

Game Theory

Department of Electronics

EL-766

Spring 2011

Lecture 2

March 9, 2011

Nash Equilibrium

- Many problems (especially in resource allocation) are not solvable by iterated strict dominance
- A broad class of games are characterized by the Nash equilibrium solution.
- Nash equilibrium is a profile of strategies such that each player strategy is an optimal response to the other players' strategies
- Definition: A mixed strategy profile σ^* is a Nash equilibrium, if for all players i

$$u_i = (\sigma_i^*, \sigma_{-i}^*) \geq (s_i, \sigma_{-i}^*), \forall s_i \in S$$

- Similar def. for the pure strategies
- Note: If a player uses a nondegenerate strategy in a Nash eq., he must be Indifferent between all the pure strategies to which he assigns positive probability

Strict Nash equilibrium

- NE is strict if each player has a unique best response to his rivals' strategies
 - No mixed strategies
- s^* is a strict Nash equilibrium iff is a Nash equilibrium, and for all i , and all s_i

$$s_i \neq s_i^* \Rightarrow u_i(s_i^*, s_{-i}^*) > u_i(s_i, s_{-i}^*)$$

- Existence
 - Pure strategy Nash equilibrium may not exist
- Uniqueness
 - Nash equilibrium need not to be unique
- Efficiency
 - Pareto Optimality?

Example Games

- Prisoners Dilemma
- Auctions
- Traveling Salesman

- Matching Pennies
- Friend or Foe

Example Games: Matching pennies

- 2 players
 - simultaneously announce head or tails.
- If match
 - player 1 wins, otherwise player 2 wins.
- Equilibrium: randomize with prob $1/2$

	H	T
H	1,-1	-1,1
T	-1,1	1,-1

Game: Friend or Foe

- **Game:** Game show: prize 1000\$.
- To win the money, the 2 members of the team must go in separate rooms and vote Friend or Foe.
- It is assumed that the two team mates are complete strangers.
- If both choose Friend, then the two players split the money equally.
- If one chooses Foe, while the other chooses Friend, then the former takes all the money.
- If both choose Foe, then both players lose, and they get nothing.

Friend or Foe: Nash equilibrium

	Friend	Foe
Friend	500,500	0,1000
Foe	1000,0	0,0

How to get to the Nash Equilibrium?

- Rational Introspection
- Focal Point
- Trial and Error
- Pre-play communication

Which equilibrium is the best?

- **Pareto efficiency:**

- A strategy profile is *Pareto optimal* if some players must be hurt in order to improve the payoff of other players

- Definition: A strategy profile s^* is said to be *Pareto optimal* iff there exists no other strategy profile s' , such that if for some j

$$u_j(s') > u_j(s^*), u_i(s') \geq u_i(s^*), \forall i \in I \setminus J$$

- **Observations:**

- A strategy profile that is a Nash equilibrium may not be Pareto optimal (efficient).
 - A strategy profile which is Pareto efficient, is not necessarily a Nash equilibrium.
 - We would like Nash equilibrium to be Pareto efficient.

Pareto Efficient: Example Game

	a_1	a_2
a_1	<u>2,3</u>	<u>-2,7</u>
A_2	<u>6,-5</u>	0,-1

	a_1	a_2
a_1	2,3	<u>-2,7</u>
A_2	<u>6,-5</u>	<u>0,-1</u>

Pareto Efficient: _____