two proposed strategies. Strat. #1 and strat. #3 offer a throughput similar to that provided by the AT strategy which can be taken as an upper bound for the throughput achievable on the considered relay link. If the energy efficiency is taken into consideration, the results are reversed and the performance offered by strat. #2 is slightly better that of strat. #1. However, both are outperformed by strat. #3, whereas the AT strategy shows a very low efficiency and, once more, exhibits a performance strongly dependent on the number of nodes able to decode the source transmission. Therefore, these results evidence that: a) the proposed strategies can achieve a superior energy efficiency at the price of a slight decrease in the achievable throughput with respect to a link management ignoring the number of transmitting nodes; b) these strategies tend to achieve better performance in the presence a large number of potential relay nodes, i.e. when the AT strategy becomes less efficient.

The performance difference between strat. #1 and strat. #2 can be explained as follows. In the scenario considered for the development of strat. #1, a node can expect a positive payoff from a transmission attempt only if it contributes to a correct transmission accomplished by a cluster; for this reason, even if a large number of nodes within the cluster decides to transmit, a node experiencing a good channel will be prone to transmit too in order to earn a positive (even if low) payoff. On the contrary, in the scenario referring to strat. #2, each node earns a fixed payoff irrespective of its contribution in

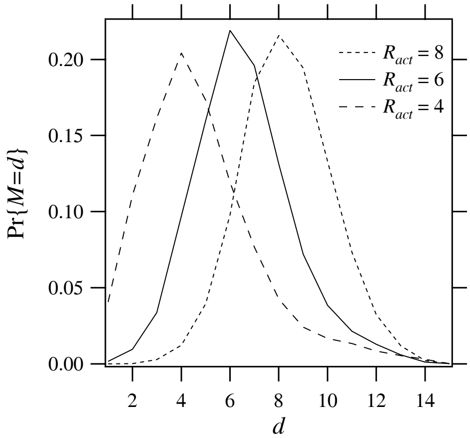

Fig. 7. Estimated probability mass function of the diversity degree achieved by a link managed in the presence of a control of the involved nodes when $R = 6$ and $R_{act} = 4, 6, 8$.

a cluster transmission; therefore it will be less favourable to transmit when it deems that the remaining nodes are able to accomplish the transmission without its contribution, so that an useless energy expense is avoided. This results in a superior energy efficiency of strat. #2, especially for slowly varying channels, at the price of a mean throughput reduction.

It is also worth pointing out that, in principle, strat. #3 cannot be directly compared with strat. #1 and strat. #2, since it is based on a completely different rationale. Despite this, its superior performance can be motivated by its promptness to respond to the system state estimated by each node; in this case, in fact, the best response can be instantaneously played, whereas in strat. #1 and strat. #2 is approached through a continuous adjustment of the transmission probability.

The performance offered by the proposed strategies in the presence of an *imperfect knowledge* of the degree of diversity R needed on a cooperative link has been also assessed. Fig. 7 illustrates the estimated probability mass function of the degree of diversity achieved by a link when the nodes establishing it compute their strategy under the assumption that $R = 6$, but the degree of diversity actually needed (denoted R_{act} in the figure) is 4, 6 or 8. The results referring to the two scenarios characterized by $R_{act} \neq R$ evidence that the proposed approach allows a given cluster to reach a degree of diversity close to that really needed. The low impact of the estimation error represent an interesting property of the proposed solutions. In fact, it allows the cluster behavior to follow the changing needs of a wireless link so that its efficiency is always maximised in the presence of channel variations.

VII. Conclusions

In this paper the problem of node management in cooperative data transmissions based on a DR-OSTC scheme has been investigated. Some solutions have been developed resorting to various tools provided by game theory. The devised strategies for node management are *fully distributed*, since are characterized by autonomous choices made by each potential relay node between the following two simple alternatives: transmitting a data packet (so actively contributing to a cooperative link) or remaining silent (leaving the transmission burden to the neighbours). This choice is based on the status of the involved node (hence, on the amount of its available resources and on its experienced channel conditions) and on the behavior of the other nodes. This allows to coordinate the transmissions among the potential relays *without any explicit information exchange among them*, so avoiding the drawback of the transmission overhead that usually characterizes cooperative transmission protocols. The proposed solutions are of significant practical interest since they allow, on the one hand, to guarantee the participation of a proper number of nodes to a transmission cluster (hence avoiding an energy waste associated with an excessive number of transmissions) and, on the other hand, to avoid a considerable throughput decrease with respect to unmanaged solutions.

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Appendix A

The evaluation of the payoffs listed in Tables I-II requires the knowledge of probabilities of the events $\{M(t) \geq R\}$ and $\{M(t) < R\}$, conditioned on $N(t)$. These probabilities can be evaluated as

$$\Pr\{M(t) < R|N(t)\} = \sum_{m=1}^{R-1} \Pr\{M(t) = m|N(t)\} \quad (32)$$

and

$$\Pr\{M(t) \geq R|N(t)\} = \sum_{m=R}^{L} \Pr\{M(t) = m|N(t)\}. \quad (33)$$

If L distinct codewords are available and a *randomic codeword selection* is assumed, the probability $\Pr\{M(t)|N(t)\}$ is given by (see [13, Pag. 710])

$$\Pr\{M(t)|N(t)\} = \binom{L}{L-M(t)} \sum_{r=1}^{M(t)} \left[\left(\frac{r}{L}\right)^{N(t)} \binom{M(t)}{r} (-1)^{M(t)-r} \right] \quad (34)$$

if $M(t) \leq N(t)$ or $\Pr\{M(t)|N(t)\} = 0$ otherwise.

Appendix B

Let us rewrite (19) as

$$\Pr(M(t) < R) = k \Pr(M(t) \geq R), \quad (35)$$

from which it is easily inferred that (see (16))

$$\sum_{x=0}^{R-1} F(x; \lambda_M(t)) = k \sum_{x=R}^{\min(L,N(t))} F(x; \lambda_M(t)), \quad (36)$$

where $\min(L, N(t))$ denotes the number of distinct codewords that can be actually selected by the nodes of the relay cluster. Since $\sum_{x=0}^{\min(L,N(t))} F(x;\lambda) = 1$, (36) leads to

$$\sum_{x=0}^{R-1} F(x;\lambda_M(t)) = \frac{k}{k+1}, \quad (37)$$

which can be rewritten as

$$Q(R,\lambda_M(1)) = \frac{k}{k+1}. \quad (38)$$

Finally, inverting (38) produces (18).

Appendix C

In our approach the mean number of nodes $\bar{N}(R)$ is identified by the value of $N(t)$ maximizing the probability of having R distinct codewords in a cooperative transmission; in other words this parameter it solution of the equation

$$\frac{\partial \Pr\{M = R|N\}}{\partial N} = 0. \quad (39)$$

To solve this problem, an approximate version of (34), originally developed by R. von Mises [15], can be exploited; then $\Pr\{M|N\}$ can be expressed as

$$\Pr\{M|N\} = \frac{\lambda^{(L-M)}}{(L-N)!}e^{-\lambda} \quad (40)$$

with

$$\lambda \triangleq Le^{-\frac{N}{L}}. \quad (41)$$

The expression (40) has been derived for $N, L \to \infty$; despite this, the errors introduced by its use can be deemed acceptable even for small value of these parameters. Thus, substituting (40) in (39) yields

$$\frac{\left(\begin{array}{c} e^{-\frac{N}{L}-Le^{-\frac{N}{L}}}\left(Le^{-\frac{N}{L}}\right)^{L-R} \\ -e^{-\frac{N}{L}-Le^{-\frac{N}{L}}}\left(Le^{-\frac{N}{L}}\right)^{L-R-1}(L-R) \end{array}\right)}{(L-R)!} = 0. \quad (42)$$

Solving (42) in the variable N leads to

$$\bar{N}(R) = -L\log\left(\frac{1}{(L-R)!} - \frac{R}{L(L-R)!}\right) - L\log((L-R)!). \quad (43)$$

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Simone Sergi was born in Modena, Italy, in April 1982. He received the Dr. Ing. Degree in Telecommunication Engineering in 2006 and the Ph. D. degree in 2010, both from the University of Modena and Reggio Emilia, Italy. His main research interests lie in the areas of cooperative communication techniques for wireless ad hoc and sensor networks, with particular emphasis on the development of distributed protocol for the organization and exploitation of virtual antenna array, resource management and distributed signal processing.

Giorgio M. Vitetta (S'89, M'91, SM'99) was born in Reggio Calabria, Italy, in April 1966. He received the Dr. Ing. Degree in Electronic Engineering (cum Laude) in 1990 and the Ph. D. degree in 1994, both from the University of Pisa, Italy. In 1992/1993 he spent a period at the University of Canterbury, Christchurch, New Zealand, doing research for digital communications on fading channels. From 1995 to 1998 he was a Research Fellow at the Department of Information Engineering of the University of Pisa. From 1998 to 2001 he has been holding the position of Associate Professor of Telecommunications at the University of Modena and Reggio Emilia. He is now Full Professor of Telecommunications in the same university. His main research interests lie in the broad area of communication theory, with particular emphasis on coded modulation, synchronization, statistical modeling of wireless channels and channel equalization. He is serving as an editor of both the IEEE Transactions on Communications (editor for Channel Models and Equalization in the area of Transmission Systems), and the IEEE Transactions on Wireless Communications (in the area of Transmission Systems).