Game Theory

Department of Electronics

EL-766

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Lecture 13 (Game Theory)

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Learning in games

- Why learning?
  - For introspection, the rules of the game, rationality of the players, payoff functions – all common knowledge
  - Another problem: for multiple equilibria, how players come to expect the same equilibrium?

- Applicability
- Repeated games
- Teach opponent to play a best response to a particular action, by repeating it over and over again

Example of sophisticated learning

- How would you play this game, if you were player 1?

	L	R
U	1,0	3,2
D	2,1	4,0

Sophisticated learning?

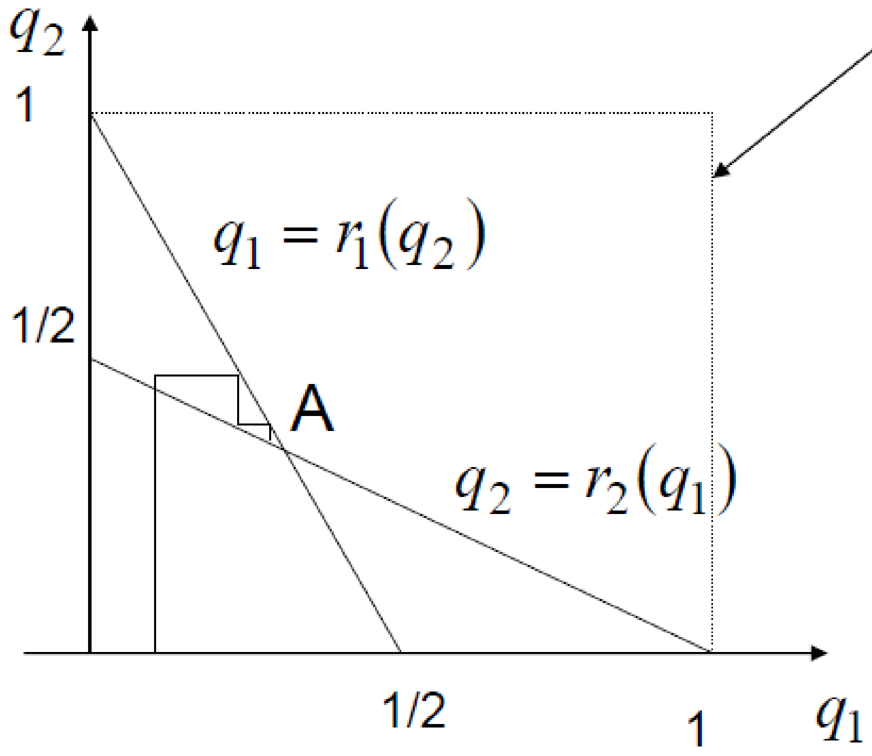
- Most learning theory → models for which the incentive is small to alter the future play of the opponents.
  - Examples:
    - large anonymous population: population size large compared to the discount factor
    - Players locked in their choices and discount factor small compared to maximum speed at which the system can possibly adjust

Common models for learning

- **Fictitious play**
  - Players observe only their own matches and play a best response to the historical frequency of play
- **Partial best-response**
  - A fixed portion of users switches each period from its current action to a BR to the aggregate statistics from the previous period
- **Replicator Dynamics**
  - The fraction of the population using a given strategy, grows proportionally to that strategy's current payoff.

One type of learning: Cournot adjustment

- Unique Nash eq. is at the intersection of the reaction curves



The process converges to Nash equilibrium from any Starting point →eq. globally stable

Fictitious play

- Repeated game
- Stationary assumption
- Each player: belief of opponents "strategy" by looking at what happened
- Player then plays best response (BR) according the his belief
- Belief: a prediction of the opponent action distribution, i.e. the degree to which player i believes player j will play a certain action.
- Players choose their actions in each period , s.t. to maximize their expected payoff, with respect to their belief for the current period.

Updating beliefs

- Player i: initial weight function

$$K_0^i : S^{-i} \rightarrow \mathcal{R}^+$$

- Game iteratively repeated → K updated:

$$K_t(s^{-i}) = K_{t-1}(s^{-i}) + \begin{cases} 1, & \text{if } s_{t-1}^{-i} = s^{-i} \\ 0, & \text{ow.} \end{cases}$$

- Given the frequency vector K → updates beliefs
  - The belief player i has at time t about its opponent to play s^{-i} at time t:

$$\gamma_t^i\left(s^{-i}\right) = \frac{K_t^i\left(s^{-i}\right)}{\sum_{\hat{s}\in S^{-i}} K_t^i\left(\hat{s}^{-i}\right)}$$

← Simple normalization