• Repeated Games
• Infinitely Repeated Games
• Folk Theorem
• Strategies for Repeated Games
• Incomplete Information Games
• Bayesian Games
Repeated Games

• Review: repeated games may introduce new equilibrium points and may motivate players for cooperation

• Finitely Repeated Games
  – Backward Induction

• Finite Horizon Games

• Infinite Horizon Games
Alternate representation

Extensive Form of Games: Review

- Equivalence between extensive form game and normal form games

Contingent strategies for player 2

<table>
<thead>
<tr>
<th></th>
<th>C,C'</th>
<th>C,D'</th>
<th>D,C'</th>
<th>D,D'</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2,-1</td>
<td>2,-1</td>
<td>0,3</td>
<td>0,3</td>
</tr>
<tr>
<td>B</td>
<td>2,2</td>
<td>1,5</td>
<td>2,2</td>
<td>1,5</td>
</tr>
</tbody>
</table>
Folk Theorem for Infinitely Repeated Games

• If players are sufficiently patient, then any feasible, individually rational payoffs can be enforced by an equilibrium.

• In the limit of extreme patience, repeated play allows virtually any payoff to be an equilibrium outcome.

Theorem: For every feasible payoff vector $v$ with $v_i > v_i$ for all players $i$, there exist $\delta < 1$, such that for all $\delta \in (\delta, 1)$, there is a Nash equilibrium of $G(\delta)$ with payoffs $v$.

$$v_i = \min_{\alpha - i} \max_{\alpha_i} g_i(\alpha_i, \alpha_{-i})$$
Folk Theorem II

• \( v_i \)-player i’s reservation utility of minmax value.
• This is the lowest payoff player i’s opponents can hold him to by any choice of alpha

Some definitions: \( v_i \) = the lowest payoff, players’ i opponents can hold him to

\[
\begin{align*}
    m^i_{-i} &= \text{minimax profile against player } i \\
    V &= \text{convex hull}\{v \mid \exists a \in A, \text{ with } g(a) = v\}
\end{align*}
\]

Proof idea: based on “unrelenting strategy”, a player who deviates will be minmaxed in every subsequent period.
- Assume there exists a pure action profile \( a \), s.t. \( g(a) = v \)
- Strategy of players: play \( a \) if action \( a \) was played in the previous period, or the action played differed in two or more components; if in the previous period, player one was the only one to deviate, then play \( m^i_j \) for the rest of the game
- Question: can player i gain by deviating from this strategy profile?
Folk Theorem III

If player 1 deviates at period $t$:

$$(1 - \delta^t)v_i + \delta^t (1 - \delta) \max_a g_i(a) + \delta^{t+1} v_i$$

This payoff is less than $v_i$, if $\delta > \delta_i$

$$(1 - \delta_i) \max_a g_i(a) + \delta_i v_i = v_i$$

$$\delta = \max_i \delta_i$$
Credible Equilibrium?

- Strategies used for the proof require all the opponents to play the minmax profile → can be costly → are the opponents threats credible
- The chosen strategies are not subgame perfect
- Do the conclusions apply to the payoffs of perfect eq.
  - YES. (perfect folk theorem)
- Nash –threats folk theorem: Let $\alpha^*$ be a static equilibrium (an equilibrium for one stage of the game) with payoffs $e_i$. Then for any $v \in V$ with $v_i > e_i$, for all players $i$, there is a $\delta$, such that for all $\delta > \delta$ there is a subgame perfect equilibrium of $G(\delta)$ with payoffs $v$. 
### Classic strategies repeated games

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Description</th>
<th>Advantages and Disadvantages</th>
</tr>
</thead>
<tbody>
<tr>
<td>ALLC</td>
<td>Always cooperate</td>
<td>Susceptible to exploitation</td>
</tr>
<tr>
<td>ALLD</td>
<td>Always defect</td>
<td>No cooperation</td>
</tr>
<tr>
<td>Tit For Tat (TFT)</td>
<td>Cooperate on the first stage of the game, then do as the other player previously did</td>
<td>Highly robust as a general strategy but when playing against another TFT, cannot recover from an erroneous defection.</td>
</tr>
<tr>
<td>Contrite Tit For Tat (CTFT)</td>
<td>Both players start with &quot;good standing.&quot; Cooperate if your opponent is in good standing, or if you are not. Otherwise defect.</td>
<td>Maintains a record of an opponent’s &quot;standing.&quot; Can recover from an opponent's erroneous defection</td>
</tr>
<tr>
<td>Generous Tit For Tat (GTFT)</td>
<td>As TFT but cooperate after an opponent's defection with a certain probability</td>
<td>Superior to TFT because it can recover from an erroneous defection. Exploitable by ALLD</td>
</tr>
</tbody>
</table>
# Classic strategies repeated games

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<td>PAVLOV</td>
<td>Cooperate if and only if both protagonist and opponent played identically in the last round</td>
<td>Adapts by changing strategies when unsuccessful.</td>
</tr>
<tr>
<td>Prudent PAVLOV (P-PAVLOV)</td>
<td>Similar to PAVLOV, but only resume cooperation after two rounds of mutual defection</td>
<td>Can recover from an erroneous defection.</td>
</tr>
<tr>
<td>REMORSE</td>
<td>Cooperate if in &quot;bad standing&quot; or if both players cooperated in the last round</td>
<td>Maintains a record of an opponents &quot;standing.&quot; Can recover from an opponent's erroneous defection</td>
</tr>
<tr>
<td>Suspicious Tit For Tat (STFT)</td>
<td>Defect on the first move, otherwise do as the other player last did</td>
<td>If plays against TFT the result is continual defection thereafter</td>
</tr>
</tbody>
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## Additional Strategies

<table>
<thead>
<tr>
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<th>Advantages and Disadvantages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tit For Two Tats (TF2T or TFTT)</td>
<td>Cooperate on first move and defect after two consecutive defections by the opponent</td>
<td>Exploitable by a strategy which alternately cooperates and defects</td>
</tr>
<tr>
<td>GRIM</td>
<td>Cooperate if both players cooperated previously. Change to ALLD if the other player defects.</td>
<td>Unforgiving. Cannot recover from an erroneous defection</td>
</tr>
</tbody>
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