Game Theory

Department of Electronics

EL-766

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Lecture 13 (Communications)

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- Power control
  - Classical approach
  - Game theoretic solutions

Power control for wireless systems

- Recall our wireless design example last class:
  - Physical layer performance measure: bit error rate (BER)
  - Based on the used modulation scheme: BER target can be mapped into an SIR (signal to interference ratio) target
  - Reliable communication → meet target BER/SIR
    - **How can you achieve this?**
      - **WIRELESS SYSTEMS: INTERFERENCE LIMITTED**
      - **Dynamically adjust to the current interference pattern (level):**
        - » **Change powers**
        - » **Change transmission rate**
        - » **Waveform adaptation**
        - » **MAC: schedule transmission**
        - » **Routes: affect interference distribution in an ad hoc network**

Power control

Select your power level that you exactly meet your target SIR, γ_0

- If SIR > γ_0, use too much power → battery drain; → interference with others

- If SIR < γ_0, packets cannot be received correctly → → retransmissions – energy inefficient

Power Control cont.

- Assume that: Q transmitters use the same channel C_0

  They have power: $P = (p_1, p_2, \dots, p_Q)^T$

  p_i the power at the i^{th} transmitter

  $i = 1, 2, \dots, Q$

- The expression for SIR at receiver i is

$$SIR_i = \frac{g_{ii} p_i}{\sum_{j=1, j \neq i}^{Q} g_{ij} p_j + n_i}$$

g_{ij} - link gain

n_i - noise power at receiver i

Power Control cont.

- Transmitter i is supported if :

$$SIR_i \geq \gamma_0 \qquad \gamma_0 \text{ - target SIR}$$

=> $$p_i \geq \gamma_0 \left(\sum_{j=1, j \neq i}^{Q} \frac{g_{ij}}{g_{ii}} p_j + \frac{n_i}{g_{ii}} \right) (*)$$

power to select if all other powers are kept fixed

-Denote $\frac{g_{ij}}{g_{ii}} = h_{ij} \quad \frac{n_i}{g_{ii}} = \eta_i$

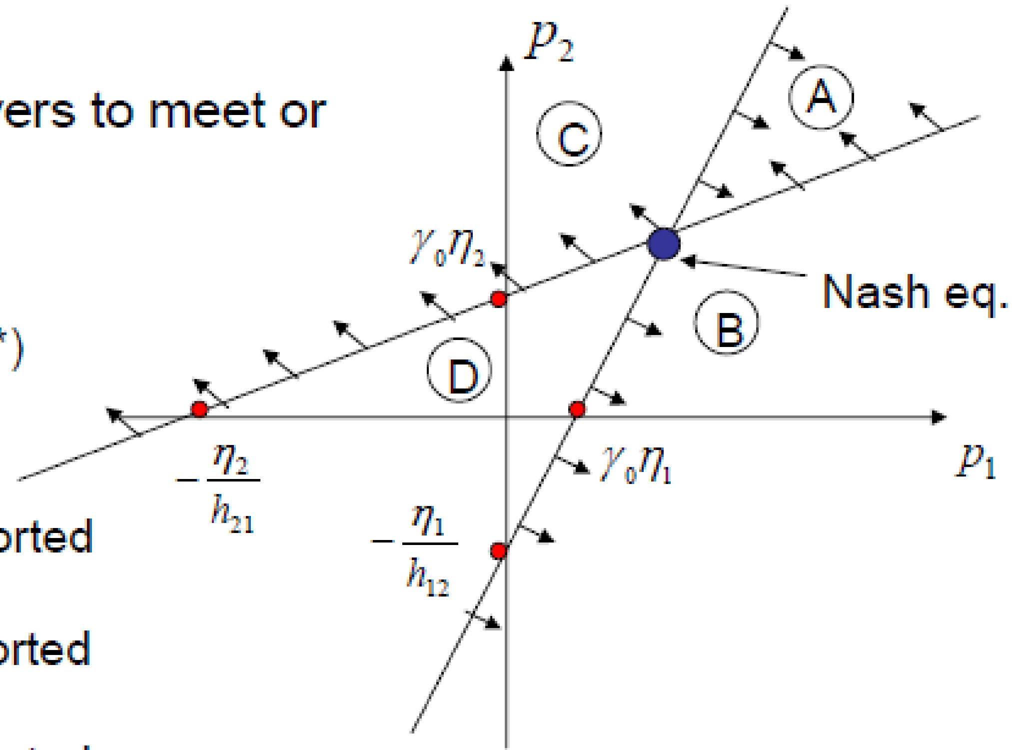
- In a system $(*)$ has to be hold for all $i = 1, 2, \ldots, Q$

Game: strategy – power
utility - SIR

Simple 2 user example

- Users adjust their powers to meet or exceed target SIR:

$$\begin{cases} p_1 \geq \gamma_0 (h_{12} p_2 + \eta_1) \\ p_2 \geq \gamma_0 (h_{21} p_1 + \eta_2) \end{cases} \quad (*)$$



Ⓐ - both users can be supported

Ⓑ - only user1 can be supported

Ⓒ - only user2 can be supported

Ⓓ - none can be supported

Reaction functions

Minimum power solution: achieved for equality in (*) $\begin{cases} p_1 = \gamma_0 (h_{12} p_2 + \eta_1) \\ p_2 = \gamma_0 (h_{21} p_1 + \eta_2) \end{cases}$

Nash equilibrium and minimum power solution

- **Game theoretic solution:**
- **Classic approach**

- **Nash equilibrium**
- **Minimum power solution**

- Existence?
- Uniqueness?
- Pareto efficiency?

↔ Feasibility condition for power control

- Power efficiency???

Power Control Feasibility

- How many users can you support to maximize capacity, while maintaining SIR requirement?

- Feasibility conditions:

  For Q users:

$$(I-H)P \geq \eta$$

$$H_{Q\times Q} \longrightarrow H_{ij} = (h_{ij}) \qquad h_{ij} = \begin{cases} \gamma_0 \dfrac{g_{ij}}{g_{ii}} & i \neq j \\ 0 & i = j \end{cases}$$

$$\eta = (\eta_1, \eta_2 \ldots\ldots \eta_Q)^T$$