Information Theory

Department of Electronics

Assignment # 01 **Submission Date**
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Q No.1

A random variable *X* is define by the CDF (Cumulative Distribution Function)

$$F_X = \begin{cases} \mathbf{0}, & x < 0 \\ \frac{1}{2}x, & 0 \leq x < 1 \\ K, & x \geq 1 \end{cases}$$

1. Find the value of K.
2. Is this random variable discrete, continuous, or mixed?
3. What is the probability that $\frac{1}{2} < X < 1$?
4. What is the probability that $\frac{1}{2} < X \leq 1$?
5. What is the probability that X exceeds **2?**

Q No.2

A communication receiver filters and amplifies the voltage across the terminals of an antenna. The final output of the receiver, sampled at a certain time t, is a random variable X. when no signal, but only background noise is present at the input to the receiver, the probability density function of X is

$$f_X(x|H_o) = (2\pi\sigma^2)^{-1/2} \exp\left(-\frac{x^2}{2\sigma^2}\right).$$

When a signal has also arrived, the probability density function of X is

$$f_X(x|H_1) = (2\pi\sigma^2)^{-1/2} \exp\left(-\frac{(x-m)^2}{2\sigma^2}\right), \quad m>0.$$

The prior probability that signal is present equal $\frac{1}{2}$.
Given that a particular value x of this random variable X has been observed at the output of the receiver, what is the conditional probability that a signal is present? That is, calculate $P_r\,(signal\ present|X = x)$.

Q No.3

A message source generates one of four messages randomly. The probabilities of these messages are **0.4**, **0.3**, **0.2**, and **0.1**. Each emitted message is independent of other message in the sequence. What is the source entropy?

Q No.4

Let X and Y be binary random variables with,
$P(X = 0, Y = 0) = \frac{1}{3}$, $P(X = 0, Y = 1) = \frac{1}{6}$, $P(X = 1, Y = 0) = \frac{1}{6}$ and $P(X = 1, Y = 1) = \frac{1}{3}$.
Find $I(X;Y)$.

<u>Q No.5</u>

A fair coin is flipped until the first head occurs. Let ***X*** denote the number of flips required.

a) Find the entropy ***H(X)*** in bits. The following expressions may be useful:

$$\sum_{n=1}^{\infty} r^n = \frac{r}{1-r} \quad , \quad \sum_{n=1}^{\infty} nr^n = \frac{r}{(1-r)^2}$$

b) A random variable ***X*** is drawn according to this distribution. Find an ***"efficient"*** sequence of *yes-no* questions of the form, ***"Is X contained in the set S?"*** Compare ***H(X)*** to the expected number of questions required to determine ***X***.

<u>Q No.6</u>

The World Series is a seven-game series that terminates as soon as either team wins four games. Let ***X*** be the random variable that represents the outcome of a **World Series** between teams *A* and ***B***; possible value of ***X*** are ***AAAA, BABABAB,*** and ***BBBAAAA.*** Let ***Y*** be the number of games played, which ranges from 4 to 7. Assuming that ***A*** and ***B*** are equally matched and that the games are independent, calculate ***H(X), H(Y), H (Y|X)*** and ***H (X|Y).***