Game Theory

Department of Electronics

EL-766

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Lecture 7

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Games of Incomplete Information

- Recall:
  - Games of perfect information – sequential games
  - Games of imperfect information – simultaneous move games
  - Games of incomplete information?
    - Some players do not know the payoffs of the others

Bayesian game characterization

- Type of the players → contains any initial private information that a player might have
- Knowledge about the types is characterized by a pdf

$$p(\theta_1, \theta_2, \ldots, \theta_I)$$

$p(\theta_{-i} \mid \theta_i)$= player's 1 probability about its opponents types, given its own type

$$p(\theta_i) > 0, \quad \forall \theta_i \in \Theta_i$$

- Given the pure strategy space S_i, the payoff function of each player i, will depend on players' types:

$$u(s_1, \ldots, s_I, \theta_1, \ldots, \theta_I)$$

Bayesian Equilibrium

- **Definition:** A Bayesian equilibrium in a game of incomplete information with a finite number of types θ_i for each player i, prior distribution p, and pure strategy spaces Si is a Nash equilibrium of the "expanded game", in which each player's i space of pure strategies is the set $S_i^{\theta_i}$ of maps from θ_i to S_i.

Define a strategy profile: s(.) and $s'_i \in S_i^{\theta_i}$
The profile s(.) is a pure strategy Bayesian equilibrium if, for each player i

$$s_i(.) \in \underset{s'_i(.) \in S_i}{\arg\max} \sum_{\theta_i} \sum_{\theta_{-i}} p(\theta_i, \theta_{-i}) u_i(s'_i(\theta_i), s_{-i}(\theta_{-i}), \theta_i, \theta_{-i})$$

$$s_i(.) \in \underset{s'_i(.) \in S_i}{\arg\max} \sum_{\theta_{-i}} p(\theta_{-i} \mid \theta_i) u_i(s'_i, s_{-i}(\theta_{-i}), \theta_i, \theta_{-i})$$

Example: Cournot competition

- Firms select the quantities of production si = qi
- Payoffs defined as

$$u_i = q_i(\theta_i - q_i - q_j)$$

- Common knowledge: firm 1 has type $\theta_1 = 1$

- Firm 2 – private information about θ_2
- Firm one beliefs:

$$\theta_2 = 3/4, \quad p = 1/2$$
$$\theta_2 = 5/4, \quad p = 1/2$$

- Belief of Firm 1 is common knowledge
- Firms choose their outputs simultaneously

Cournot competition: equilibrium

- q_1 = firm ones output
- For firm 2: for $\theta_2 = 3/4, \quad q_2^L$

$$\theta_2 = 5/4, \quad q_2^H$$

$$\frac{\partial u_2}{\partial q_2} = 0 \Rightarrow q_2(\theta_2) = \frac{\theta_2 - q_1}{2}$$

$$\frac{\partial u_1}{\partial q_1} = 0 \Rightarrow \frac{1}{2} q_1(1 - q_1 - q_2^H) + \frac{1}{2} q_1(1 - q_1 - q_2^L) = 0 \Rightarrow q_1 = \frac{2 - q_2^H - q_2^L}{4}$$

Unique Bayesian equilibrium: $(q_1 = 1/3;\ q_2^L = 11/24;\ q_2^H = 5/24)$

Example

- An industry with 2 firms: incumbent (player 1) and potential entrant (player 2)
  - Player 1: Build new plant ?
  - Player 3: Enter?

	Enter	Don't
Build	0,-1	2,0
Don't	2,1	3,0

Building cost HIGH

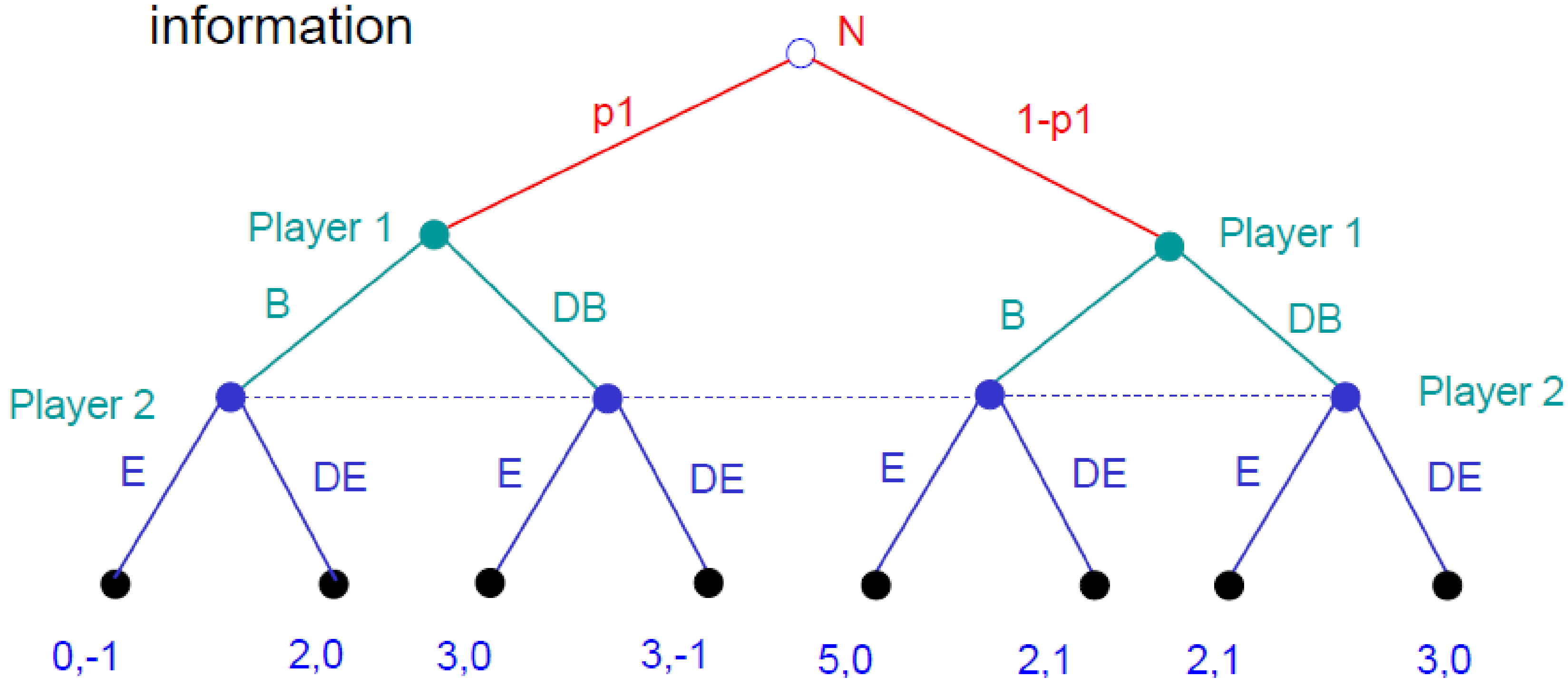
	Enter	Don't
Build	3,-1	5,0
Don't	2,1	3,0

Building cost LOW

Player 1 knows its cost for building (HIGH or LOW)
Player 2 does not know
→ assign a probability p1 for HIGH

Equivalent game

- Introduce prior move by nature: determines player 1 "type"
- Player's 2 incomplete information → imperfect information



Equilibrium solution

	Enter	Don't
Build	0,-1	2,0
Don't	2,1	3,0

Building cost HIGH

	Enter	Don't
Build	3,-1	5,0
Don't	2,1	3,0

Building cost LOW

If cost is HIGH: Don't build → dominant strategy for player 1

If cost is LOW; → dominant strategy for player 1: build

How about player 2?

- strategy for 2:

$$\text{enter if} : -1 * (1 - p_1) + 1 * p_1 > 0 \Rightarrow p_1 > 1/2$$

Equilibrium solution

	Enter	Don't
Build	0,-1	2,0
Don't	2,1	3,0

Building cost HIGH

	Enter	Don't
Build	1.5,-1	3.5,0
Don't	2,1	3,0

Building cost LOW

If cost is HIGH: Don't build → dominant strategy for player 1

If cost is LOW; no dominant strategy

→ y = probability that player 2 enters

→ x = probability that player 1 builds (given the type of player 1)

Building better than not building: $1.5y+3.5(1-y) > 2y+3(1-y) \rightarrow y< ½$

Enter better than not enter: $(-1)\, x(1-p_1) + 1[1-x(1-p_1)]>0$

$\rightarrow x<1/[2(1-p_1)]$