

Game Theory

Department of Electronics

EL-766

Spring 2011

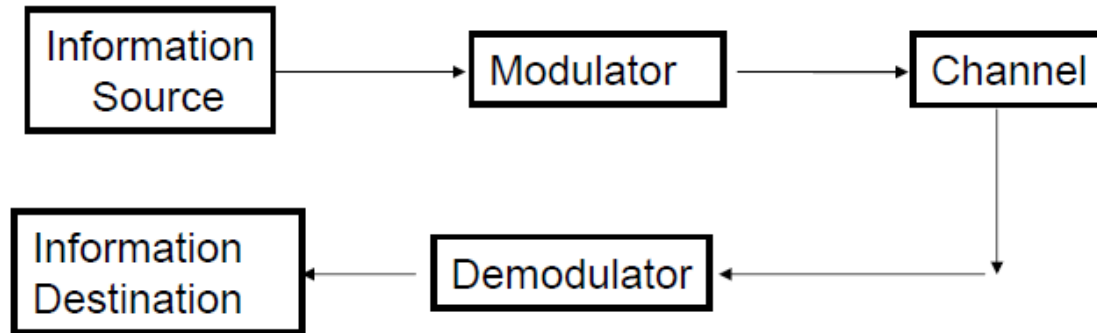
Lecture 9

April 7, 2011

Wireless Communications

- “Wireless Communications: Principles and Practice”,
T.S. Rappaport, December 2001, Prentice Hall

Wireless Communication: Simple Model



Assume information source is digital: generates a string of bits that must be transmitted using electromagnetic waves (no wires)

- modulates a carrier

- sinusoidal signals – suitable carriers, $A\sin(2\pi ft + \theta)$

characterized by

- amplitude: amplitude modulation

- frequency: frequency modulation

- phase: phase modulation

Example: BPSK: $s_0(t) = A\sin(2\pi f_c t + \pi) = -A \sin(2\pi f_c t)$, $0 \leq t \leq T$

$s_1(t) = A\sin(2\pi f_c t)$, $0 \leq t \leq T$

Different Modulation Methods

- QPSK: modulate both the sine and cosine (the quadrature) carrier

$$A_1 \sin(2\pi ft + \theta) + A_2 \cos(2\pi ft + \theta)$$

- Better spectral efficiency:

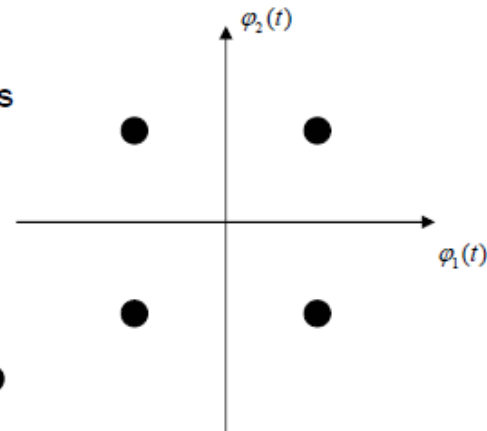
$$\text{Spectral Efficiency} = \frac{\text{Bit rate}}{\text{Transmission Bandwidth}} \quad (\text{bps} / \text{Hz})$$

- Can you improve further the spectral efficiency?
 - M-ary modulation
 - Example M - QAM $A_1, A_2 = \pm 1, \pm 3, \dots, \pm \sqrt{M} - 1$
 - $\log_2 M$ bits encoded into one symbol
 - Large M – higher rates!!!
 - Question: can we get unlimited high rates for a given bandwidth by increasing M ?

Signal Constellation and Detection

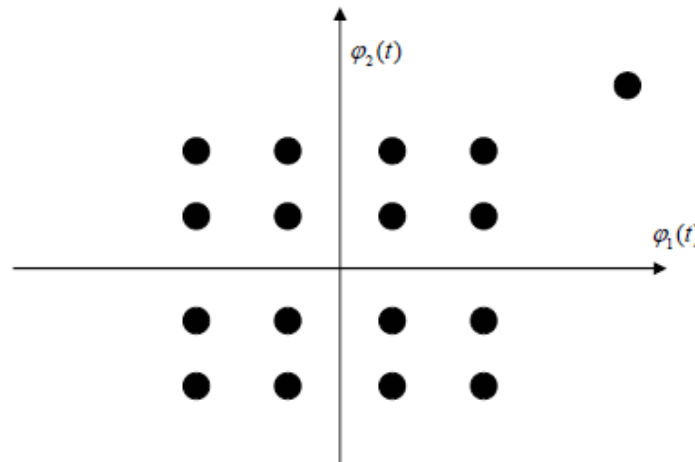
- 4-QAM: $A_1 \sin(2\pi ft + \theta) + A_2 \cos(2\pi ft + \theta) = A_1\varphi_1(t) + A_2\varphi_2(t)$

$$\left\{ \begin{array}{ll} A_1 = \pm 1 & \varphi_1 = \sin(2\pi ft) \\ A_2 = \pm 1 & \varphi_2 = \cos(2\pi ft) \end{array} \right\} \text{ bases functions}$$



- 16-QAM

$$\left\{ \begin{array}{l} A_1 = \pm 1, \pm 3 \\ A_2 = \pm 1, \pm 3 \end{array} \right.$$



- Higher constellation, less room for errors
- Problem: channel introduces noise, fading, distortions

The Physical Channel

- Higher-order (M-ary) → increased spectral efficiency
- Rate of **reliable** data transmission
 - limited by impairments due to physical properties of the channel:
 - **noise** (receiver & background)
 - **path losses** (spatial diffusion & shadowing)
 - **multipath** (fading & dispersion)
 - **interference** (multiple-access & co-channel)
 - **dynamism** (mobility, random-access & bursty traffic)
 - **limited transmitter power**

Noise

- **Noise** present in all communication systems.
 - **White** Gaussian noise:
 - **spectrum constant for all frequencies**
 - pdf is Gaussian
 - Key parameter of noise:
 - zero mean
 - spectral height $N_0/2 = \sigma^2$ (variance of the noise)
 - Key performance parameter when no interference is present:
 - **SNR = E_b/N_0** (signal-to-noise ratio) (E_b = received energy per bit)
 - Determines **BER** (bit error rate)
 - Different BER for different modulation types

Propagation Models

- Two basic types of propagation effects:
 - **Large-scale** (spatial diffusion & shadow fading)
 - **Small-scale** (multipath fading)
- Propagation in free space: ignores any interactions
 - Antenna radiates a sine wave with the carrier frequency

$$f = \frac{c}{\lambda} \quad c = 3 \times 10^8 \quad \text{speed of light}$$

- Friis free space equation:

$$P_r = P_t \left(\frac{\lambda}{4\pi r} \right)^2 g_t g_r$$

P_t = transmit power

P_r = received power

g_t, g_r = transmit/receive antenna gains

r = distance between the antennas

Attenuation

Propagation along the earth's surface: 2-ray model

- Flat earth assumption
- Ground wave reflected
 - Delay
 - Phase shift
 - Attenuation

$$P_r = P_t \left(\frac{h_t h_r}{r^2} \right)^2 g_t g_r$$

Approximation path loss model with n = path loss coefficient:

$$P_r = P_t g_t g_r \frac{const}{r^n}$$

-for omnidirectional antennas: $g_t = g_r = 1$

Large and Small Time Scale Fading

Fading effects - different at different time scales

- the instantaneous signal envelope (**short time scales** (ms)) is
 - **Rayleigh** distributed (NLOS)
 - **Rice** distributed (LOS)
- the mean value of the Rayleigh (or Rice) distribution can be considered a constant for the shorter time scales, but in fact it is a **random variable** with a **lognormal** distribution (**large time scales** (seconds))
 - caused by the changes in scenery (occur on a larger time scale)
- the mean of the Lognormal distribution varies with the distance from the transmitter according to the path loss law
 - If the mobile moves away or towards the transmitter (e.g. base station) the received signal will also vary in time, according to the appropriate power law loss model (e.g. free space: decreases proportional with the square of the distance, etc.)

Sharing the Spectrum: Multi-user Communications

Multiple users' transmissions interfere with each other

- Users need to be separated
 - Frequency (FDMA)
 - Time (TDMA)
 - Using different codes (CDMA)
 - In space (spatial separation in cellular and ad hoc networks)
- Performance measure: BER (Bit error rate)
 - characteristic for the type of service (e.g. req. voice BER $\cong 10^{-2}$).
 - Can be mapped into a SIR (SINR) (Signal to Interference plus noise ratio) requirement

Network architecture: Users may all transmit to the same access point (cellular, wireless LAN), or they can use peer-to-peer communication (ad hoc network)

- no wires → soft link concept – depends on the reception quality

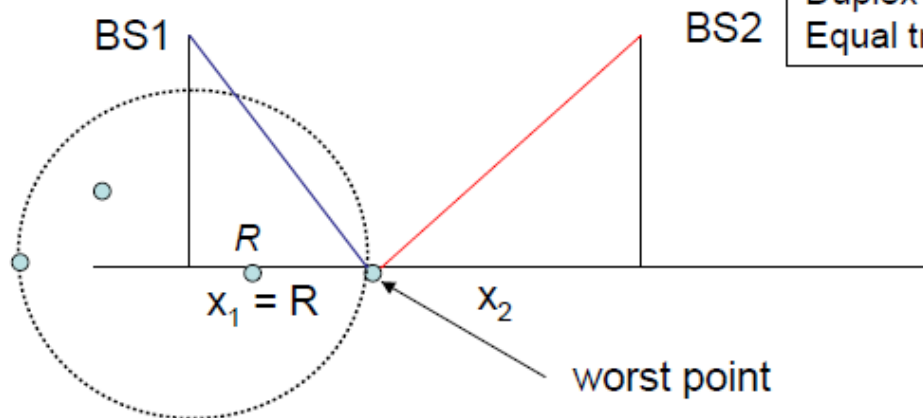
Interference Example: Channel Reuse

- How should we design a cellular system
 - One base station (BS), high power and large coverage?
 - Split into cells to accommodate a larger density of users?

Example: 2 BS use the same channels and are situated at distance D
Question: how should we choose D/R (R is the coverage radius for one cell,
Such that all users meet their target SIR (signal-to-interference ratio)
- SIR maps the bit error rate performance (BER)

- Example – continuation

Assume noise is 0
 Channel impairment is determined by other users using the same channel
 Duplex communication
 Equal transmission powers



$D = x_1 + x_2$
 $T = \text{target SIR in dB}$

$$\left. \begin{aligned} \overline{P_S} &= \frac{\text{const}}{x_1^n} P_t \\ \overline{P_I} &= \frac{\text{const}}{x_2^n} P_t \end{aligned} \right\} \Rightarrow \overline{SIR} = \frac{\overline{P_S}}{\overline{P_I}} = \left(\frac{x_2}{x_1} \right)^n = \left(\frac{D - R}{R} \right)^n$$

$$\overline{SIR}_{dB} = 10 \log_{10} \overline{SIR} = 10n \log_{10} \left(\frac{x_2}{x_1} \right) \geq T$$

- Example – continuation

$$\frac{x_2}{x_1} \geq 10^{\frac{T}{10^n}} \Rightarrow \frac{D}{R} \geq 1 + 10^{\frac{T}{10^n}} \rightarrow \text{want small or big number?}$$

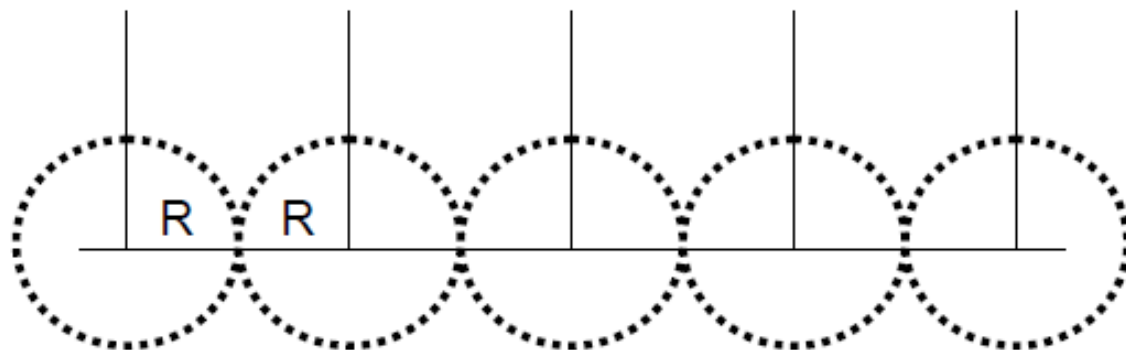
From cellular efficiency point of view -> want small numbers

The effective number of channels/cell = total number of channels/ cell reuse factor (N)

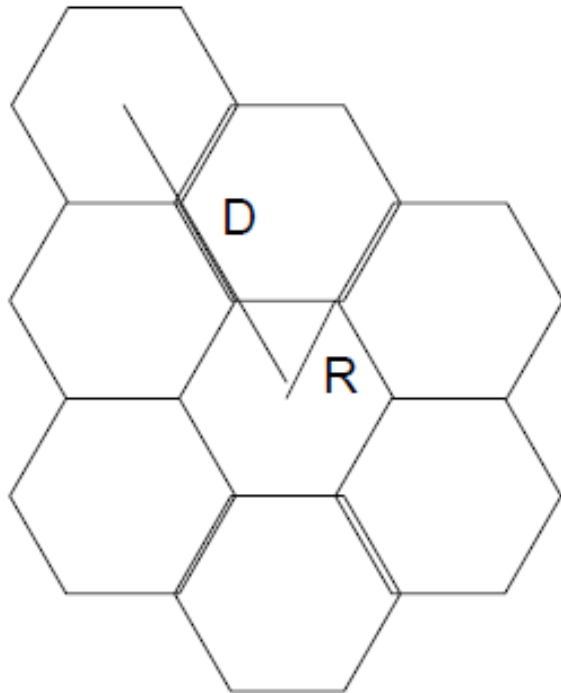
How to determine N for given SIR requirement ?

For base stations situated in a straight line, it can be shown that

$$N = \frac{1}{2} \frac{D}{R}$$



- For hexagonal cells



$$N = \frac{1}{3} \left(\frac{D}{R} \right)^2$$

For CDMA systems: $N=1$

We can define the cellular efficiency:

$$\eta = \frac{B_s}{B_c N} \text{ channels/cell}$$

B_s = spectrum allocated to the cellular system

B_c = bandwidth/channel

N = channel reuse

Sharing the spectrum: CDMA

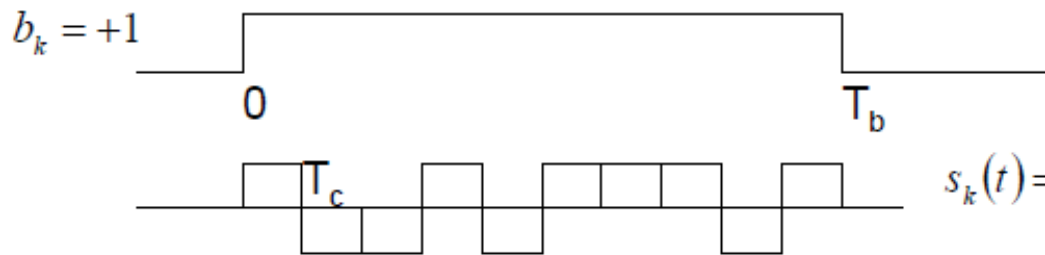
- Basic CDMA principle: all users transmit simultaneously using the same frequency band and are characterized by different **signature sequences codes** $\mathbf{s}_i, i=1,2,\dots, K$ (K = number of users)
 - Signature codes can be selected to be **orthogonal**: users are completely separated from each other $\mathbf{s}_i^T \mathbf{s}_j = 0; i \neq j$ $\mathbf{s}_i^T \mathbf{s}_i = 1$

Disadvantages:

- number of users that can be supported in the system is limited by the number of orthogonal codes
 - If length of code is N , $\max\{K\} = N$
- Orthogonality cannot be maintained for asynchronous transmission
- Codes can also be selected **non-orthogonal** but with small cross-correlations
 - Random codes – all entries are -1 or 1 with equal probability (coin flips)
 - Pseudo-random codes (IS-95 cellular CDMA) – m-sequences
 - very long sequences cyclically repeated (generated by linear shift registers) – appear as random
 - different statistical properties than random codes

Simple single user CDMA system

$$b_k(t) = b_k \times p_T(t) \quad \text{- bit waveform}$$



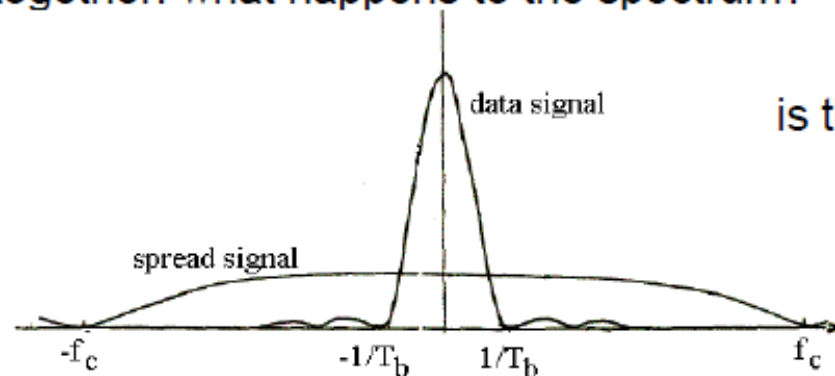
$$N = \frac{T_b}{T_c} = \text{spreading gain}$$

$$s_k(t) = \frac{1}{\sqrt{N}} \sum_{j=1}^N s_{ij} p_{T_c}(t - (j-1)T_c)$$

- signature sequence waveform

-signature sequence code: $s = [+1, -1, -1, +1, -1, +1, +1, +1, -1, +1]$

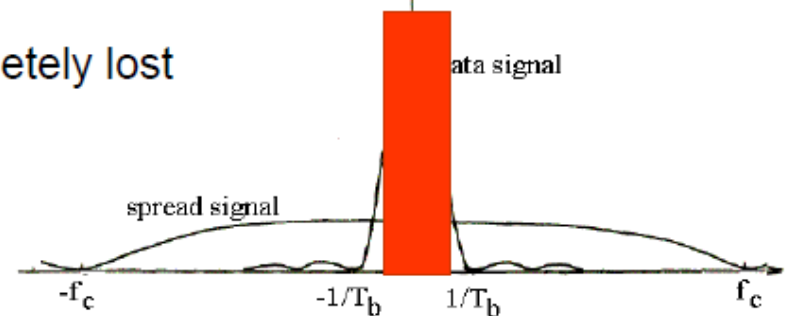
- Multiply them together: what happens to the spectrum?



is this good or bad?

Properties of spread spectrum

- At a first glance: bad – uses more bandwidth for the same transmission rate
- Very important advantages:
 - Resistant to frequency selective fading
 - A deep fade affects only partially the signal on that particular frequency – signal can still be recovered
 - Without spreading – the signal is completely lost
 - Exploits multipath: rake receiver
 - Low probability of intercept
 - Low signal level – noise like
 - Hard to eavesdrop
 - Creates reduced interference to other users
 - Resistance to jamming and interference
 - Narrow band jamming and interference affects only partially the signal
- The last two properties are particularly attractive for unlicensed bands
- Because of its resistance to interference: can have frequency reuse 1 - big capacity advantage

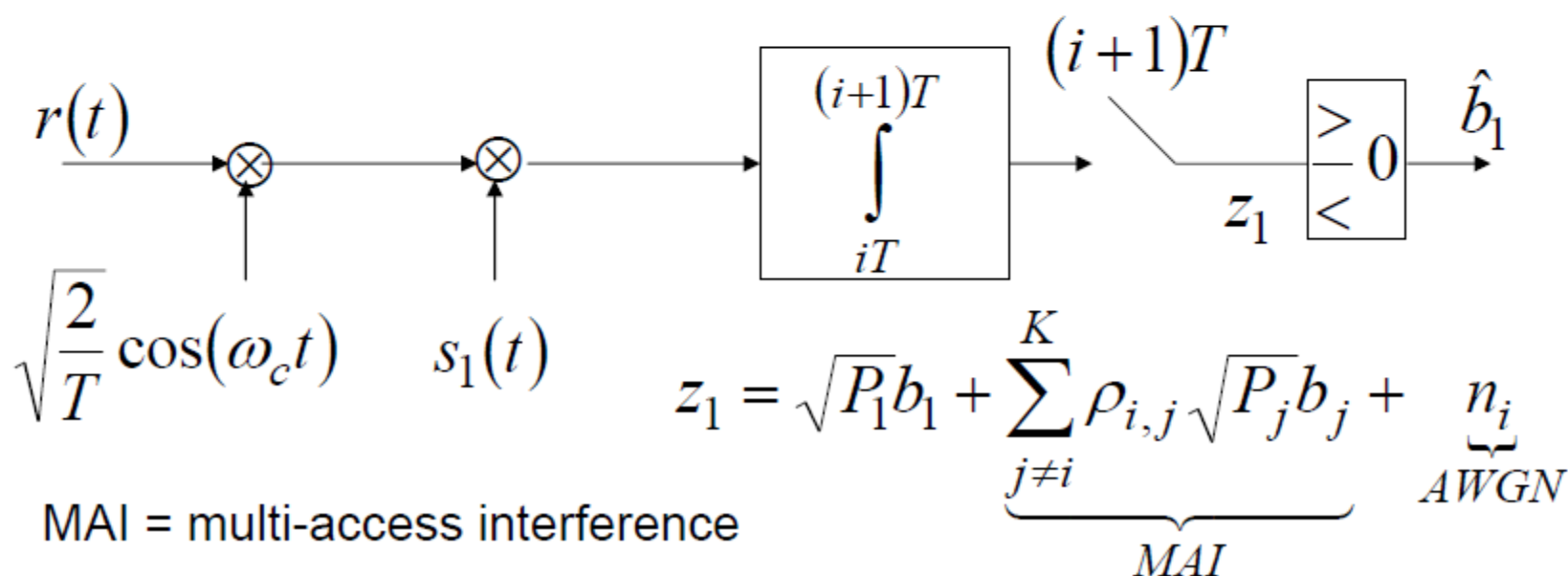


Multiple users

- Every user has a different signature sequence
- The received signal

$$r(t) = \sum_{k=1}^K A_k b_k s_k(t) + n(t) \quad A_k = \sqrt{P_k}$$

- To detect signal 1: matched filter receiver



Performance and optimality

- Performance depends on
 - Powers: implement power control
 - Cross-correlations:

$$\rho_{i,j} = \int_0^T s_i(t)s_j(t)dt = \sum_{n=1}^N s_{in}s_{jn} = \mathbf{s}_i^T \mathbf{s}_j$$

- For random sequences $\begin{cases} E[\rho_{ij}] = 0 \\ \text{var}[\rho_{ij}] = \frac{1}{N} \end{cases}$
- Matched filter: optimal for **Gaussian** noise
 - Assumption: central limit theorem – interference is Gaussian

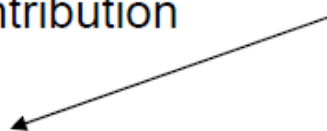
$$\begin{cases} E[I] = 0 \\ \text{var}[I] = P(K-1)\frac{1}{N} = \text{interference power} \end{cases}$$

Probability of error

BPSK/QPSK: $P_e = Q(\sqrt{SIR})$

If K is large – neglect the noise contribution

All received powers equal

$$P_e = Q\left(\frac{\sqrt{P}}{\sqrt{\frac{P}{N}(K-1)}}\right) = Q\left(\sqrt{\frac{N}{K-1}}\right)$$


SIR – key measure for performance – determines capacity: **soft capacity**

Note: a K user asynchronous system \sim (2K-1) synchronous system (virtual users)
for asynchronous systems:

$$P_e = Q\left(\sqrt{\frac{3N}{K-1}}\right)$$

Resource allocation for wireless networks

- Transmission power assignment
- Channel allocation (frequency, time, codes)
- Rate assignment
- Route selection

Static or **dynamic** ?

- Mobility, burstiness, fading, etc → interference conditions change in time

Centralized or **distributed** ?

- centralized: works for downlink in cellular systems
- distributed:
 - uplink in cellular – centralized would require a lot of signaling overhead
 - ad hoc networks – self-organizing networks, no infrastructure

Game theory for resource allocation

- Users interact with each other by creating interference
- Game theoretic formulation:
 - N users, choose actions:
 - Transmitted power level
 - Channel (frequency, time, code – waveform selection)
 - Transmission rate
 - Transmission route (for peer-to-peer connections in multi-hop ad hoc networks)
 - Rewards (utilities) – associated with transmission quality
 - Achieved BER (SIR)
 - Energy expenditure
 - Transmission delay
 - Throughput

Game theory for resource allocation

- Defining the interactions between users, their strategies and their utilities → different game formulations
- Study the performance of these games
 - **Convergence**
 - Existence and uniqueness of Nash equilibrium
 - Conditions for convergence
 - **Efficiency (optimality)**
 - Best possible performance?
 - Pareto efficiency?
 - **Fairness**
 - Are resources shared equitably between users?