Game Theory

Department of Electronics

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Lecture 12

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An example of a signaling game

- **Two period reputation game**
  - 2 firms on the market

  Period 1: both firms of the market; only firm 1 action a1
  - Actions for firm 1: prey or accommodate
    - If prey: firm 2 gets P2<0
    - If accommodate: firm 2 gets D2>0
  - Type of firm 1: “sane” or “crazy”
    - “crazy” – always prey
    - “sane”
      - If accommodates: payoff for 1: D1>0
      - If preys: payoff for 1: P1< D1 → prefers to accommodate
      - If 2 exits→ M1>D1 (monopoly)

Two period reputation game: cont.

- Period 2 - player 2 selects action a2: stay or exit
  - If exits, it gets a 0 payoff, and player 1 gets M1> D1
- Assumptions:
  - Player 1 knows his type
  - Player 2 believes that player 1 is sane with probability p
  - δ = discount factor between the two periods
- Building reputation for the sane player
  - Player 1 may try to convince player 2 that he is crazy, to get M1>D1 in the second period of the game.

Taxonomy of PBE

- **Separating equilibrium:** the two types of player 1, choose two different actions in period 1
  - Firm 2 has complete information for the second period

$$\mu(\theta = \text{sane} \mid a_1 = \text{accomodate}) = 1$$
$$\mu(\theta = \text{crazy} \mid a_1 = \text{prey}) = 1$$

- **Pooling equilibrium:** the two types of player 1, choose the same action in period 1

$$\mu(\theta = \text{sane} \mid a_1 = \text{prey}) = p$$

- **Hybrid (semi-separating equilibria):** the sane type may randomize between preying and accommodating

$$\mu(\theta = \text{sane} \mid a_1 = \text{prey}) \in (0, p)$$
$$\mu(\theta = \text{sane} \mid a_1 = \text{accommodate}) = 1$$

What type of equilibrium? Existence.

- Separating eq. existence: sufficient and necessary condition

$$D_1(1+\delta) \geq P_1 + \delta M_1 \Leftrightarrow \delta(M_1 - D_1) \leq (D_1 - P_1)$$

- Pooling eq. → to enforce exit for player 2. Condition:

$$pD_2 + (1-p)P_2 \leq 0$$

- If the above two conditions do not hold → hybrid PBE

Note: uniqueness of the eq. in this case, is due to the fact that the "crazy" type is assumed to always prey.

Multi-stage games with observed actions and incomplete information

- Each player i has type θ_i, and types are independent

$$p(\theta) = \prod_{i=1}^{I} p_i(\theta_i)$$

- At each period t (t=0,1,2,… T), players choose their actions simultaneously, and the actions are revealed at the end of the period
- Players' action set at a date t is type independent
- Behavior strategy: $\sigma_i(a_i \mid h^t, \theta)$
- Payoffs $u_i(h^{T+1}, \theta)$

- Subgame perfection → BNE not only for the whole game, but also for the "continuation game" starting at period t after all possible histories h^t
  - Continuation games → proper subgames?
    - No. They do not stem from a singleton information set

Continuation games → true games

- Need to specify the players' beliefs at the start of each continuation game.
- **Definition:** A perfect Bayesian equilibrium is a (σ,μ) that satisfies (P) and (B(i) – B(iv)).

B(i) Posterior beliefs are independent, and all types of player i have the same belief.

- For all θ, t and h^t:

$$\mu_i(\theta_{-i} \mid \theta_i, h^t) = \prod_{j \neq i} \mu_i(\theta_j \mid h^t)$$

- even unexpected events will not change the independence assumption for the type of the opponents

Perfect Bayesian equilibrium: cont

- B(ii) Beliefs are updated according to Bayes' rule:
  - For all i,j, h^t, and a_j^t, If there exist

$$\hat{\theta}_j,\ \text{s.t. }\ \mu_i\left(\theta_j \mid h^t\right) > 0,\ \sigma_j\left(a_j^t \mid h^t, \theta_j\right) > 0,\ \text{then}$$

$$\mu_i\left(\theta_j \mid h^t, a^t\right) = \frac{\mu_i\left(\theta_j \mid h^t\right)\sigma_j\left(a_j^t \mid h^t, \theta_j\right)}{\sum_{\hat{\theta}_j} \mu_i\left(\hat{\theta}_j \mid h^t\right)\sigma_j\left(a_j^t \mid h^t, \hat{\theta}_j\right)}$$

- B(iii) Don't signal what you don't know
  - For all i,j, h^t, and a^t and $\hat{a}^t$

$$\mu_i\left(\theta_j \mid (h^t, a^t)\right) = \mu_i\left(\theta_j \mid (h^t, \hat{a}^t)\right),\ \text{if } a_j^t = \hat{a}_j^t$$

Perfect Bayesian eq. – cont.

- B(iv) All players have to have the same belief about the type of another player
  - Imposed because of the req. of eq. analysis: players have the same belief about each other's strategies.
  - For all θ_k, and h^t

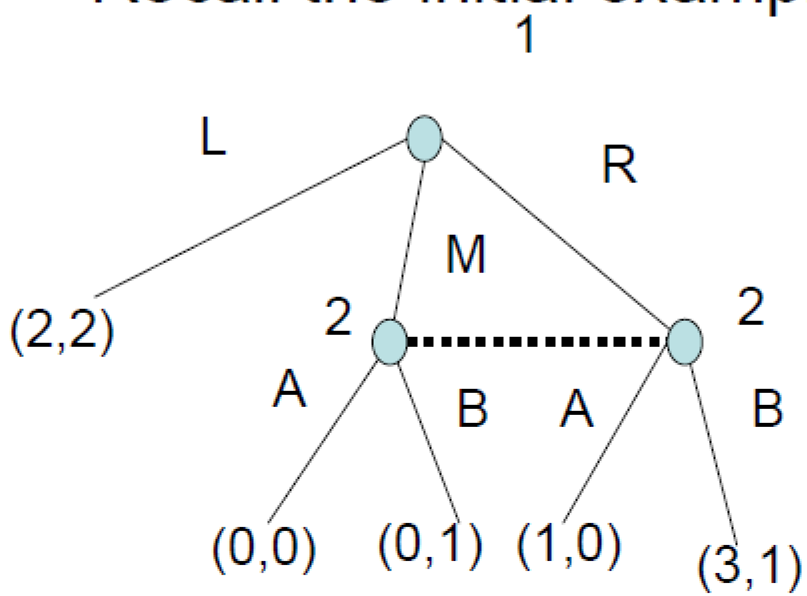
$$\mu_i\left(\theta_k \mid h^t\right) = \mu_k\left(\theta_k \mid h^t\right) = \mu\left(\theta_k \mid h^t\right), \text{ for } i \neq j \neq k$$

- (P) For each player i, type θ_i, alternative strategy σ'_i, and history h^t

$$u_i\left(\sigma \mid h^t, \theta_i, \mu(. \mid h^t)\right) \geq u_i\left(\sigma'_i, \sigma_{-i} \mid h^t, \theta_i, \mu(. \mid h^t)\right)$$

Sequential equilibrium

- We saw already that the requirement that the players' strategies form a Nash equilibrium is too weak → formally the only proper subgame for the games of incomplete, or imperfect information is the whole game.
- Recall the initial example



Is (LA) equilibrium plausible?

- Nash equilibria: (L,A) and (R,B)
- Both subgame perfect

Sequential equilibrium: cont.

- (L,A) is not plausible
  - Whatever player's 2 beliefs on player's 1 move (M or R), he must chose B if he has an opportunity to move.

- Need to generalize the previous condition (P) → given the system of beliefs, no player can gain by deviating at any information set.
- (s) An assessment (σ,μ) is *sequentially rational* if, for any information set h, and alternative strategy $\sigma'_{i(h)}$,

$$u_{i(h)}(\sigma \mid h, \mu(h)) \geq u_{i(h)}(\sigma'_{i(h)}, \sigma_{-i(h)} \mid h, \mu(h))$$

Sequential eq. cont.

- Consistency condition on beliefs is also introduced

- (C) An assessment (σ,μ) is consistent if

$$(\sigma,\mu) = \lim_{n\to+\infty} (\sigma^n, \mu^n)$$

For some sequence $(\sigma^n, \mu^n) \in \Psi^0$ ← The set of all assessments

- **Definition:** A sequential equilibrium is an assessment (σ,μ) that satisfies (S) and (C)

Existence: For any finite extensive-form game, there exist at least one sequential equilibrium.