# Game Theory

Department of Electronics

EL-766

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# Nash Equilibrium

- Many problems (especially in resource allocation) are not solvable by iterated strict dominance
- A broad class of games are characterized by the Nash equilibrium solution.
- Nash equilibrium is a profile of strategies such that each player strategy is an optimal response to the other players' strategies
- Definition: A mixed strategy profile σ* is a Nash equilibrium, if for all players *I*

$$u_i = (\sigma_i^*, \sigma_{-i}^*) \geq (s_i \, , \sigma_{-i}^*), \forall s_i \in S$$

- Similar def. for the pure strategies

- Note: If a player uses a nondegenerate strategy in a Nash eq., he must be Indifferent between all the pure strategies to which he assigns positive probability

# Strict Nash equilibrium

- NE is strict if each player has a unique best response to his rivals' strategies
  - No mixed strategies
- s* is a strict Nash equilibrium iff is a Nash equilibrium, and for all *i*, and all $s_i$

$$s_i \neq s_i^* \Rightarrow u_i(s_i^*, s_{-i}^*) > u_i(s_i, s_{-i}^*)$$

- Existence
  - Pure strategy Nash equilibrium may not exist
- Uniqueness
  - Nash equilibrium need not to be unique
- Efficiency
  - Pareto Optimality?

# Example Games

- Prisoners Dilemma
- Auctions
- Traveling Salesman

- Matching Pennies
- Friend or Foe

# Example Games: Matching pennies

- 2 players
  - simultaneously announce head or tails.
- If match
  - player 1 wins, otherwise player 2 wins.
  - Equilibrium: randomize with prob1/2

| | H | T |
|---|---|---|
| H | 1,-1 | -1,1 |
| T | -1,1 | 1,-1 |

# Game: Friend or Foe

- **Game:** Game show: prize 1000$.
- To win the money, the 2 members of the team must go in separate rooms and vote Friend or Foe.
- It is assumed that the two team mates are complete strangers.
- If both choose Friend, then the two players split the money equally.
- If one chooses Foe, while the other chooses Friend, then the former takes all the money.
- If both choose Foe, then both players lose, and they get nothing.

# Friend or Foe: Nash equilibrium

| | Friend | Foe |
|---|---|---|
| Friend | 500,500 | 0,1000 |
| Foe | 1000,0 | 0,0 |

# How to get to the Nash Equilibrium?

- Rational Introspection
- Focal Point
- Trial and Error
- Pre-play communication

# Which equilibrium is the best?

- **Pareto efficiency:**
  - A strategy profile is *Pareto optimal* if some players must be hurt in order to improve the payoff of other players
- Definition: A strategy profile *s** is said to be *Pareto optimal* iffthere exists no other strategy profile *s',* such that if for some *j*

$$u_j(s') > u_j(s^*), u_i(s') \geq u_i(s^*), \forall i \in I \setminus J$$

- **Observations:**
  - A strategy profile that is a Nash equilibrium may not be Pareto optimal (efficient).
  - A strategy profile which is Pareto efficient, is not necessarily a Nash equilibrium.
  - We would like Nash equilibrium to be Pareto efficient.

# Pareto Efficient: Example Game

| | $a_1$ | $a_2$ |
|---|---|---|
| $a_1$ | 2,3 | -2,7 |
| $A_2$ | 6,-5 | **0,-1** |

| | $a_1$ | $a_2$ |
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Pareto Efficient: _____